

Modelling and Optimal Control of the Cascaded Kainji and Jebba Hydroelectric Power System in Nigeria

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(05/30GC151)

B.Eng., M.Eng. Unilorin.

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Jebba Hydroelectric Power System in Nigeria**

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Faculty of Engineering and Technology,

University of Ilorin, Ilorin, Nigeria

in

Partial Fulfillment of the Requirements for the Award of Doctor of
Philosophy (PhD) Degree in Electrical Engineering

April 2018

Declaration

I, OGUNBIYI Olalekan, hereby declare that the work reported in this thesis titled “Modelling and Optimal Control of the Cascaded Kainji and Jebba Hydroelectric Power System in Nigeria” is a record of my research work in the Department of Electrical & Electronics Engineering, University of Ilorin, Ilorin, under the supervision of Professor. B. J Olufeagba. I further declare that to the best of my knowledge, no part of this thesis has been submitted elsewhere in a previous application for award of a degree. All sources of knowledge used have been duly acknowledged.

.....

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Certification

This is to certify that the thesis titled “Modeling and Optimal Control of the Cascaded Kainji and Jebba Hydroelectric Power System” by OGUNBIYI, Olalekan meets the requirements and regulations governing the award of the Doctor of Philosophy (PhD) Electrical and Electronics Engineering of the University of Ilorin, Ilorin, Nigeria.

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Dedication

*In loving memory of my mother Mrs. Lydia Arinola Ogunbiyi and my siblings for
being there through the thick and the thin.*

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“I would have lost heart, unless I had believed that I would see the goodness of the Lord in the land of the living” Psalm 27:13 NKJV. My sincere and supreme gratitude goes to the *TRINITY* for HIS infinite and unquantifiable mercy, grace, inspiration and for being my stronghold. I can’t imagine how it would have been if not that HE was there throughout.

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Thanks.

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Abstract

The cascaded hydroelectric power stations at Kainji and Jebba on the River Niger play a vital role in the Nigerian electricity supply that has been in a state of crisis for over four decades. Sustainable solutions to this problem require strategic solutions like the Roadmap for Power Sector Reform of 2010 and tactical components that address knotty issues of operations and management that have so far been either outsourced or completely omitted. The latter devolves into addressing the nearly intractable problem of determining control strategies that optimize suitable performance indices constrained by the dynamical equations describing the reservoirs. The aim of this research is to develop dynamical models and control strategies for optimal power generation at Kainji and Jebba hydroelectric power stations. The objectives were to: (i) develop a mathematical model (incorporating reservoir characteristics, inputs such as inflows with outflows due to power generation, flood-control discharges and evaporation); (ii) numerically solve the resulting equations and determine the parameters using measured data; and (iii) compute control laws for optimal operation of the system.

A nonlinear dynamical model derived using energy conservation and flow continuity conditions was solved numerically employing an Adams-Moulton-Bashforth predictor-corrector integrator with a Runge-Kutta starter (AMBPCRKS). Reservoir parameters were then estimated using information obtained from a database created using National Control Centre (NCC) data and observations of evaporation and power generation during the study period. The optimal control problem, formulated as the minimisation of a quadratic cost functional subject to the system's dynamical equations, was then solved by direct and indirect methods. The indirect method, which is based on Pontryagin's Minimum Principle (PMP), resulted in a two-point boundary value problem (TPBVP) which was subsequently solved by an innovated multiple shooting technique referred to as the Progressive Domain Extension Method (PDEM) and a conjugate gradient algorithm.

The findings from the study were that:

- i. integrating model equations using the AMBPCRKS in an EXCEL-VBA® environment achieved an accuracy of the fifth order;
- ii. the calibrated model using reservoir parameters with actual inflows and outflows were within 2% of observed values, thus making it a dependable tool for planning, operations and training;
- iii. approximate finite-dimensional optimal controls computed by the steepest descent and conjugate gradient approaches converged to nearly identical values although the latter technique took fewer iterations; and
- iv. PMP-based controls extracted through the TPBVP can be solved using PDEM and conjugate gradient approaches with a stop criterion of 10^{-6} , with the results being almost identical and practically realisable.

In conclusion, the research successfully addressed an extant frustrating difficulty confronting the operators of the cascaded power stations by solving an optimal control problem using available information and innovative computational methods realized on the ubiquitous EXCEL-VBA® platform. It is recommended that the ownership of the stations and government be encouraged to adopt the results for planning, operations, training of engineers and management as well as to support further work to complete the control process.

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CHAPTER ONE

Introduction and Research Overview

1.1 Preamble

The socio-economic activities of any nation and better quality of life for citizens are greatly influenced by cheap and reliable supply of electrical energy. Electrical energy is an easily controlled form of energy. It is easier to transmit and distribute with less losses and pollution. It is also the most efficient way of consuming energy; hence electricity is replacing other forms of energy (Ogunjobi, 2015; Lasode, 2017). Electricity supply facilitates the means of eradicating poverty in the society and ensure sustainable urbanization (IFC, 2016; CBN, 2015).

Electricity production in the world increased by almost 3.4% between the years 1974 to 2015. The contribution of Africa to this increase is minimal, hence economic growth in sub-Saharan Africa is highly constrained by chronic power shortage and inadequate network. Two-third of the people in the region lives without access to electricity and the cost of power shortage and running backup generators in the industries was estimated around 1% - 4% of Gross Domestic Product (GDP) (IEA, 2016).

Nigeria is a country located in western part of Africa. It lies between latitude 4° and $14^{\circ} N$, longitude 2° and $15^{\circ} E$ with its capital in Abuja ($9^{\circ} 4' N, 7^{\circ} 29' E$). There are two main rivers flowing across it, river Niger and Benue. It has a total area estimated at $923,768 \text{ Km}^2$ and population around 185,989,640 as at 2016 (BBC, 2017). Nigeria is ranked highest among the countries without electricity. The supply is inadequate, unreliable and not sustainable. The negative impact on the economy is severe as larger

population still lives in darkness and abject poverty. (Udeme, Sebastine, & Okafor, 2017; Akinwale, Jesuleye, & Siyanbola, 2013). As at 2001, out of the over 150 million population, roughly 40 % had access to electricity and the associated authorities are faced with high running cost and energy losses (Nnaji, 2011). The power sector snapshot of 2015 estimated 55% of the population are not connected to the national grid, showing that there was no significant improvement in a decade (Power Africa & Advisory Power Team, 2015).

Effort to join the rest of the world in the provision of cheap and reliable electricity, the government of Nigeria developed a Roadmap for Power Sector Reform in 2010 (Power Africa & Advisory Power Team, 2015). The Roadmap aimed at restructuring the power sector and achieves stable electricity in the country through the removal of obstacles to private sector investment, clarifying the government's strategy on the divestiture of the Power Holding Company of Nigeria (PHCN) successor companies and improving service delivery throughout the transition. The technical component of the action plans were to increase generation to 40,000 *MW* by 2020, raise transmission capacity from 330 *kV* to 700 *kV*, increase capability of distribution network to 20 % and reduce associated distribution losses by at least 5% by April 2011 (PTFP, 2010; Biobaku, 2010). Despite all effort to ensure compliance with the Roadmap and with vast resources for generating electrical energy, the country is still confronted with enormous challenges in its power sector. Sambo, et al(2012) identified the critical challenges of Nigeria power sector as poor utilization of the existing assets and deferred maintenance. In 2015, the operational capacity of the generating resources was less 1/3 of installed (Power Africa & Advisory Power Team, 2015). The supply problem will be greatly minimised if the installed units are maintained and operated to function effectively. In 2016, an

amendment was therefore made to the Roadmap such that it was called “Roadmap for Change”. The goal was to harness all possible power sources however small. The nation is considering the available generation, transmission and distribution resources and how to make them function effectively (Nigeria Electricity Hub, 2016). This necessitates the need for research on all generating resources such that they perform optimally.

1.2 Electrical Power Status of Nigeria

The Nigeria national grid is powered from twenty-eight (28) generating stations with total installed capacity of 12,522 MW. The stations are either hydro-turbine, steam turbine or gas-fired turbine. The plants are run by generation companies classified as Privatized (those formerly under the PHCN), National Integrated Power Project (NIPP), and Independent Power Producers (IPPs). Figure 1.1 shows the contribution of the different generating stations to the installed capacity, generating capacity and peak generation on 1st August 2016.

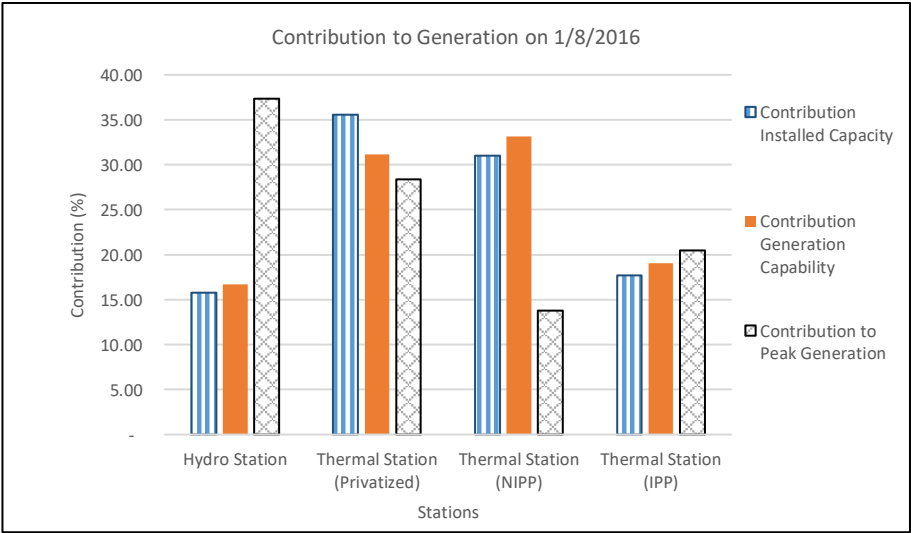


Figure 1.1: Contribution of Power Stations to Total Generation on 1/8/2016 (TCN-NCC, 2016)

Hydropower has the highest contribution to peak generation despite about 16 % of the installed capacity. The thermal stations on the other hand are confronted with challenges

due to inadequate supply of their prime mover and apparent failure rate. The failure rate is because of their inadequate prime mover as compared to the free water in hydropower.

Figure 1.2 and Figure 1.3 presents a similar observation of the contribution of the power stations to the energy generation on 24th August 2017. Hydropower station contributes 32% of the energy generated. The indication of this is that hydropower provides the base load to the grid.

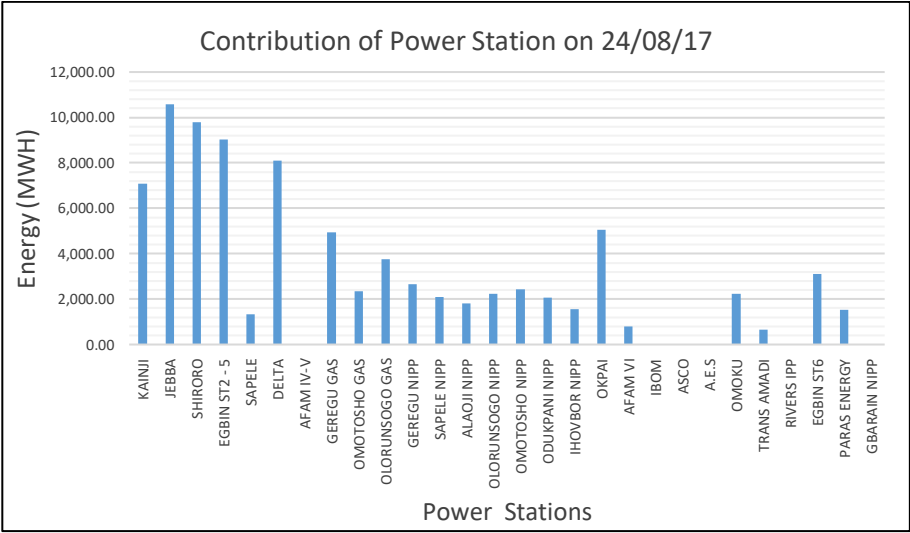


Figure 1.2: Contribution of Power Stations on 24/08/17 (TCN-NCC, 2017)

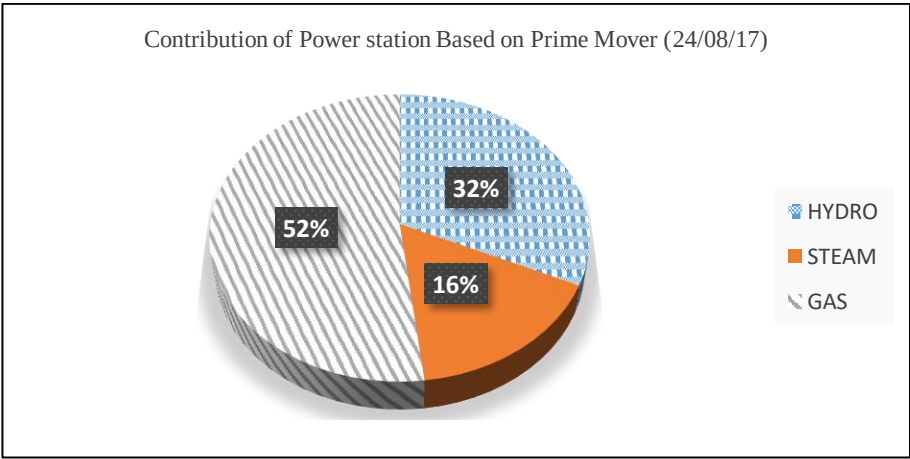


Figure 1.3: Contribution of Power Stations Based on Prime Mover (24/08/17) (TCN-NCC, 2017)

1.3 Hydroelectric Power in Nigeria

Hydroelectric power plants produce electric power by converting the kinetic and potential energy of water falling from a high potential, using different types of turbine to generate electricity. The kinetic energy drives a shaft which is directly coupled to an alternator. It is one of the most efficient and cost-effective systems for producing electricity from renewable energy source. Although the construction cost may be high but the running cost over a long period of time is far smaller as compared to either thermal or nuclear fired station. The energy conversion process is clean, it gives no energy fallout like smoke or ash. However, reservoir associated with the stations can affect the ecosystem and as such proper environmental impact study with mitigation technique must be carried out. Taking together the high reliability and available resources, hydropower stations are suitable for providing base load the national grid (Gupta J. B., 2013; Kundur, 2010; Gupta B. R., 2011).

As shown in Figure 1.4, hydropower generation in Nigeria is currently provided at three major locations. The Kainji hydroelectric power station (KHEPS) and the Jebba hydroelectric power station (JHEPS) are located on the River Niger about 103 km from each other with the latter downstream. The Shiroro hydroelectric power station (SHEPS) is built on Kaduna River in Niger State, Nigeria.

River Niger is the longest river in Nigeria and the third longest river in Africa, following the Nile and Congo. It extends about 4,180 km from its source at Futa Djallon Mountains of Guinea, flowing through Mali, Niger and Nigeria before entering the Atlantic Ocean (Jimoh O. D., 2015). The Hydrology of River Niger at Kainji is characterized by two almost distinct inflow peaks named white and black flood.

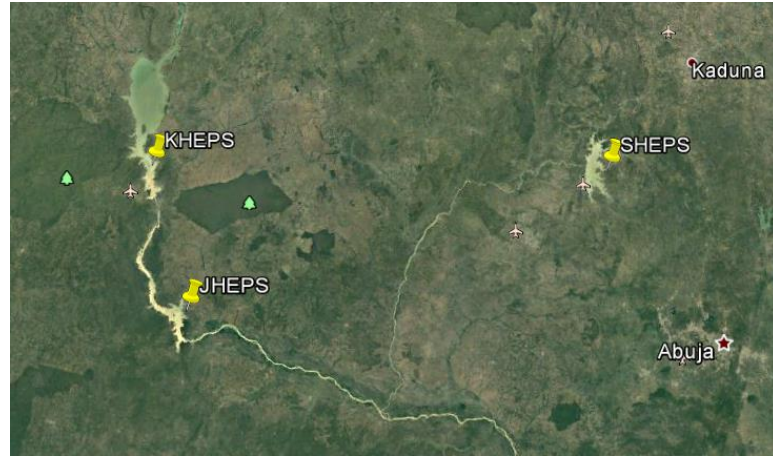


Figure 1.4: Cascaded KHEPS and JHEPS

White flood is associated with rainfall within the catchment area of Nigeria between May to October. The peak flow occurring sometime in the month of September or October (5628 m³/s, 4665 m³/s). Black flood on the other hand originates from the river's source and arrives at Kainji between November and December (Ale, Alowolodu, Babatola, & Olufeagba, 2011; Jimoh O. D., 2015).

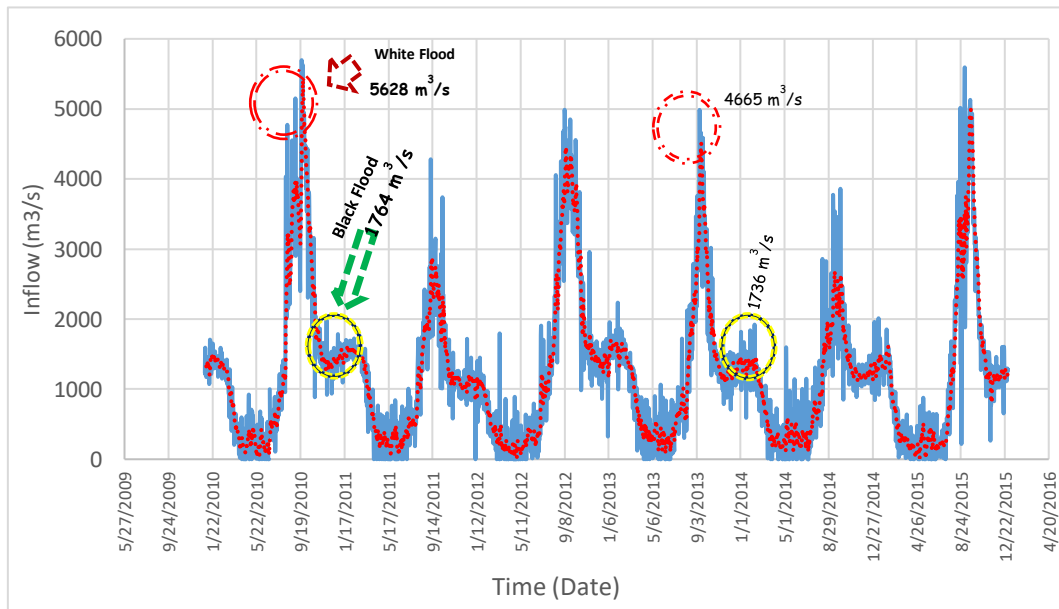


Figure 1.5: Kainji Inflows between Jan. 2010 - Dec. 2015 (TCN-NCC, 2016)

1.4 Kainji and Jebba Hydroelectric Power Stations

The Kainji hydroelectric power station is located at $09^{\circ}51'45''N$, $04^{\circ}36'48''E$ was officially commissioned in February 1969 with a capacity of 320MW, produced by four Kaplan driven turbo-alternator designated as units 7, 8, 9 & 10, each rated at 80 MW. In 1976, two additional units of Kaplan turbine, units 11& 12 each rated at 100 MW were added, increasing the installed capacity of the station to 520 MW. In 1978 the installed capacity was increased to 760 MW with the installation of two additional sets of fixed blade machines unit 5 and 6, each related 120 MW.

Kainji dam is 85.5 m in height and about 8 km in length. The reservoir lake stretches some 136 km upstream and has a breadth of 24 km at its widest point. The maximum head elevation is 141.7 m and maximum tail elevation is 104m. The lake has a total capacity of 15 billion cubic meters covering an area of 1270 square kilometres (Ale, Alowolodu, Babatola, & Olufeagba, 2011; Ale T. O., 2014; Aribisala, 1989; Salami, 2007).

The Jebba hydropower Station is located 64 km downstream of the Kainji Dam on the Niger River in Nigeria. The station is on $09^{\circ}08'08''N$, $04^{\circ}47'16''E$. It was commissioned on April 13, 1985, with a rated capacity of 578.4 MW. It has six (6) fixed blade propeller type turbo-alternator each at 96.4MW. The net operating head is about 27.6 m and its maximum flow per unit is around $380 \text{ m}^3/\text{s}$. Each turbo-alternator is connected to a step-up transformer rated at 119 MVA maximum continuous rating but 103.50 MVA base load rating (Salami, 2007; Nwajinka, Akekere, & Yousuo, 2013).

The Jebba Reservoir depends on discharge from the Kainji power station during the black flood but during the white flood (Rainy season) some rivers downstream of Kainji flows into River Niger and hence into Jebba reservoir in addition to Kainji discharge. As a result, the Jebba reservoir is much smaller and must be operated in cascade with the Kainji reservoir. This arrangement imposes the need for better water management if the units at Jebba are to operate efficiently all the year.

1.5 Investigation of Operational Characteristics of the KHEPS and JHEPS

An aspect of the problems affecting the operation of KHEPS and JHEPS in cascade is the reliance on intuitive (trial and error) water release rules instead of scientifically motivated policies (Salami, 2007; Jimoh, 2008). KHEPS turbo-alternators have variable blades and internal governors; this allows the reservoir head to vary between 24 m and 42 m above sea level, depending on the time of the year. Clearly under such conditions, fixed blade turbine cannot operate.

JHEPS on the other hand employs fixed bladed turbines and as a result the head has to be maintain strictly between the range of 99 m and 103 m. Achieving this objective poses some very serious challenges to the operator since the two reservoirs are in cascade separated by 64 km and the discharge from KHEP depend on the condition of machine, inflow into Kainji and the rainfall in between.

Maximum rainfall between August to November of the year usually causes flooding of Kainji reservoir around September. When the spillways are opened in September during high inflows, the water flows into Jebba reservoir. Sometimes this also results in flooding of the Jebba reservoir because water also flows into it from the catchment area. During

the period of low inflows from December to May, the head of water in the reservoirs often falls below the nominal level. Any attempt to operate the machines at low head usually causes vibration of the turbo-alternators.

The status of the individual turbo-alternator also has a direct effect on the reservoir head. Whenever a unit fails at KHEPS, it reduces the inflow into JHEPS. Operators of JHEPS may be forced to reduce the number of operating machines which in turn reduces the energy on the grid. The head at KHEPS keeps increasing and a significant percentage of the water is lost to evaporation. A reverse situation occurs where machines fail at JHEPS while KHEPS keeps releasing water, the excess water spill away instead of being utilised at KHEPS.

Managers operate the two reservoirs based on experience and intuition and this has resulted into some unpleasant consequences. The situation can be greatly improved with a proper study of the system dynamics. The usual practice is for the operators to interact through phone calls and power line communication. The manner whereby the water is released, and the amount required is usually through a guess work. The net effect is that the units operate below their capabilities and the water that should be converted to energy is either spilt at JHEPS or left to evaporate at KHEPS. A review of the daily report of operations from the two stations shows that there are several days when the units are shut down due to the inability to manage resources appropriately.

1.6 Automatic Control System at KHEPS and JHEPS

The cascaded KHEPS and JHEPS can be better managed if the behaviour of the system is comprehensively studied and management strategies are developed using formal

approaches. From system study and literature review, it is obvious that neither an optimal control model nor automatic control mechanism exists for optimal usage of resources. Aminu and Kangura (2011) recommended the development of a model with a microprocessor-based control system to improve hydropower generation management and hence efficiency.

This research, therefore, focuses on developing a model for the two stations which can be used to determine the optimal control laws for the released of water from KHEPS such that the reservoir head at JHEPS remains relatively constant. The optimal control algorithm is to be incorporated into an embedded controller.

Unfortunately, the design of a system that ensures the optimal use of hydropower resources to maximize power generation within a cascaded system is challenging. The problem has to be properly posed in a standard form before it can be solved (Robert & Michaud, A Simple Multi-objective Control for Cascaded Hydro Power Plants, 2011; Rauschenbach, 2005; Zheng, Fu, & Wei, 2013). Solutions of optimal control problems are often analytically intractable and computationally complex (Kirk, 1998; Taraba, 2013). The task in this research required a thorough understanding of the system structure and dynamics, the formulation of a proper model, setting up an optimal control problem and solving the resulting equations to obtain the control law.

1.7 Motivation

The need to improve the electric power situation in Nigeria demands that all aspects of the power equation be optimized. Since the Kainji and Jebba hydropower Stations play an important role in both supplying and stabilizing the national grid, it is imperative to

explore and design efficient, reliable and optimal power for its operation. The problems that can lead to such desirable results can be posed and solved within the context of the control system where the controlled system is the combination of the two reservoirs operated in tandem.

1.8 Problem Statement

The problem addressed in this research is to develop a suitable framework for dynamic model and control of hydropower generation at the Kainji and Jebba hydroelectric power systems operating in cascade.

1.9 Aim and Objectives

Aim

The aim of this research is to develop dynamical models and control strategies (algorithms) for optimal power generation based on the utilization of the water resources at Kainji and Jebba Hydroelectric power stations.

Objectives

The objectives of this research are to:

- i. develop a model, incorporating reservoir characteristics such as inflow, discharge, evaporation, topology and status of the unit's power generation at KHEPS and JHEPS;

- ii. compute the numerical solution for the resulting model, calibrate the parameters and validate the model using measured data and hence study the system behaviour under changing conditions of inflow, load, head and unit availability; and
- iii. compute control laws (strategies) for optimal operation of the system.

1.10 Scope of the Study

The scope of this research covers the modeling, the optimal utilization and control of the potential energy stored in the reservoirs. Alternator dynamics, turbine dynamics and mechanical losses were not included in the system model, but their effects were embedded in the conversion efficiency. Some parameters of the reservoirs were estimated from measurement because there was inconsistency in values found in literature.

There can also be several optimal control problems as posed by Lagrange, Mayer and Bolza but problem solved in this research is the Lagrange problem.

1.11 Outline of Report

The thesis is divided into five chapters. In chapter two, review of related work and relevant concepts are discussed. In chapter three, the methodologies for achieving the stated objectives are presented and developed. It includes the development of mathematical models and the optimal control techniques. Chapter four presents the results obtained by tuning the model to obtain some of the system parameters, results from model validation, simulations carried out for the optimal control techniques and discussions of the results. Conclusion, recommendation and suggestion for further research are contained in chapter five.

CHAPTER TWO

Literature Review and Relevant Concepts

2.1 Preamble

In chapter One the problem to be addressed in this work was defined in very general terms. However, the process of solving the problem requires a more detailed definition of the context - which shall be outlined here as a precursor to collating relevant research and laying a foundation for the effort.

The cascaded dams KHEPS and JHEPS are made feasible by two reservoirs constructed on the international river, the Niger, as it wends its way finally to the Ocean. The two reservoirs differ in that the primary one at Kainji encompasses a large lake that extends from the dam to its entrance over 120 km. The secondary reservoir located at the Jebba is much smaller, less disruptive of the ecosystem yet capable of generating electric power of almost the same magnitude as its much larger counterpart.

The two reservoirs head are vulnerable to problems such as possible increase or decrease in impoundments upstream, ecology, environmental, inflows, evaporation, reliability, stochastic factors that are weather related, availability of the turbo-alternators, and numerous other constraints that are influenced by the system dynamics. To ensure the stations gives satisfactory performance at all time, a control system is therefore required to tackle each of these influencing factors.

Optimal control theory solves the problem of determining the optimum control signal that will cause a dynamic system to satisfy its physical constraints and minimize a certain performance index (Kirk, 1998; Paraskevopoulous, 2002). Unfortunately, a system that

is not properly understood cannot be control. Therefore, the determination of optimal control requires a systematic study of the dynamics of the system to be controlled, its mathematical representation, formulation of the problem in standard form and solving the associated equations.

Hence this chapter presents the review of hydroelectric power generation in Nigeria, the associated problems and how it can be operated for optimal performance. To facilitate the determination of optimal control, the chapter focuses on presentation of some available information about the stations and related work by other researches. It considered the justification of how the management of resources becomes an optimal control problem and the review of techniques for solving the associated equations.

2.2 The Kainji and Jebba Hydroelectric Power Systems in Nigeria

Generation of electricity involves the conversion of energy available from natural source to electrical energy. Commercial sources of such energy in Nigeria include water at high pressure head, solid fuel such as coal, liquid and gaseous fuel from hydrocarbon. Other sources that are not in commercial use are energy from the wind, water flowing at high velocity, geothermal energy, ocean tides and atomic energy released during nuclear reaction (Manohar & Adeyanju, 2009; Gupta J. B., 2013; Akorede et al, 2017). In most cases, these energies are converted to electricity through a prime mover that is coupled to a turbo-alternator. The prime mover can be a steam turbine, gas turbine, hydraulic turbine, petrol engine, diesel engine etc. The prime mover converts the energy in its natural form to a mechanical energy that drives the rotor of an alternator. There are other methods of generating electricity without the prime mover, among them are solar energy from the sun, thermionic converters, thermoelectric generation etc.

Hydroelectric power (HEP) plants produce electric power by converting the kinetic energy and potential energy of water falling from a high potential to generate electrical energy. The kinetic energy drives a mechanical shaft which is directly coupled to an alternator. It is one of the most efficient and cost-effective systems for producing electricity from renewable resources like water.

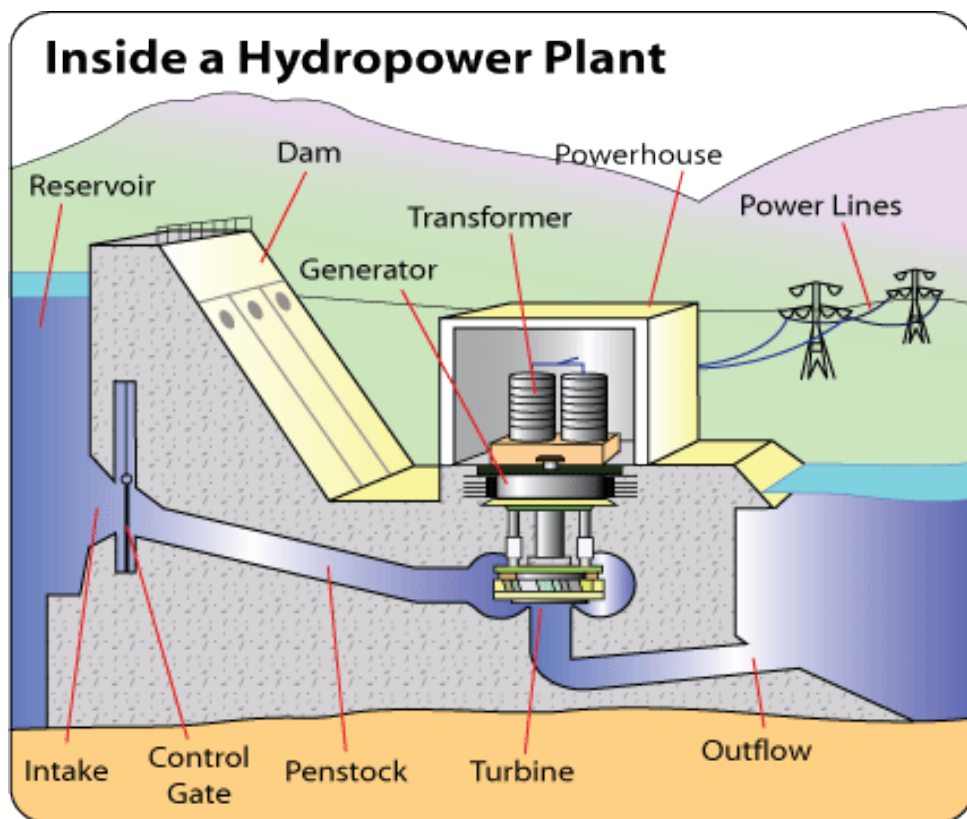
Nigeria has a huge feasible hydropower potential of which only 23% has just been developed (Jimoh O. D., 2015). As mentioned earlier in Chapter One, hydroelectric power in the country is currently provided by three projects- the cascaded KHEPS (760 MW) and the JHEPS (578 MW) about sixty Kilometers from each other are on the River Niger and Shiroro hydroelectric power plant (600 MW) built on Kaduna River in Niger State, Nigeria. Other hydroelectric power project under construction (or Planned) are Zungeru (950 MW), Mambilla Hydro (3960 MW), Makurdi Hydro (1062 MW), Ikom (730 MW), Lokoja (1050 MW), Onitsha Hydro (1050 MW) and Gurara (or Abuja Hydro 300 MW) (Zarma, 2006; Sambo, Gabar, Zarma, & Gaji, 2012).

Despite the enormous construction cost, longer take off time and the need for larger areas of land, investment in hydroelectric power has been motivated by its low running cost over a long period of time as compared to the thermal and nuclear plants that requires constant supply of fuel. It produces a clean energy as it does not give out smoke or ash. Its startup time is fast since it doesn't require heat and can therefore be synchronized with the grid quickly. The governors can be designed to give a high precision in performance, hence it makes relatively constant speed and constant frequency possible. They are robust in operation and are characterized by longer life span. Their efficiencies only fall slightly with age. KHEPS has been in operation since 1969 while JHEPS started operation in

1985, yet the two are still contributing greatly to the nations generating capacity.
(Kundur, 2010; Gupta, 2013; Gupta, 2010).

2.3 Component Parts of Hydroelectric Power System

Looking into the hydroelectric power system of Figure 2.1, it consists of some major component parts and these parts are discussed below.



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Figure 2.1: Major Parts of Hydroelectric Power System

- i. **Water Reservoir:** Generally, hydroelectric power station can take its source of energy from the potential energy of water falls or the kinetic energy of water flow. KHEPS and JHEPS are classified under the former. The reservoir is used to store water during high flow and to create hydraulic head for the turbine. The reservoir

also allows the release of water in a controlled manner to the penstock. KHEPS Reservoir Lake stretches almost 136 km upstream and has a breadth of 24 km at its widest point. The maximum head elevation is 141.7 m and maximum tail elevation is 104 m. The lake has a total capacity of 15 billion m^3 covering an area of 1270 square kilometre (Sarafinchin, 2017). The available water head ranges between 23.8 m and 41.2 m. The site map is shown in Figure 2.1.

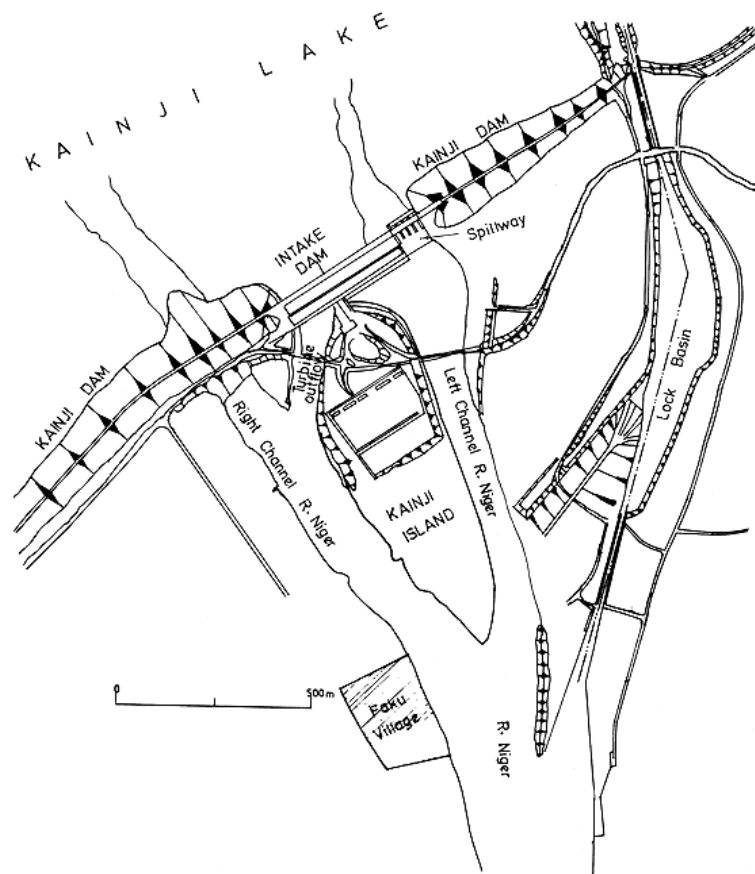
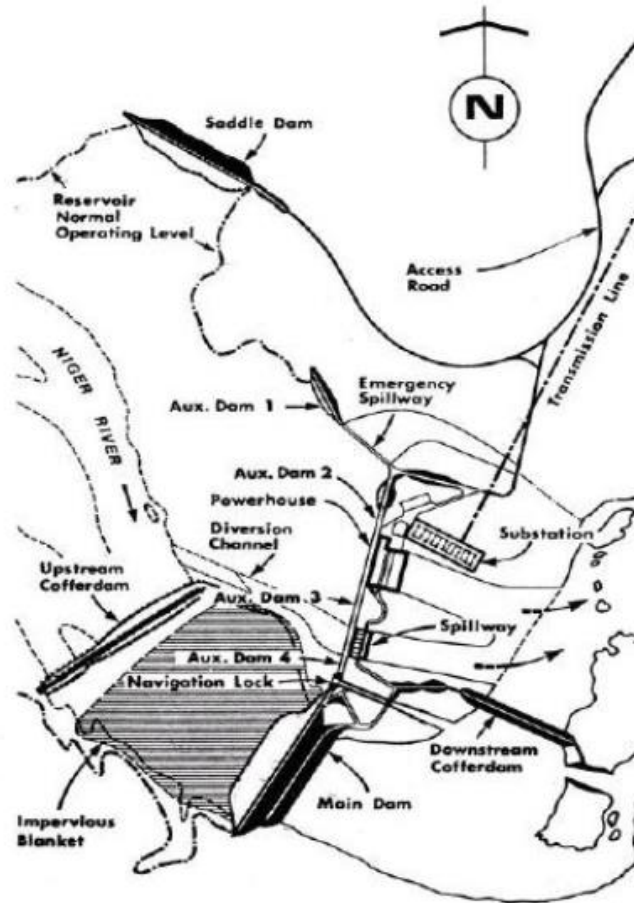


Figure 2.2: Site Map of KHEPS (Chiromo, Amu, Otonglo, & Balderamma, 2016)

JHEPS reservoir is approximately 25% of the size of KHEPS, this can be seen in the site map of Figure 2.2. It has a length of 100km and life storage approximated to 1,000,000,000 m^3 . The surface area is approximately 303 km^2 at an elevation of

103.0 *m* measured to seal level. The Normal operating head is at an elevation of 103 *m* and minimum of 99 *m*.



© JHEPS

Figure 2.3: Site Map of JHEPS

- ii. **Dam:** A reservoir can be natural (e.g. Lake) or artificial. Dams are used to create an artificial reservoir. The also provides a means of challenging water into conduits (penstock).

The KHEPS dam has a concrete main dam and a saddle dam. The saddle dam protects the main dam during flooding. The dam is 85.5 *m* in height and roughly about 8 *km* in length (Sarafinchin, 2017).

JHEPS dam is located approximately 100 *km* south of KHEPS dam and is followed by a lock and three auxiliary concrete dams noted as dams 4, 3 & 2. The main dam comprises of an earth and rock-fill dam wall and several auxiliary dams. It has a length of 670 *m* and is 42 *m* high (Chiromo, Amu, Otonglo, & Balderamma, 2016). The huge cost and long duration of building a dam for HEP station constitute part of the limitations to its spread as compared to thermal power station.

- iii. **Intake:** This is the entrance into the penstock. Trash racks are fitted prior to it in order to prevent debris into the turbine. Hoist operated intake gates (or Control gates) are provided to block flow into the turbine for maintenance and scheduling. JHEPS has six intake gates with a height of 12.4 *m* and with of 10 *m* each.
- iv. **Penstock:** This is the conduit that connect the intake to the scroll case of the turbine. In the case of KHEPS and JHEPS, each turbine is provided with its own penstock. The thickness of the conducts are design to withstand hydrostatic pressure and surges caused by fluctuation of load. The internal diameter of each penstock at JHEPS is estimated 10 *m*.
- v. **Turbine:** The essence of the potential energy in the reservoir is to drive a mechanical system called turbine. Turbine is a rotating mechanical engine that converts kinetic energy of falling water into rotational energy which is converted into electricity energy by generator. (Douglas J. F., Gasiorek, Swaffied, & Jack, 2012). Turbines are of different types depending on the type of flow, action of water on moving blades, head, quantity of water, according to name of originator and speed. Generally, they consist of set of blades to catch running water. The flowing water is directed to the blades of turbines runner which creates force on the blade. Shaft or

axle are connected to the blade and it rotates as the water turns the blades. The rotating axle is attached to rotor of an alternator (Woodford, 2008; Mishra, 2015).

Hydro turbines are of two types, impulse and reaction machines. The difference is lying in the manner of head conversion to kinetic energy. In the reaction turbine, only part of the available head is converted into kinetic energy. The fluid fills the blade passages and the head change or pressure drop occurs within the impeller. Reaction designs are of the radial-flow, mixed-flow, and axial-flow types and are essentially designed to extract the momentum of the high-energy fluid.

Only the total available head is converted into kinetic energy in the impulse turbine. It first converts the high head through a nozzle into a high velocity jet, which then strikes the blades at one position as they pass by. The impeller passages are not fluid-filled, and the jet flow past the blades is essentially at constant pressure (Gupta J. B., 2013; White, 2015; Douglas J. F., Gasiorek, Swaffied, & Jack, 2012)

There are three major types of water turbine used: the Kaplan and Francis turbines, both of which are reaction turbine, the Pelton wheel which is an impulse turbine.

A Pelton wheel is an impulse turbine used in high head and low flow stations. Its major component parts are nozzle, runner with bucket, brake, jet, flow regulating arrangement and casing. The energy in the penstock is converted into kinetic energy by a bucket receiving water from a nozzle. The amount of water striking the buckets is controlled by the motion of spear. As water strike into the runner, it starts to rotate along the axle.

The Francis turbine is a reaction turbine used in medium head and medium flow. It is an inward mixed flow turbine where the water under pressure enters the runner from guide vanes radially and discharge out of the runner axially. The motion of

water is controlled by movable inlet wicket gates, fixed around the runner, through which the water passes on its way from the spiral casing to the runner. Usually the pressure at inlet is more than that at the outlet, the water flows in a closed circuit and the runner is always full of water. After doing work, water is discharged into the tailrace through a draft tube.

Kaplan Turbines are used for low heads with large quantity of water available. They are design with adjustable blade such that it increases their efficiency even under part load. They are having 3 to 6 blades as compared to Francis runner that can have up to 16- 24 blades. They run at speed between 400 – 1500 rpm, higher than of Francis turbine. They are capable of reversed operation as pump, hence applicable in pump storage scheme (Gupta R. B., 2011).

The powerhouse of KHEPS has an installed capacity of 760MW comprised of eight turbine-alternator groups that include four Kaplan type 80MW, two turbines also Kaplan type rated 100 MW and two Francis propeller turbines rated at 120 MW. The station is designed to further accommodate four more turbines, but this has not materialised till date.

The powerhouse is equipped with six fixed blade Kaplan turbines with a 96.4 MW capacity each to give a total installed capacity of 578.4 MW. All turbines have a clockwise rotational speed of 93.75 rpm; they were originally manufactured by Andritz (Chiromo, Amu, Otonglo, & Balderamma, 2016).

- vi. **Synchronous Generator:** The synchronous generator (also called the alternator) are the major source of supplying electrical energy in large quantity. They are electromechanical device that convert mechanical energy from prime movers to electrical energy. In the case of HEP, the water power from hydro-turbine is

converted to electrical energy. The arrangement is usually vertical, and the turbine is coupled directly to the alternator, hence the name turbo-alternator (Theodore, 2000). As earlier mentioned in the introduction, there are eight alternators at KHEPS. Two are rated 120 MW, another two units are rated 100 MW while the remaining four units are rated 80 MW. JHEPS comprises of 6 identical vertical shaft generating units each having output capacity of 103,500 kVA at base load rating with net head of 27.6 m , discharge of 385 m³/s, 16 kV and 0.85 power factor. Generating units have a maximum continuous output of 119,000 kVA at head of 29.3 m.

- vii. **Transformer:** This is an electromagnetic device that transform electrical energy from one voltage level to another. In the case of JHEPS, synchronous generators output 16 *kV* of energy. For the purpose of transmission, the transformer raises this energy to 330 *kV* before being connected to the switch yard. Although most generating stations have one or more step down transformer that converts the 16 *kV* generated voltage to 415 V for in-house and local use.
- viii. **Outflow (Tailrace):** Water coming from the turbine discharge through the tailrace into the downstream. The size has a significant effect on the energy generated.
- ix. **Spillway:** Hydropower dams are provided with a means of discharging excess water during high inflows and floods. Spillways are designed to discharge major flood easily without any damage to the dam. KHEPS has four (4) spillways with hydraulic operated gates of 15.2 x 15.2 m (50 x 50 ft.), which could be opened to control flood and to release water for use at the Jebba Dam downstream. (Sarafinchin, 2017). There are two spillways at JHEPS; the main and emergency spillway. The Main spillway has a sill elevation of 70 m, six numbers of radial gates each of 12 *m* in

length and height of 9.5 m. The emergency spillway is a free overflow type with a length of 220 *m* and crest elevation of 103.15 *m*.

2.4 Constraints on the Operation and Management of the KHEPS and JHEPS

The operation and management the two hydropower systems have attracted some attention, especially in trying to optimize the individual stations. This is because there are many constraints on the management of generating resources. The fundamental constraints are as follows: there is a maximum amount of water that can be stored in the reservoir, there is maximum and minimum head for the reservoirs, the amount of water in the reservoir is affected by environmental factor such at evaporation and absorption, there may be constraints on the release of water from upstream or to downstream and the generating capacity of the stations are constrained by the installed and available turbo-alternators. Hence, some previous works are reviewed in this section. Apart from generic studies of the systems, relevant work involving each or the two hydropower systems are reviewed in this section. The works can be classified according to different aspects of hydropower. Inflows (analysis, forecasts, impact), power generated, reliability (plant failure rates, maintainability), plant refurbishing and joint operation.

2.5 Review of Research Work on Inflows into KHEPS and JHEPS

Water inflow is like the life of any hydropower projects. Ale et al. (2011) presented the results of an exponentially smoothed time series for forecasting inflows into the KHEPS reservoir. The technique is based on digital control technique. It is applied to develop a non-causal representation of the inflows into the Kainji dam. Inflow data from 1992 through 2008 were used to validate the models and provide short term forecasts. The

results reported a high degree of compliance of forecasts for biannual periods. The technique can play a significant role in this project since it can provide a useful approximation for determining long term water management protocols which can be further optimized as more accurate forecasts are obtained.

Nwobi-Okoye and Igboanugo (2012) employed a transfer function model obtained from a time series study of the reservoir inflows to initialize an input/output model of hydropower generation at KHEPS. Using the inflows and power generated time series for a 10-year period, the work obtained that the efficiency of the power generation facility was worst in the year 2006 and best in 2003. A common thread among the 10-year annual plot of the hydrological time series is that there is a general decline of water level and flow rate as from March or thereabout, reaching the lowest ebb in June and July of every year, and then it starts building up and reaching a peak in December and January. This approach has limitations that render it limited. The fact that the power generated by the station depends on the condition of its turbo-alternators make this input approach quite unreliable. Unfortunately, the time series of power generated may not provide a clean picture of the model simply because the power output is also dependent on the reliability of the turboalternators. The transfer function is simply the z-transform of the time series equation and it is clear that there are at least three stochastic processes – turbo-alternator failures, inflows, evaporation and other losses. It becomes even less reliable when the request for water by JHEPS is considered. The time series approach is a noncausal study which may veer away from reality if for example the failures of TAs become very erratic.

Assessment of impact of hydropower dams' reservoir outflow on the downstream river flood regime in Nigeria was the focus of Olukanni and Salami (2012) research. Data on

reservoir inflow, turbine discharge and reservoir level were collected. Statistical, time series, flow duration curve and probability distribution analyses of the hydrological variables were applied to data collected at the three hydropower stations. The main cause of flood in the downstream regime was identified to be the sudden release of water from the hydropower dams located upstream of the study area. The study revealed that the sudden release of flood water is not due to normal operation at the hydropower stations in Nigeria, but is due to sudden discharges at the reservoirs located in the Niger Republic and the Republic of Mali

2.6 Review of Research Work on Reliability of KHEPS and JHEPS

In the work of Adamu et al. (2012), the reliability of KHEPS was examined. Using the Frequency and Duration (F&D) approach; forced outage rate, MTTF, MTTR, MTBF, the overall station reliability was evaluated by the convolution of the generation and load models. The study shows that the generating units at Kainji hydropower stations have not been adequately maintained leading to frequent and delayed forced outage indicating unreliable performance of the individual units and the entire station.

The reliability of the KHEPS turbo-alternators was considered in (Ale T. O., 2014). The results were a summary of Markov model for the seven turbo-alternators using failure and repair parameters derived from the observation data. The performance of the station indicated an availability that was unsatisfactory, but which still enabled the station to be useful since the number of functional machines was about five. Nonetheless, the observed availability point at the need for extensive refurbishing of the plants.

Thomas & Olufeagba (2016) worked on the performance analysis of the JHEPS in its third decay. The operational data over a period of ten years were collected, analyses of time to failure and repair were carried out to determine the availability and reliability of the turbo alternators. It was observed that the turbo-alternators lack proper routing maintenance, though the units are in their useful life performance.

2.7 Review of Research Work on Power Generated at KHEPS and JHEPS

Abdulkadir et al. (2013) worked on modelling of hydropower reservoir variables for energy generation: neural network approach. Reservoirs' variables and energy generated were collected from Power Holding Company of Nigeria for a period of (1970-2011) and (1984-2011). The methodology used was the multiple perception neural network. The neural network summary yielded a good forecast for Kainji and Jebba hydropower reservoirs with correlation coefficients of 0.89 and 0.77 respectively. These values of the correlation coefficient showed that the networks are reliable for modelling energy generation as a function of reservoir variables for future energy prediction.

2.8 Review of Research Work on Plant Refurbishing and Joint Operation of KHEPS and JHEPS

Rehabilitation of Kainji hydropower station was the focus of Blanié, Seret & Renaud (2014). The approach was that of a refurbishment approach as it revisited the project, starting with the inception report and continuing through the production of specifications, review of contractual offers from possible contractors and issuance of the contract then, new design, manufacture and site erection. The work emphasizes what went right and what went wrong during the rehabilitation. The paper develops more specifically the

choice and characteristics of the new Kaplan units to replace the units 5 and 6 propellers thus giving a greater flexibility and efficiency of operation across the head and load range.

Obadote (2009) discusses the energy crisis in Nigeria: technical issues and solutions. The paper pointed out that the current infrastructure of the hydro plants needs rehabilitation and the actual energy output of the plants is far below their projected capacity. It enumerated some technical issues and challenges of hydropower plants: no overhaul of the plants, the output of the hydro plants is highly oscillatory according to the seasonal droughts, the trends of climate change have led to a continual loss of water, since the power output of hydro plants is dependent upon the flow of the river, with less water, there is less potential energy to harness. The major problem at hand is the need for proper maintenance, upgrade and additional generation station.

2.9 Review of Research Work on Joint Operation of KHEPS and JHEPS

In Jimoh (2008), a stochastic dynamic programming technique was employed to determine optimal rules for the release of water at KHEPS reservoir. It pointed out that the operating rules adopted by managers between 1996 and 1999 could not cope with high inflows occurring between August and September because the water level in the reservoir was high prior to the arrival of such flow. Thus, there was high spillage to avoid structural failure of the dam. It was also observed that volume of water released from the reservoir remained high after the occurrence of high inflow. The results recommended that the managers should adopt an operation policy that could handle inter-annual variation in inflow to the reservoir to maximize the benefit of impounding the water of River Niger and minimize the annual flooding of the river plain.

Ajao, Olabode, & Sule (2014) developed an application in Visual Basic express edition as a computational interface for hydropower plants. The program interface was reported to have the capability of computing the output of hydropower plants in a short time, develop a reliable expected flow and power characteristics of the site and generate a comprehensive report of the computation that is printable. The limitations of the program interface include its inability to compute the renewable energy delivered to an off-grid load.

2.10 Review of Research Work on Cascaded Hydroelectric Power System

Hydro plant control is an application area with an interesting set of problems for control engineers. The hydro turbine is a non-linear, non-stationary multivariable system whose characteristics vary significantly with the unpredictable load on it and this presents a difficulty in designing an efficient and reliable controller (Kishor, Saini, & Singh, 2007).

A design concept of complex hydro cascaded scheme was presented by (Fustik & Kiteva, 2006). It was proposed that a proper control architecture should involve measurement of real time data using ICT. This approach is more appropriate for the design of control system as compared to the presentation of (Salami, 2007) where the optimal policy is based on extraction of information from previous data. A model for time optimal control system should be based on physical and measurable features of the cascaded HEPS instead of statistical model whose parameters are not available for measurement. Meanwhile (Fustik & Kiteva, 2006) worked on the “Treska” cascade hydro system in Macedonia, the outcome of (Salami, 2007) form a background for further studied and development of optimal strategy for the cascaded KHES and JHEPS.

The control model is not only useful for system design, but it can also be the back bone of a decision support system. They are based on the use of relevant data and model analysis by direct integration with software. Such system was presented by (Zhong, Song, & Tan, 2009) and previously design one are the “Engineering Optimal Operation System of California” and “US central drainage area (CVP) and the water optimal operation system of the Tennessee Valley Organizations (HYDROSIM). The serve to assist the dispatcher of cascaded HEPS making operational decision. The operators and management could therefor simulate operational decision, alternatives, and assumptions and make amendment to give a good precision.

(Kishor, Saini, & Singh, 2007) categorized the research work done so far on the hydro plant models and its controller design into three. There are simple linearized first-order models of the plant which can predict the behaviour of the plant and thus its performance under different operating conditions. Non-linear models are often required for study of large signal transient stability in isolated hydroelectric plants, their controller tuning. The long-term dynamics of such nonlinear model have not been yet comprehensively studied. In recent time, contributions are being made to explore the potentials of artificial intelligence technique in modelling and control of the plant.

Sattouf (2014) worked on simulation model of hydro power plant using Matlab / Simulink. The model on consists of hydro turbine connected to synchronous generator, which is connected to public grid. The Simulink program in Matlab is used to obtain a schematic model of the hydro plant by means of basic function blocks. This approach better than using a compilation of program code as in other software programs. The library of Simulink software programs includes function blocks which can be linked and

edited to model. The main objectives of this model are aimed to achieve some operating modes of the hydro plant and some operating tests.

In a similar work by Guedesa et al (2012), studies were conducted on the optimal control of series of cascaded hydroelectric power stations. The case study was different from the KHEPS and JHEPS, Kainji-Jebba own because some of the power stations have reversible turbines. The solution solved the problem by posing it as a discrete-time optimal control with an objective function constructed to maximize the profit of power production.

(Zheng, Fu, & Wei, 2013) focused on the evaluation of power generation efficiency of a cascade of hydropower plants comprising of the three Gorges and Gezhouba cascade complex in China. The research investigates the major factors that could affect power output of the cascade complex. The results showed that the current power output of the Three Gorges and Gezhouba cascade complex had already reached around 90% of the maximum theoretical value. Compared to other influencing factors evaluated in this study, the accuracy of hydrological forecasts and flood control levels can have significant impact on the power generating efficiency, whereas the navigation has a minor influence. This research provides a solid quantitative methodology to assess the operation efficiency of cascade hydropower plants, and more importantly, proposes potential methods that could improve the operation efficiency of cascade hydropower plants.

Some authors worked on previously measured data and came up with what could have been an optimal use or a better operation policy. Nevertheless, others considered the problem rooted in control system engineering. The later soon deviate from the main energy source because of the difficulties in modelling the reservoir dynamics to suit

control engineering methodologies. They focus more on the governor and the hydraulic turbines whereas the major problem is the control of the stochastic variation in the reservoir head and stochastic failure of the units. This research will consider the problem as an issue for control engineers and attempt to model the reservoir nonlinear dynamics in such a way that modern and sophisticated control methodology will bring about an optimal generation policy.

2.11 Summary of Previous Work and Research Gap

Based on the available literature, it was discovered that there has been no research that considered KHEPS and JHEPS in cascade. Likewise, no author has also worked on the design of control system they can manage or optimise the available generating assets. Some authors worked on previously measured data and came up with what could have been an optimal use or a better operation policy. Nevertheless, others considered the problem rooted in control system engineering. The later soon deviate from the main energy source because of the difficulties in modelling the reservoir dynamics to suit control engineering methodologies. They focus more on the governor and the hydraulic turbines whereas the major problem is the control of the stochastic variation in the reservoir head and stochastic failure of the units.

Therefore, this research considered the problem modelling and operation of the two stations as an issue for control engineers. Effort was made to model the reservoir nonlinear dynamics in such a way that modern and sophisticated control methodology will bring about an optimal generation policy

2.12 Need for a Model and Optimal Control System on the Cascaded KHEPS and JHEPS

KHEPS turbo-alternators have internal governor such that the head can vary between 24 m to 42 m but JHEPS depends on operating at constant head held within tightly control limit and all units operates at base load. Due to system design, the reservoir head of JHEPS is highly sensitive to water release from KHEPS and units' availability. The amount of water that must be released from KHEPS and the duration of the release such as to maintain JHEPS reservoir at relatively constant head is the problem yet unsolved.

The effect of the trial and error approach currently being used is that JHEPS operators often shutdown some units intentionally if the release from KHEPS is low. Sometimes the water from KHEPS is spilled at JHEPS reservoir due to unit's failure. There are also occasions where JHEPS can only operate few units and instead of KHEPS to run all its available units, they allow the reservoir head to rise while the water is also evaporated. All these results in poor utilization of available energy and lower power generation.

Based on the available information gathered, the problem of real time optimal management of resources between KHEPS and JHEPS has not been solved. Despite all the research efforts made so far, operators still relied on intuitive water release rules to maximise power generation regularly. There are no proper scientifically motivated techniques for the management of resources between the two stations. Most methods being proposed are mere parameter optimization technique which are only meant for optimizing static and not dynamic system. The system dynamics is also nonlinear, hence most linear techniques may not be applicable.

Therefore, the problem is purely an optimal control problem where by a control signal is desired that will force the reservoir head at JHEPS to move from an initial point to a desired point in finite time and subject to constraints imposed by the system dynamics. Unfortunately, an optimal control problem cannot be solved without a proper model of the system dynamics. From available literature, such a control model is also not available. Hence the focus of this work has been to come up with models of the system dynamics and determine the optimal control laws.

Unfortunately, many problems that are rooted in nonlinear optimal control theory do not have computable solutions or they have solutions that may be obtained only with a great deal of computing effort (Anderson & Moore, 1989). Hence this research work considered the modelling of the system, proper presentation of the optimal control problem and provision of a suitable technique for determining the optimal control.

2.13 Mathematical Modelling of a Hydropower Station

Numerous physical phenomena and problems can be described with mathematical equations. Therefore, it is a norm in science and engineering to express a physical observation with a mathematical description called model. Kreyszig (2011) defines a mathematical modelling as the process of setting up a model, solving it mathematically and interpreting the result in an understandable form.

Mathematical model is a nontrivial and the starting point of solving a control problem. It is a necessity in the design of control systems to formulate the simplest mathematical description that predict the response of a physical system to an expected input signal.

The model does not only help in the control system design but can be used to study the system performance under change conditions of input and uncertainties.

It is worthy of note that the mathematical models of most physical systems and problems are characterised by differential equation. The differential equation may be linear or nonlinear, ordinary or partial and it can even be of higher order (Kirk, 1998; Nagrath, 2006).

Hydroelectric power system consists of various dynamics, among them are: inflow dynamics, reservoir dynamics, turbine dynamics, governor dynamics, alternator dynamics, and voltage and frequency stability dynamics. Usually the precise knowledge of the structure and parameters of these dynamics may not be known. The mathematical modelling and parameter estimations of these dynamics are very useful for the operation and management of the system. The model with also helps in predicting the sensitivity of the controlled system to random fluctuations in system parameters.

2.14 Optimal Control System

Optimal control theory solves the problem of determining the best possible control strategy that will cause a dynamic system to satisfy its physical constraints and minimize a certain performance index. The theory is based on calculus of variations and argued to have officially started in 1697 with the work of Johann Bernoulli (1667- 1748), when solving the “Brachystochrone” problem (Sussmann & Willems, 1997).

Later on comes some important contributions to the theory by the work of Hamilton and Jacobi, Euler and Lagrange, Lev Pontryagin who developed the minimum principles, Richard Bellman who also developed the dynamic programming approach to solution of

optimal control and the design of linear quadratic regulator by Rudolf Kalman (Becerra, 2008) (Bryson & Ho, 1969; Becerra, 2008).

2.15 Setup of an Optimal Control Problem

Formally, a controlled system is characterized by the state variables of equation 2.1 (e.g. distance, velocities, concentration etc.) and a control variable of equation 2.2 (e.g. flow rate, voltage, force, temperature etc.).

$$\mathbf{x}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \mathbf{x}_3(t), \dots, \mathbf{x}_f(t)]^T \quad (2.1)$$

$$\mathbf{u}(t) = [\mathbf{u}_1(t), \mathbf{u}_2(t), \mathbf{u}_3(t), \dots, \mathbf{u}_f(t)]^T \quad (2.2)$$

The setup of an optimal control problem comprises of the following:

- i. A mathematical model of the system to be controlled. The model can be an ordinary differential equation, discrete equations, partial differential equations, stochastic differential equations etc. A typical model of an ordinary differential equation form can be expressed as equation 2.3.

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \quad (2.3)$$

The model must be able to predict the system response to all foreseeable control input. Hence the engineer must first of all determine the solution to the model, such as to establish that a unique response exist for a given control input and initial conditions.

A model can be linear or nonlinear, this determines the type of solutions. Most linear model can be solved by the methods established in literatures but the solution to the nonlinear models are usually challenging. It is therefore a difficult

task since the engineer must provide a solution to the model before computing the optimal control.

- ii. A functional that will serve as the performance index. The performance index (J) can be a function of the state variable, control variable, initial and final time. There are three standard formulation when defining a performance index for an optimal control problem; the Bolza, Mayer and Lagrange forms (Becerra, 2008). The basic form is the Bolza form expressed in equation 2.4

$$J = K(x(t_f), t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t) dt \quad (2.4)$$

Where t_0 and t_f are the initial and final time, f, L and K represents functions

If $\int_{t_0}^{t_f} L(x(t), u(t), t) dt = 0$, the problem is the Mayer form and

if $K(x(t_f), t_f) = 0$, the problem is the Lagrange form.

- iii. There can be constraint to be satisfied by both states, controls and specific boundary conditions can be imposed on the system.

Subject to:

$$\dot{x}(t) = f(x(t_0), u(t), t)$$

$$x(t_0) = x_0$$

The problem to be solved is to find an admissible control $u^*(t)$ (optimal control) which cause the system of equation 2.3 to follow and admissible state trajectory $x^*(t)$ (optimal trajectory) and minimizes the performance index J (Rodrigues, Monteiro, & Torres, 2014).

2.16 Solutions to Optimal Control Problems

Due to the complexity of systems and applications, analytical methods are rarely used to compute the solutions to optimal control problems. Numerical solutions are mostly used in determining optimal control. In most cases, the numerical methods are highly sophisticated, and they do task computers (Schultz & Melsa, 1967). There have been numerous numerical procedures developed over the years, these procedures can be classified into two categories, the direct methods and indirect methods (Becerra, 2008; Rao, 2009; Rao A, 2009).

The choice of the numerical approach to be used depends mostly on the type of system, model, the performance index and the associated constraints. Other factors include the time to be used in coding, the available memory space for the iterative procedure, the initial guess, convergence, computations etc.

2.16.1 Direct Optimal Control

Direct method of computing optimal control involves the discretization of the state and the control in such a way that the problem is converted to a nonlinear optimization problem or nonlinear programming problem (Rao, 2009; Belts, 2001).

Procedure:

- Subdivide the time interval into $N > 1$ control state: $t_0 < t_1 < t_2 < \dots < t_N = t_f$
- In each of the sub-interval, (t_{i-1}, t_i) approximate $u(t) = u^i(t)$
- The problem can then be solved using an appropriate nonlinear optimization (nonlinear programming NLP) method.

The word appropriate here means that the technique to be used depends on the problem being solved. Popular techniques can be classified into two: the gradient based, and the heuristic based. The heuristic based are not widely applicable for uses by engineers as the gradient based. Popular gradient based are the steepest descent and the conjugate gradient methods.

The accuracy and robustness of optimal control computed using the direct approach depends on the optimization technique used. Different techniques can respond differently, a designer should therefore try several methods to ensure that the solution is reliable (Trond, 1999; Rodrigues, Monteiro, & Torres, 2014).

2.16.2 Dynamic Programming

Another method that cannot be classified as direct or indirect method of solving optimal control is the dynamic programming. Solution of optimal control using dynamic programming is based on the Bellman Principle of Optimality which says that, *“An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision”* (Kirk, 1998; Belts, 2001; Bertsekas, Dynamic Programming and Optimal Control, 2005).

To use the dynamic programming approach, the continuous control problem system is approximated by quantizing the admissible control, state and time to a finite level with optimal control required at each time interval (Jimoh O. , 2008; Sun & Huang, 2011).

There are three major drawbacks associated with the use of dynamic programming. It requires large memory space for storing history, the execution time can be long

depending on the processor speed and suffers the “curse of dimensionality” as the number of state variables, control variable and time interval increases.

2.16.3 Indirect Optimal Control

The indirect method applies calculus of variation to set up necessary conditions that must be satisfied by the optimal control. Calculus of variation, together with Pontryagin’s minimum principles are used to setup optimality conditions. These conditions produce optimal control canonical equations such that their solution ensures that an optimum point has been reached. While using this approach, it is usually necessary to calculate the Hamiltonian, co-state equations, the optimality and transversely conditions (Rodrigues, Monteiro, & Torres, 2014).

An alternative to the Pontryagin’s method is the Hamilton-Jacobi approach. It removes the necessity for solving the two-point boundary value problem and it produces a closed loop optimal control law, such that the control is a function of the state. Nevertheless, there is a necessity for solving nonlinear partial differential equation called the Hamilton-Jacobi equation. The solution of Hamilton-Jacobi equation is seen difficult by engineers such the method is applicable for only few special cases. Engineers are forced to adopt the Pontryagin minimum principle despite it produces open loop solution via numerical techniques (Schultz & Melsa, 1967).

2.17 Optimization Using Calculus of Variations

Calculus of variations solves the problem of determining the maximum and minimal of a functional with or without constraints. For an unconstrained optimization, the problem is to find the minimum of a functional $J(x)$ where;

$$J(x) = \int_{t_o}^{t_f} L(x(t), \dot{x}(t), t) dt \quad (2.6)$$

Applying the method of variation of variable leads to the necessary condition for optimality called the *Euler-Lagrange Equation* stated below,

$$\frac{\partial}{\partial x} L(x(t), \dot{x}(t), t) - \frac{d}{dt} \left[\frac{\partial}{\partial \dot{x}} L(x(t), \dot{x}(t), t) \right] = 0 \quad (2.7)$$

For a constrained optimization, the problem is to find the minimum of a functional $J(x)$ where $x(t)$ is constrained by;

$$f(x(t), \dot{x}(t), t) = 0 \quad (2.8)$$

A method of Lagrange multiplier ($\lambda(t)$) can be used such that,

$$J(x) = \int_{t_o}^{t_f} L(x(t), \dot{x}(t), t) + \lambda(t) f(x(t), \dot{x}(t), t) dt \quad (2.9)$$

$$J(x) = \int_{t_o}^{t_f} \psi(x(t), \dot{x}(t), \lambda(t), t) dt \quad (2.10)$$

The condition for optimality still reduced to the Euler –Lagrange equation,

$$\frac{\partial}{\partial x} \psi - \frac{d}{dt} \left[\frac{\partial}{\partial \dot{x}} \psi \right] = 0 \quad (2.11)$$

2.18 Optimal Control based on Minimum Principle of Pontryagin

The minimum principle of Pontryagin is used to facilitate the solution to optimal control design problem where $x(t)$ is constrained by,

$$\dot{x}(t) = f(x(t), u(t), t)$$

The problem is to determine an admissible control $u^*(t)$ that cause the dynamic system $\dot{x}(t) = f(x(t), u(t), t)$ to follow an admissible trajectory $x(t)$ that minimizes that performance measure $J(u)$.

If $L(x(t), u(t), t)$ and $K(x(t_f), t_f)$ are sufficiently differentiable such that we have the functional;

$$J(u) = K(x(t_f), t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t) dt$$

Where $x(t)$ is the state trajectory starting from $x(t_0) = x_0$ and generated by the control $u(t)$. The problem is to determine the admissible control which minimizes the performance functional $J(u)$. Assuming that the admissible control and state are not bounded and t_0 is specified.

If $u^*(t)$ is an optimal control and $x^*(t)$ correspond to the optimal state trajectory, then $u^*(t)$ and $x^*(t)$ must satisfy the following conditions

- i. $\dot{x}^*(t) = f(x^*(t), u^*(t), t)$
- ii. $x^*(t_0) = x_0$
- iii.
$$\begin{aligned} J^* = J(u^*) &= K(x^*(t_f), t_f) + \int_{t_0}^{t_f} L(x^*(t), u^*(t), t) dt \\ &\leq J(u) = K(x(t_f), t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t) dt \end{aligned} \quad (2.12)$$

Introduction of the Hamiltonian

Let $\lambda(t)$ be some piecewise continuous vector –valued function called the Lagrange multiplier or co-state or adjunct vector.

Then;

$$\begin{aligned} f(x(t), u(t), t) - \dot{x}(t) &= 0, \text{ and} \\ f(x^*(t), u^*(t), t) - \dot{x}^*(t) &= 0 \end{aligned}$$

and,

$$\begin{aligned} J(u) &= K(x(t_f), t_f) + \int_{t_0}^{t_f} [L(x(t), u(t), t) + \\ &\quad \lambda(t) \{f(x(t), u(t), t) - \dot{x}(t)\}] dt \end{aligned} \quad (2.13)$$

A function called the Hamiltonian $H(x(t), u(t), \lambda(t), t)$ can be defined such that,

$$H(x(t), u(t), \lambda(t), t) = L(x(t), u(t), t) + \lambda(t) f(x(t), u(t), t) \quad (2.14)$$

Then,

$$J(u) = K(x(t_f), t_f) + \int_{t_0}^{t_f} [H(x(t), u(t), \lambda(t), t) - \lambda(t)\dot{x}(t)] dt \quad (2.15)$$

$$J(u^*) = K(x^*(t_f), t_f) + \int_{t_0}^{t_f} [H(x^*(t), u^*(t), \lambda^*(t), t) - \lambda^*(t)\dot{x}^*(t)] dt \quad (2.16)$$

Applying the *Minimum Principle of Pontryagin* at the optimum point, this gives the necessary and sufficient conditions for optimality as follow;

$$\dot{x}^*(t) = \frac{\partial}{\partial \lambda} H(x^*(t), u^*(t), \lambda^*(t), t) \quad (2.17)$$

$$\dot{\lambda}^*(t) = \frac{\partial}{\partial x} H(x^*(t), u^*(t), \lambda^*(t), t) \quad (2.18)$$

$$0 = \frac{\partial}{\partial u} H(x^*(t), u^*(t), \lambda^*(t), t) \quad (2.19)$$

These three equations are called the *Canonical Hamiltonian Equation*, with the boundary conditions,

$$\lambda^*(t_f) = \frac{\partial}{\partial x} K(x^*(t_f), t_f) \quad (2.20)$$

$$x^*(t_0) = x_0$$

The indirect approach leads to a nonlinear two-point boundary-value problem. The control task then reduces to the solution of a boundary value problem. There are different approaches with their advantages and disadvantages. In all the solution techniques, an initial guess is used to obtain a solution in which one or more of the necessary optimality conditions is not satisfied. The solution is then used to adjust the initial guess in an attempt to make the next solution come closer to satisfying all of the necessary conditions. If the steps are repeated and the iterative procedure converges, the necessary conditions will eventually be reached

2.19 Methods for the Numerical Solution of Optimal Control of Systems

Essential tools for solving nonlinear optimal control problems includes: a numerical technique for solving differential equation, a method of integrating functions, means of solving nonlinear algebraic equations and finally a method for handling nonlinear optimization technique (Rao A, 2009).

Popular methods of solving differential nonlinear differential equations are presented in (Jeffrey, 2002; Stroud & Booth, 2003; Kreyszig, 2011), these are Euler method, modified Euler methods, Runge-Kutta, Runge-Kutta-Fehlberg algorithm and Adams-Moulton method. The Euler methods may be unstable for some application and step size but Runge-Kutta algorithm and Adams-Moulton are of greater accuracy in most application.

The numerical integration methods are usually the rectangular rule also called Trapezoidal rule, Simpsons Rule and Gauss Integration Formula (Gaussian Quadrature). The Gaussian Quadrature is more practical but complicated for simple problem. However, the accuracy of the earlier methods were not too poor for most application (Hoffman, 2001; Zarowski, 2004).

Many authors had proposed methods of solving an optimal control problem, these methods can be in the form of nonlinear programming for solving a finite dimensional control problem or solving boundary value problem. Among these methods are Shooting method, steepest descent, conjugate gradient, dynamic programming, variation of externals, quasi-linearization, gradient projection, collocation etc. There has been no perfect method as each has its own advantages and disadvantages. A designer must study these methods to know the appropriate one a given control problem.

2.19.1 Method of Steepest Descent

The steepest descent is an algorithm for determining the minimum of a differentiable function (Petrova & Solov'ev, 1997; Belts, 2001). Given a performance index $J(u)$ that is differentiable, the steepest descent direction is the path opposite $\nabla J(u)$. The search starts at a differentiable point $u^k(t)$ and decreases after each iteration until it reaches the minimum point $u^*(t)$. Where:

$$\nabla J^* = \frac{\partial J^*}{\partial u^*(t)} \approx 0 \quad (2.21)$$

If $\mathbf{z}(u^k)$ is a unit vector along the increasing gradient,

$$\mathbf{z}(u^k) = \frac{\left[\frac{\partial J}{\partial u^k} \right]^T}{\left\| \frac{\partial J}{\partial u^k} \right\|} \quad (2.22)$$

Then, moving in the direction $-\mathbf{z}(u^k)$,

$$u^{k+1}(t) = u^k(t) - \psi^k \mathbf{z}(u^k) \quad (2.23)$$

ψ^k is the optimum steepest descent step size. An optimum value of ψ^k must be determined because a larger value of ψ^k results in the local minimum of J being overshoot. A smaller value for ψ^k will require much time and iterations for the search. Hence, ψ^k must be determined such as to move towards the minimum at the smallest computational time, this is another optimization in itself. Equation (3.42) gives an approximate end of the search in the present direction

$$u^{k+1}(t) = u^k(t) - \psi_{(min)}^k \frac{\frac{\partial J^k}{\partial u^k(t)}}{\left\| \frac{\partial J^k}{\partial u^k(t)} \right\|} \quad (2.24)$$

Steepest descent algorithm is very fast in moving a solution from any local point within the feasible region to the vicinity of the point of convergence (Trond, 1999). If a suitable

method of solution is unknown, steepest descent is guaranteed to iterate towards the minimum point but characterised by slow convergence.

2.19.2 A Conjugate Gradient Algorithm

Conjugate gradient algorithm is a minimization technique for function of several variable and for finding solution to a simultaneous nonlinear algebraic equation (Fletcher & Reeves, 1964). The method was first applied to solve an unconstrained optimal control problem by (Lasdon, 1967). Later, the method was adapted to solve constrained optimal control problem where a penalty function is imposed on the constraints on state and control. (Willoughby, 1969; Sargent, 2000). It incorporates the fast characteristics of steepest descent method in moving to the optimum but with additional modification in ensuring quick convergence around the optimum.

The procedure for using conjugate gradient to solve an optima control problem is as follows:

- ❖ Guessing and initial control vector $u^k(t)$ and determine the performance index $J(u^k)$ with the iteration number $k = 0$.
- ❖ Calculate the gradient vector $g^k = \nabla J$ at u^k .
- ❖ Determine the conjugate gradient parameter β^k .

$$\beta^{(k)} = \frac{\int_{t_0}^{t_f} (g^k)^2 dt}{\int_{t_0}^{t_f} (g^{k-1})^2 dt} \quad (2.25)$$

$$g^{(k-1)} = 1 \text{ when } k = 0$$

- ❖ Calculate the direction of search vector s^k .

$$s^k = -g^k + \beta^k s^{k-1} \quad (2.26)$$

$$s^{(k-1)} = 0 \text{ when } k = 0$$

- ❖ Obtain the optimum step size ψ_{min} by performing a minimization search on $J(u^{(k)} + \psi s^k)$.
- ❖ Determine the next control vector $u^{(k+1)}(t)$.

$$u^{(k+1)}(t) = u^{(k)}(t) + \psi_{min} s^k \quad (2.27)$$

- ❖ Check whether $|J(u^{k+1}) - J(u^k)| \leq 10^{-n}$ to stop the iteration, where n is a constant. If the stopping criterion is not met, increment k by one and repeat the procedure using $U^{(k+1)}$ as the new guess.

The algorithm is promising provided that the performance index is differentiable and a method of performing a line search for optimum step size is obtainable.

2.19.3 Shooting and Multiple Shooting

Solution of an indirect optimal control can be obtained by shooting method. This was presented in (Bryson & Ho, 1969) and it has been used to solving different optimal control problem (Karbowski, 2016). It starts with an initial guess of the control variables which are used to determine the optimal state trajectories. The performance index and boundary conditions are checked for both the control and state trajectories by integrating from t_0 to t_f . If there are errors in terminal conditions at t_f , the initial conditions are updated (manipulated) and the iteration continues until the terminal conditions are met and the performance index is minimum (Belts, 2001; Adam & Hashim, 2014).

Shooting methods are simple to understand and can produce moderately accurate solutions. Nevertheless, there are difficulties in estimating the first initial guess at t_0 that will produce a reasonable solution at the other boundary at t_f . The solution at the other

t_f is usually sensitive to small changes in the initial guess and the solution may even diverge.

Multiple shooting technique is introduced to overcome the sensitivity of shooting method to the initial guess. Instead of shooting from time t_0 to t_f , the time interval is divided into stages, and the shooting method is applied for each stage. This implies that the trajectory will also be divided into interval and the intermediate interval can also be a guess value. Constraints are introduced at the intermediate section to enforce continuity from one stage to the other.

Multiple shooting technique was employed in the optimal control of robot (Diehl, Bock, Diedam, & Wieber, 2005), it was compared with the shooting and collocation method and proofs to be better in handling strongly nonlinear two-point boundary value optimization problems.

A shortcoming of using multiple shooting approach is that it increases the size of the problem by increasing the additional constraints enforcing continuity and the number of Newton iteration.

2.19.4 Summary and Proposed Method of Solution

In this research, the steepest descent was tried with the hope that it will yield an optimal control after several iterations, the conjugate gradient is applied to reduce the convergence time while the solution is compared with the steepest descent. It should be noted that there are slight modifications to the basic algorithm, among is the formulation of the problem such that the performance index is differentiable and the method of computing the optimal step size at each iteration. A novel approach to the solution of

boundary value problem was also introduced, it is a modification of shooting method and called a Progressive Domain Expansion Method. Finally, an iterative technique that can overcome the problem of initial guess with shooting method was developed, the method is based on conjugate gradient algorithm. A guess of the control at the first iteration converge to the optimal control at the last iteration and the solution is less sensitive to the initial guess.

2.20 Optimal Control Software

Numerous software exists for solving optimal control problem. Among them is the RIOTS_95, it is a MATLAB based toolbox that provides an interactive environment for solving general optimal control problems and its application. It is written in C, FORTRAN and M-File scripts (Schwartz & Chen, 1997). The Dynamic Optimization Toolbox with Vector Control Parametization (DOTcvp) is also a MATLAB based optimization toolbox design from direct discretization. The Interior Point Optimizer (IPOPT) is another software written in FORTRAN and C programming for solving large scale nonlinear optimization. Others include OC-ODE, Muscode-II, MISER3, SQPlab, NLPQLP, NLPQL, Maple, XPPAUT etc.

Most of the available software are based on either FORTRAN, C, MATLAB or combination of these programming languages. In developing countries, the software is expensive, not readily available, requires yearly registration and regular update to fix different bugs. It will be better to develop a toolbox that can be used on the readily available Microsoft office.

2.21 A Brief Review Microsoft EXCEL VBA® Programming Environment

The Microsoft EXCEL Visual Basic for applications (EXCEL VBA®) extends the potentials of the Microsoft EXCEL spreadsheet and enables it to be customised. It allows the use of EXCEL for solving problems beyond the capability of the standard spreadsheet. The spreadsheet is used for inputting data, the data is processed by a program that is highly related to a Visual Basic Programming Language and the results are displayed on the spreadsheet (Robinson, 2006). It also supports the creation of custom graphics user interface (GUI) for accepting data, setting constants and displaying graphs.

EXCEL 2016 and earlier versions come with inbuilt functions that can be used directly on the spreadsheet, they are grouped into financial, statistical, information, engineering, trigonometry, data base, web etc. These functions can be called inside a VBA® program while new functions can be written for solving complex problems.

The beauty of EXCEL VBA® is that tool box can be developed for solving complex engineering problem. The tool box will run easily in an EXCEL environment and can be transferable from one system to another. A user does not need to purchase any expensive software like MATLAB, LabView etc., the software comes free with every copy of Microsoft EXCEL. Usually the software is embedded, user only needs to activate it by navigating to File – Option – Custom Ribbon and Check of the developer box. Once EXCEL is activated, the inbuilt VBA is also activated without connection to any server online or renewal. It comes with a macro recorder for assisting programmers with syntax that seems challenging, this does not come with the stand-alone Visual Basic.

EXCEL VBA® is a universally available software package that is available for almost nothing and yet has the potential to be used for a very wide variety of applications. Its however, does not have prefabricated toolboxes for all but some standard applications. Olufeagba embarked on a program to develop tools for this platform that could be used for development studies. Various applications have been successfully developed and applied to practical problems. Figure 2.5 shows the EXCEL VBA® programming environment.

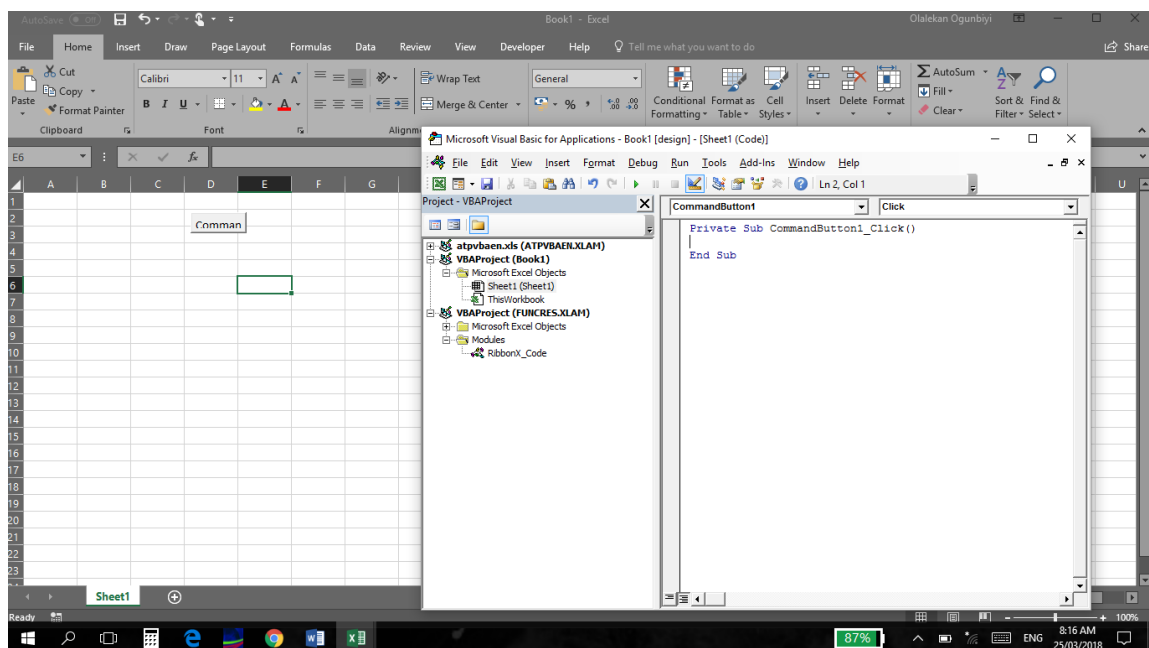


Figure 2.4 IDE for EXCEL VBA® Programming

Therefore, several important and useful results have been completed using EXCEL VBA®. Ale (2014) used EXCEL VBA in the analysis, modelling and simulation of reliability of Generation at of Kainji HEPS. The application enables a cheap and easier realization of system model and analysis. Ahmed and Ogunbiyi (2014) reported the results of the reliability analysis of a three-machine power station using a Markov model that was solved using an EXCEL VBA® realization. Durotoye & Ogunbiyi (2017) used EXCEL VBA® to develop a tool for automatic generation of root locus of a Linear Time

Invariant System (LTIS). Abolarin & Olufeagba (2016) used EXCEL VBA® to analyses the power flow on a transmission network. The difficulties with manual computation and purchase of expensive software was easily overcame. Last but not the list is the work of Thomas, et al. (2017), scripts written in EXCEL VBA® were used in analyzing the conversion efficiency of Jebba hydroelectric power station. This facilitates the easy analysis of data spanning ten years.

The above results are a small sample of what is feasible using this very versatile platform. The research reported here will similarly employ the EXCEL-VBA® platform for, modeling and control of the cascaded hydropower facilities described earlier. It is expected that all the tools needed for solving the different problems will be formally developed and verified. Naturally the results will be relatively inexpensive and relieved of web links that can be very frustrating in an environment such as Nigeria.

In the following chapters, the objectives will be unfolded and the problems that must be solved will be carefully defined, solved, verified and the results discussed.

CHAPTER THREE

Research Methodology

3.1 Preamble

In chapter two, it was observed that a necessity for control system design is the mathematical modelling of the process to be controlled. The model gives the simplest mathematical description that can predict the response of the control system to an expected input. Many systems satisfy a set of deterministic or stochastic, differential or difference equation, hence the first step in the design of an optimal control system for cascaded KHEPS and JHEPS is the development of a differential equation that describes the system dynamics.

The models are not only meant for control system design, they can be used for system studies, serve as decision tool for managers and training platform for operators.

3.2 Formulation of Mathematical Models of the KHEPS and JHEPS

To develop a model with good precision, a proper knowledge of the structure of the system is required. The model will be developed from the investigation of the mathematical and engineering principles that describes the dynamics. The precise knowledge of the system may be unknown, but the parameters of the model can be estimated from observations over time and from measurement of feedback.

3.2.1 Generating Capacity of the KHEPS and JHEPS

The basic information describing the structure of the two hydropower stations can be describe by Table 3.1.

Table 3:1: Generation Capacity of the KHEPS and JHEPS

	KHEPS	JHEPS
Location	- Latitude - Longitude	- Latitude - Longitude
Year of commissioning	1969: Units 7, 8, 9, 10 1976: Units 11 & 12 1978: Units 5 & 6.	1985: Units 1 to 6
Number of Units	8 Units	6 Units
Installed Capacity	760 MW	578.4 MW
Generator Rating	4 sets of Kaplan Turbine: 80 MW. Units 7, 8, 9, 10 2 sets of Kaplan Turbine: 100 MW. Units 11 & 12. 2 sets of Francis Turbine: 120 MW. Units 5 & 6.	6 sets of Kaplan turbine: 96.4 MW
Head (above sea level)	142 m (max) 124 m (min)	103 m (max) 99 m (min)
Max. Net Head (m)	42.2 m	29.3 m
Rated Operating Head (m)	37 m	27.6 m
Min. Net Head (m)	24 m	24 m

3.2.2 Schematic Representation of Cascaded KHEPS and JHEPS

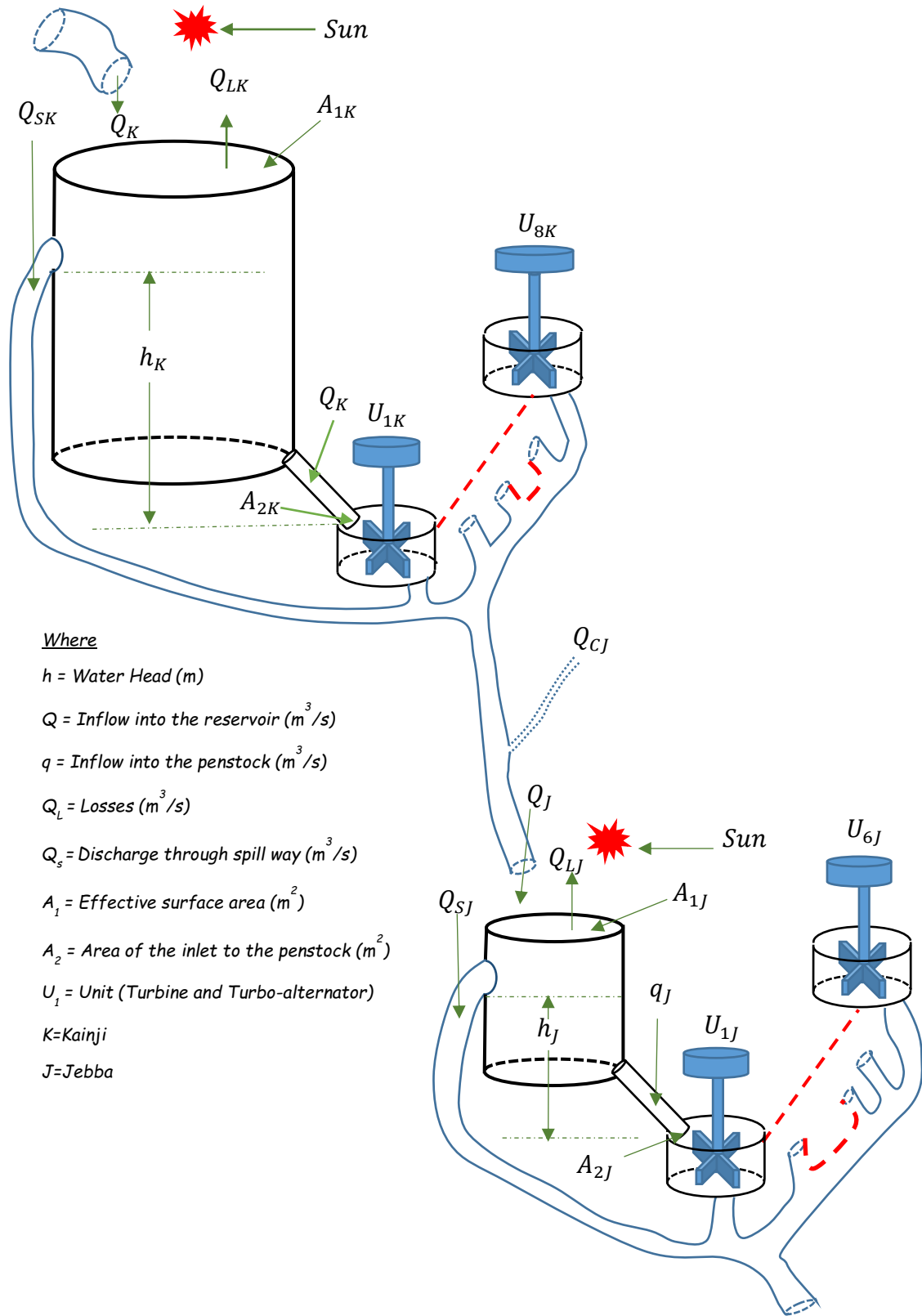


Figure 3.1: Schematic Representation of Kainji-Jebba Hydroelectric Power Stations

3.2.3 Power Output from a Hydropower Plant

Mechanical power (P_m) developed by a hydropower turbine of figure 3.1 can be expressed as:

$$P_m = \rho ghq - \text{Losses in the penstock} \quad (3.1)$$

Where ρ is the density of water in (kg/m^3);

g is the acceleration due to gravity (m/s^2);

h is the head of the reservoir (m); and

q is the flow rate in (m^3/s).

Since the conversion efficiency (turbine efficiency and generator efficiency) η must of necessity be less than 100%, the electrical power developed P_e can be expressed as:

$$P_e = \eta \rho ghq \quad (3.2)$$

And the flow rate q is related to the velocity by;

$$q = A_2 v \quad (3.3)$$

A_2 is the cross-sectional area of the penstock intake (m);

and v is the velocity of water (m/s).

$$P_e = \eta \rho gh A_2 v \quad (3.4)$$

An expression for the velocity can be obtained from the Bernoulli's energy equation when applied to the input and output of the reservoir. (Douglas et al., 2005)

$$\frac{1}{2} \rho v_1^2 + \rho g h_1 + P_1 = \frac{1}{2} \rho v_2^2 + \rho g h_2 + P_2 \quad (3.5)$$

Where P_1 and P_2 are atmospheric pressure values at the surface and the outlet, which in practice are approximately equal for Kaplan low head schemes and cancelled out in the equation (3.5). h_2 is the head at the outlet equals which equals zero. Since the velocity of water at the head of the reservoir is much less than the velocity at the penstock outlet, v_2 is far greater than v_1 and equation 3.5 becomes:

$$\rho g h_1 = \frac{1}{2} \rho v_2^2$$

$$v_2 = \sqrt{2gh_1} \quad (3.6)$$

$$q = A_2 \sqrt{2gh} \quad (3.7)$$

$$P_e = \sqrt{2} \eta \rho A_2 g^{3/2} h^{3/2} \quad (3.8)$$

$$P_e = \beta \eta h^{3/2} \quad (3.9)$$

$$\text{where } \beta = \sqrt{2} \rho A_2 g^{3/2}$$

3.2.4 Electric Power Generated at KHEPS and JHEPS

KHEPS has 8 non-homogenous units that can be classified into three groups. A set of four Kaplan turbo-alternator rated 80 MW and designated by n_{1K} . The two sets of 100 MW Kaplan turbo-alternators are designated by n_{2K} , while n_{3K} represent the two sets of 120 MW Francis turbo-alternators, hence the electric power generated at KHEPS is represented by equation 3.10. The equation is based on the fact that efficiency of conversion are different between the sets of units.

$$P_K = \left(\sqrt{2} n_{1K} \eta_{1K} \rho A_{2K} g^{3/2} \right) h_K^{3/2} + \left(\sqrt{2} n_{2K} \eta_{2K} \rho A_{2K} g^{3/2} \right) h_K^{3/2}$$

$$+ \left(\sqrt{2} \ n_{3K} \ \eta_{3K} \ \rho \ A_{2K} \ g^{3/2} \right) h_K^{3/2} \quad (3.10)$$

$$P_K = \beta_K n_{1K} \eta_{1K} h_K^{3/2} + \beta_K n_{2K} \eta_{2K} h_K^{3/2} + \beta_K n_{3K} \eta_{3K} h_K^{3/2} \quad (3.11)$$

JHEPS on the other hand has 6 identical units of 96.4 MW fixed blade Kaplan turbo-alternators and they are designated by n_J , where n_J can take an integer value from 1 to 6.

$$P_J = \left(\sqrt{2} \ n_J \ \eta_J \ \rho \ A_{2J} g^{3/2} \right) h_J^{3/2} \quad (3.12)$$

$$P_J = \beta_J n_J \eta_J h_J^{3/2} \quad (3.13)$$

Total power generated from the cascade system (P_T)

$$P_T = P_K + P_J \quad (3.14)$$

$$P_T = \beta_K n_{1K} \eta_{1K} h_K^{3/2} + \beta_K n_{2K} \eta_{2K} h_K^{3/2} + \beta_K n_{3K} \eta_{3K} h_K^{3/2} + \beta_J n_J \eta_J h_J^{3/2}$$

$$P_T = (\Psi_{1K} + \Psi_{2K} + \Psi_{3K}) h_K^{3/2} + \Psi_J h_J^{3/2} \quad (3.15)$$

$$\Psi_{iK} = \beta_K n_{iK} \eta_{iK}; i = \{1 \dots 3\}$$

$$\Psi_J = \beta_J n_J \eta_J$$

Equation 3.15 represents that the power generated, indicating that it depends on the reservoir head and number of operating units. Consequently, it is clear that the dynamical model of hydropower station is determined by the dynamical consideration of both the alternator and reservoir.

3.2.5 Dynamical Head Equations for the KHEPS and JHEPS Reservoirs

Consider a single reservoir as represented in Figure 3.2 with Q as the inflow and q as the outflow;

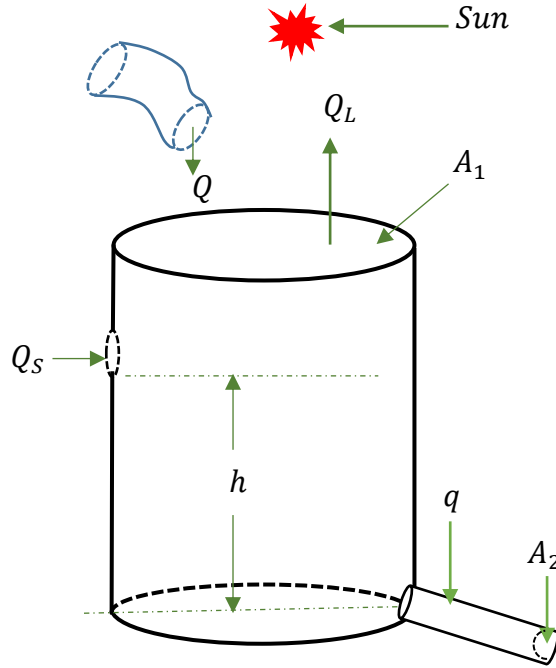


Figure 3.2: Schematic Representation of a Reservoir

$$A_1 h = (Q - Q_L - Q_s - q)t \quad (3.16)$$

$$A_1 \frac{dh}{dt} = Q - Q_L - Q_s - q = Q - Q_L - Q_s - A_2 v_2 \quad (3.17)$$

Equation 3.17 can be combined with equation 3.6 to give:

$$A_1 \frac{dh}{dt} = Q - Q_L - Q_s - \sqrt{2gh} A_2 \quad (3.18)$$

$$\frac{dh}{dt} = -(\sqrt{2g} A_1^{-1} A_2) h^{1/2} + A_1^{-1}(Q - Q_L - Q_s) \quad (3.19)$$

The head equation can be expressed as;

$$\frac{dh}{dt} = -\alpha h^{1/2} + \mu(Q - Q_L - Q_s) \quad (3.20)$$

Where

$$\alpha = \sqrt{2g} A_1^{-1} A_2$$

$$\mu = A_1^{-1}$$

Hence, the dynamical model for KHEPS with n_K number of units is expressed as:

$$\frac{dh_K(t)}{dt} = -n_K \alpha_K h_K^{1/2}(t) + \mu_K(Q_K(t) - Q_{LK}(t) - Q_{sK}(t)) \quad (3.21)$$

$$\alpha_K = \sqrt{2g} A_{1K}^{-1} A_{2K}$$

$$\mu_K = A_{1K}^{-1}$$

The dynamical model for JHEPS with n_J number of units and $Q_{CJ}(t)$ (the inflow from the catchment area in-between the two reservoirs) can be expressed as:

$$\frac{dh_J(t)}{dt} = -n_J \alpha_J h_J^{1/2}(t) + \mu_J(Q_J(t) - Q_{LJ}(t) - Q_{sJ}(t)) \quad (3.22)$$

Where

$$Q_J = q_k + Q_{sk} + Q_{CJ}$$

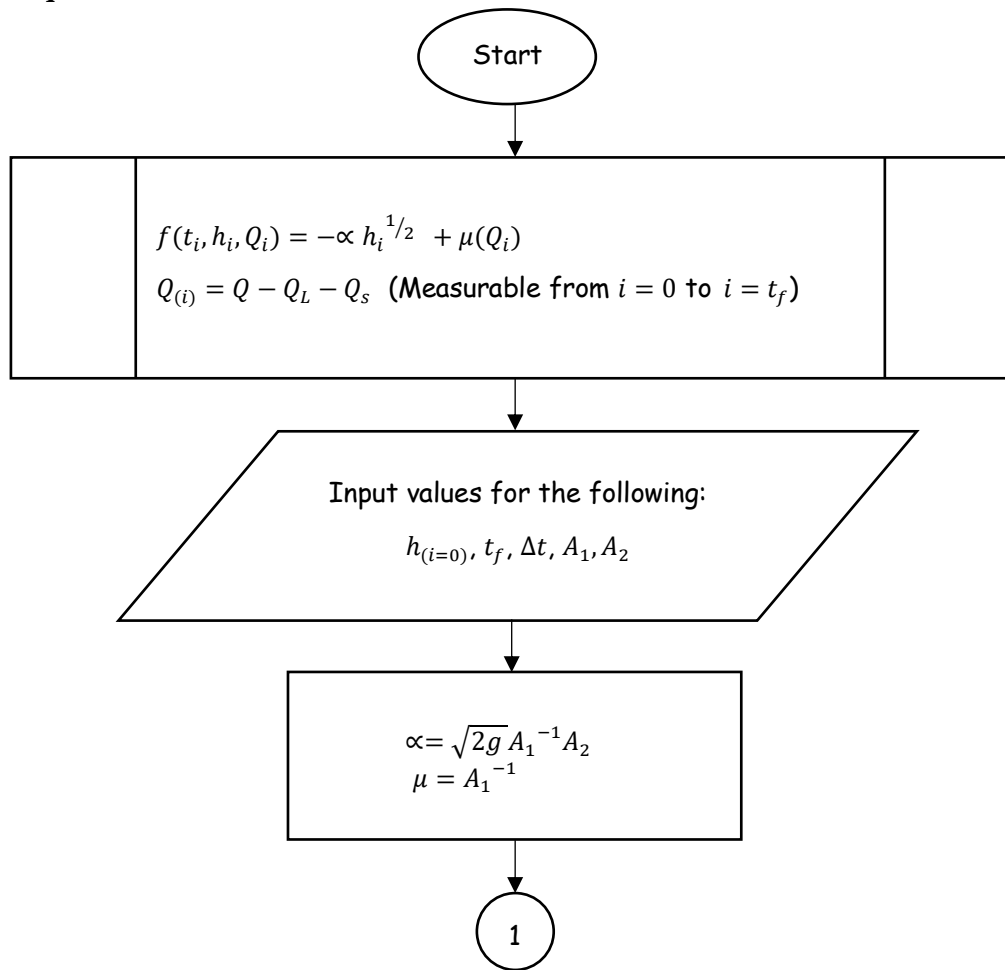
$$\frac{dh_J(t)}{dt} = -n_J \alpha_J h_J^{1/2}(t) + \mu_J((q_k(t) + Q_{sk}(t)) + Q_{CJ}(t) - Q_{LJ}(t) - Q_{sJ}(t)) \quad (3.23)$$

Where $\alpha_J = \sqrt{2g} A_{1J}^{-1} A_{2J}$ and $\mu_J = A_{1J}^{-1}$.

3.3 Numerical Solution of the Nonlinear Head Equation

Following the development of the previous equations, it is evident that the dynamical expression satisfies a nonlinear differential equation from which the closed form solution is not readily available, but which can be solve numerically. In chapter two, the review of numerical techniques for solving nonlinear ordinary differential equations were presented. As a result, three numerical techniques were used to solve the model and the results were compared. Procedure for each of these techniques are presented in the following subsection.

3.3.1 Flow Chart of the Numerical Solution of the Head Equation using Modified Euler Technique



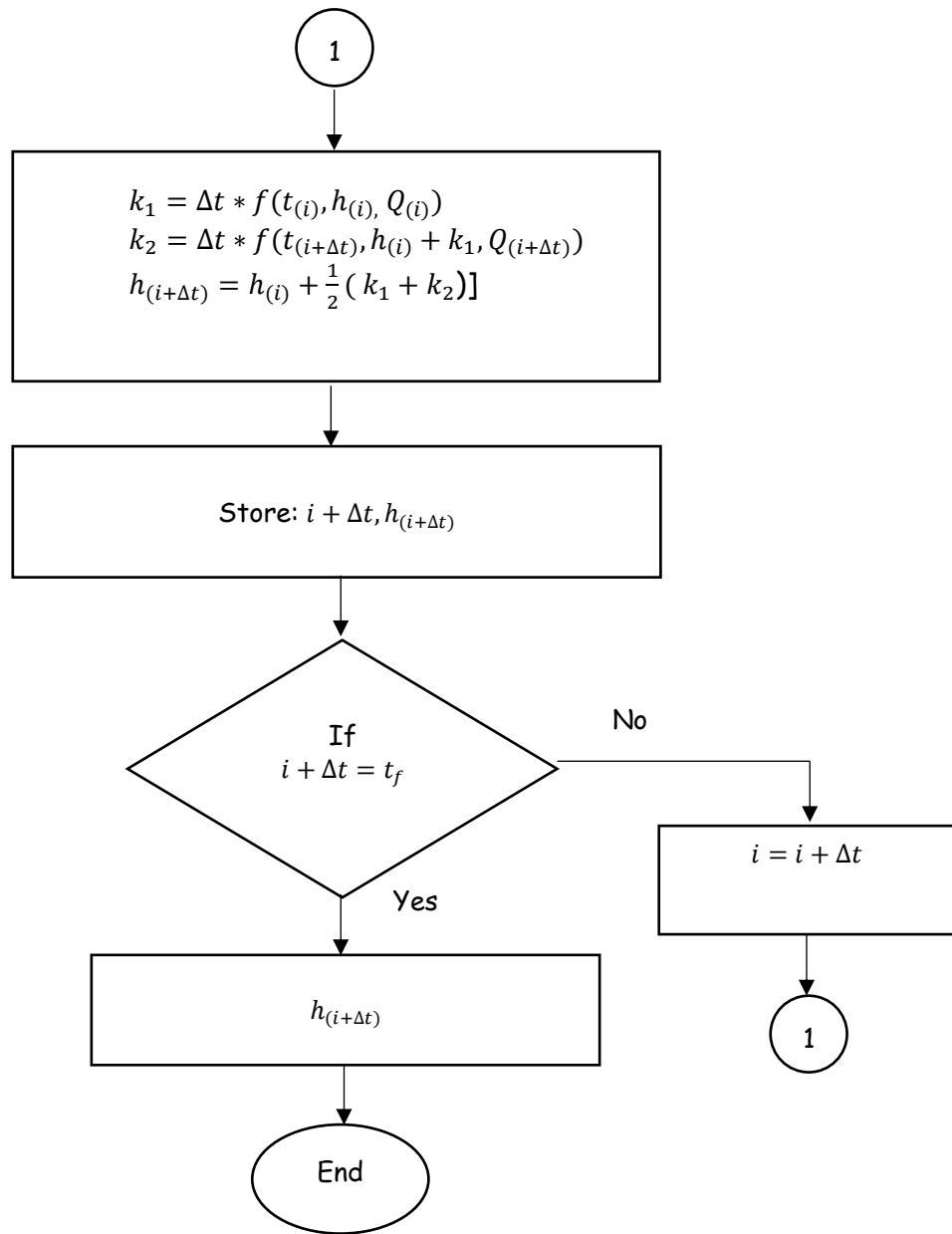


Figure 3.3: Flow Chart Numerical Solution of the Head Equation using Modified Euler Technique

3.3.2 Numerical Solution of the Head Equation using Runge - Kutta Technique

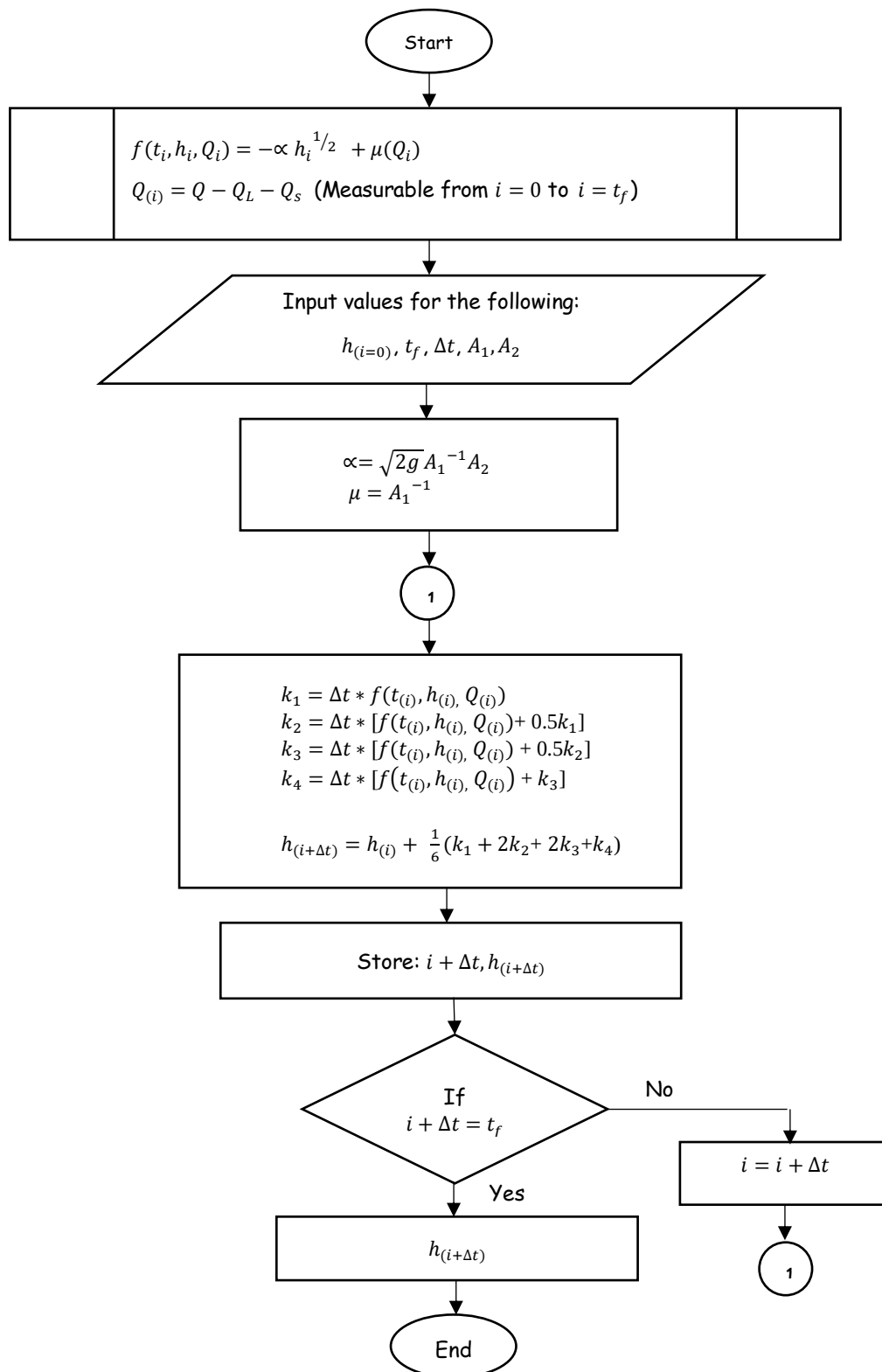
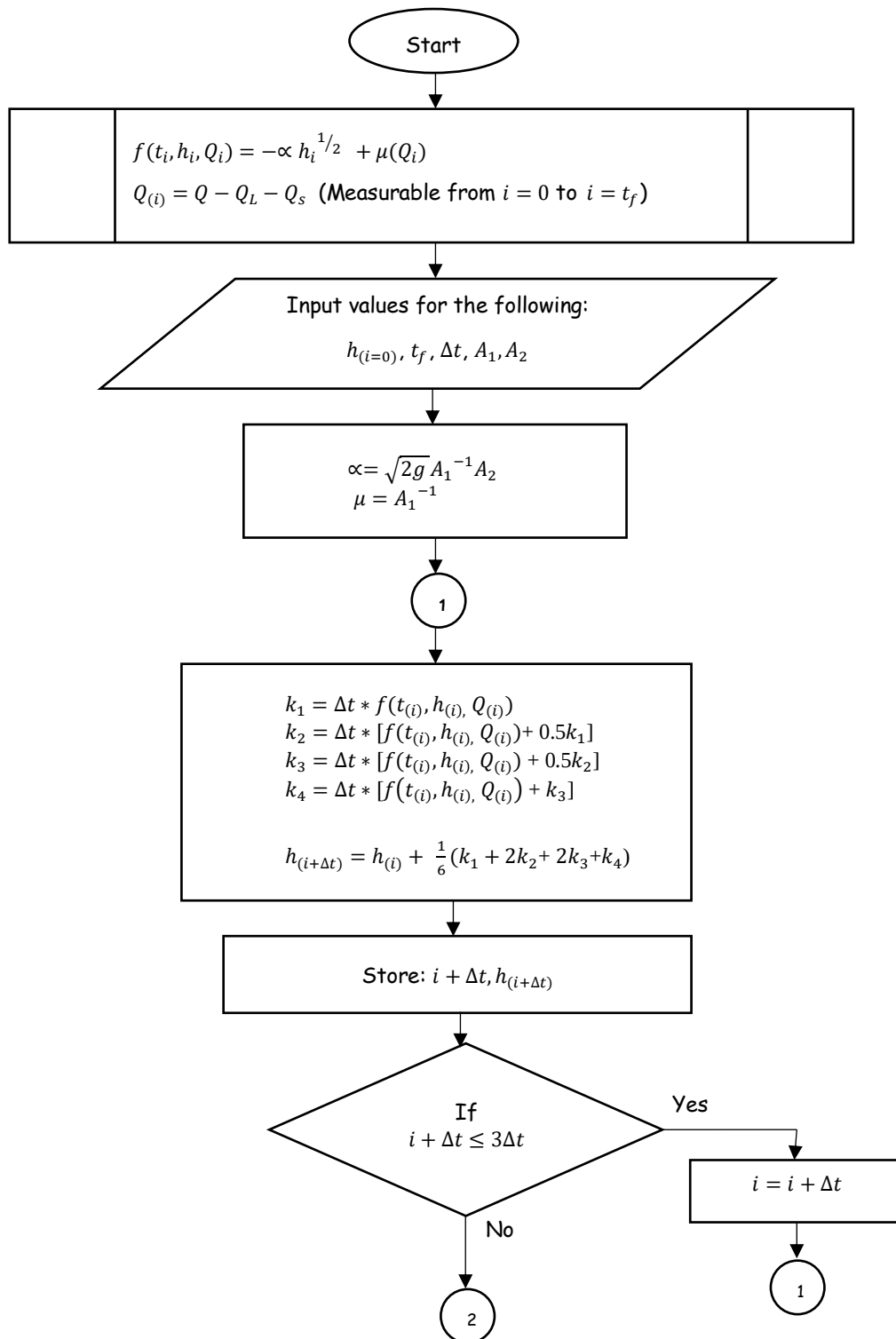


Figure 3.4: Flow Chart Numerical Solution of the Head Equation using Runge - Kutta Technique

3.3.3 Numerical Solution of the Head Equation using the Adams-Moulton Technique



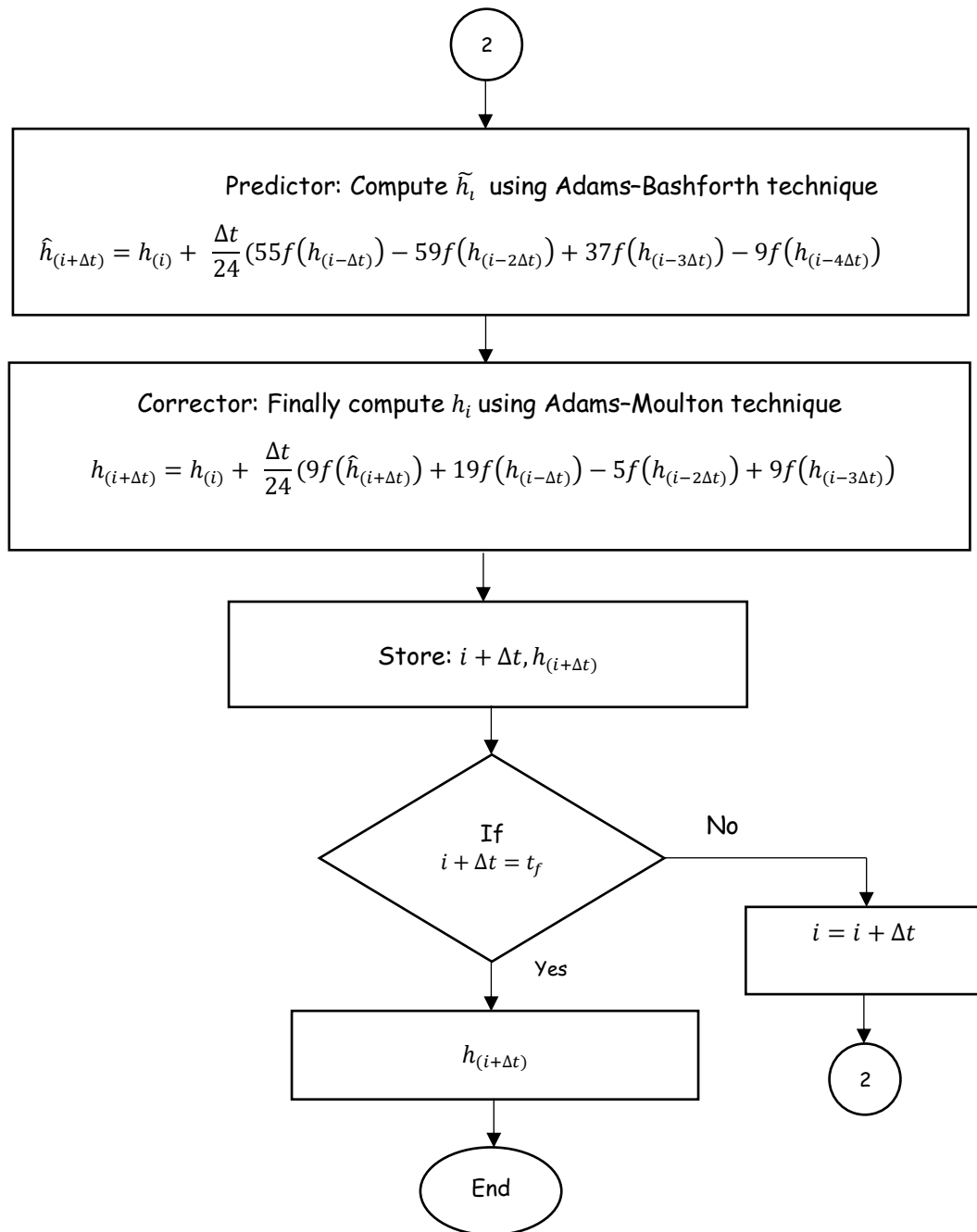


Figure 3.5: Flow Chart Numerical Solution of the Head Equation using Adams-Moulton Technique

3.3.4 Comparison of Numerical Solution of the Nonlinear Head Equation

As mentioned earlier, three numerical techniques were used to integrate the developed model. The three methods were Modified Euler, Runge-Kutta and Adams-Moulton Numerical Techniques. The results are presented and compared Figure 3.6 to 3.7.

Given that:

$$h_0 = 10 \text{ m}, \Delta t = 0.5 \text{ s}, A_1 = 1000 \text{ m}^2, A_2 = 5 \text{ m}^2, Q = Q_L = Q_s = 0 \text{ m}^3/\text{s}$$

In Figure 3.6, both Modified Euler and Runge-Kutta numerical techniques have the potentials of integrating the nonlinear model and they are both stable. Nevertheless, Runge-Kutta technique converges faster than the modified Euler technique. The shortcoming of Runge Kutta is that it is a multi-step method and four intermediate points must be determined before taking another time step.

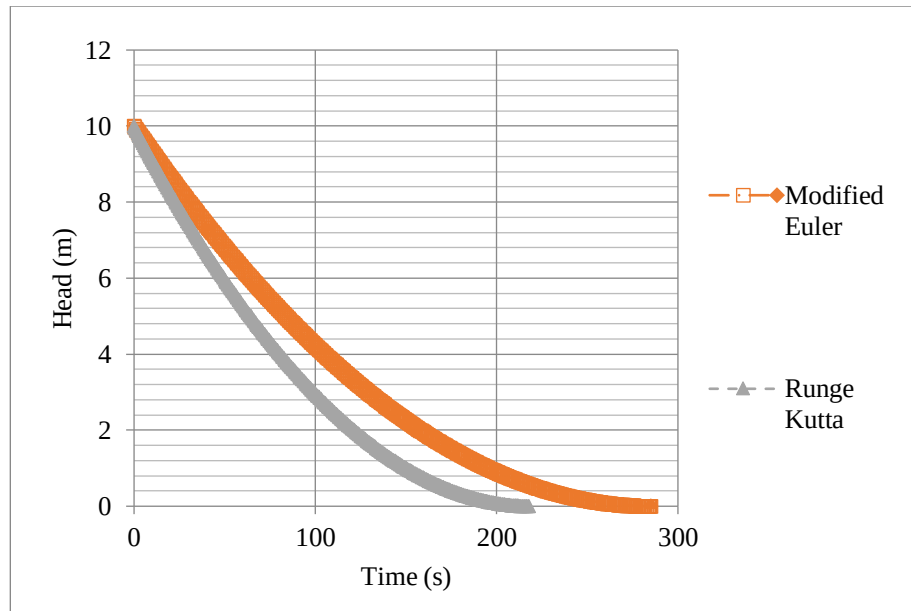


Figure 3.6: Comparison of the Numerical Solution of the Resulting Nonlinear Model Using Modified Euler and Runge-Kutta Numerical Techniques

The comparison of the performance of Runge-Kutta and Adams-Moulton numerical techniques is presented in Figure 3.7. The two multi-step techniques yielded stable and relatively equal results, but Adams-Moulton method is less computationally intensive as Runge-Kutta technique. Instead of calculating four intermediate values, solutions at four previous time steps are used to solve for a new time step.

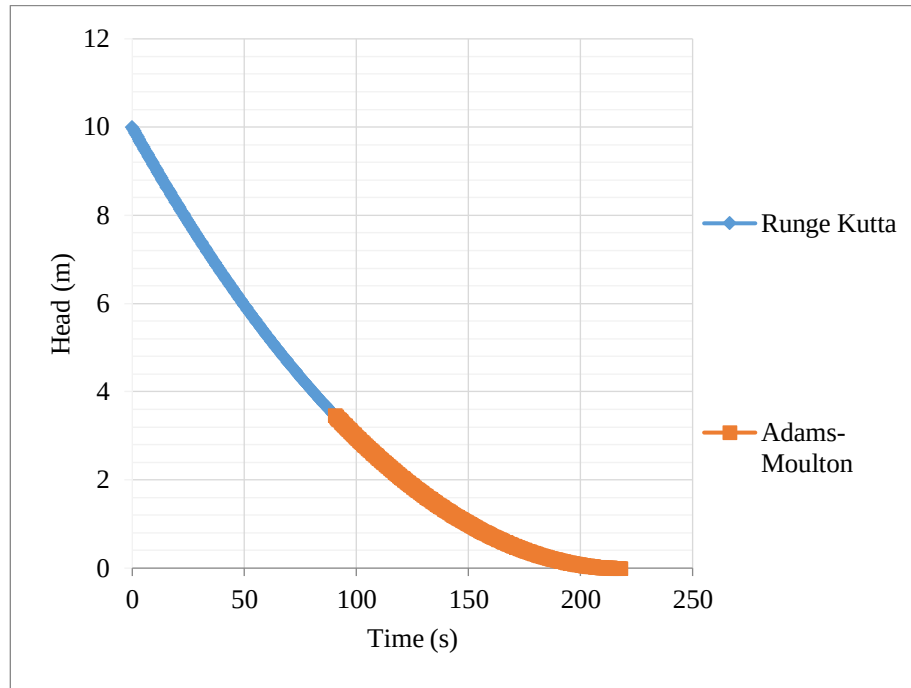


Figure 3.7: Comparison of the Numerical Solution of the Resulting Nonlinear Model Using Runge-Kutta and Adams-Moulton Numerical Techniques

The Adams-Moulton technique was chosen as the integrator in this work because it gives an accurate result like Runge-Kutta technique and only one intermediate step need be computed instead of four steps in Runge-Kutta. The only disadvantage is that it is not self-starting; hence it must be started by another technique or sufficient initial conditions are provided.

3.4 Model Calibration and Tuning Using Observations

While the model equations provide a framework for relating forces, dimensions and inputs to give a dynamical representation, the actual values that best relate the model output must be determined in a calibration / tuning process. The model format already takes care of the role of forces in the dynamics leaving the dimensions and inputs/ outputs.

As a result, the calibration process can be rationally prosecuted as two independent processes. It is noteworthy that physical dimensions do not affect the inputs/ outputs thus allowing for the separation of the calibration into two stages. The next section addresses the determination of the effective reservoir cross sectional area given observed and deduced inputs.

While certain parameters of the system are measurable, others such as the cross-sectional area of the reservoir and scroll casing are not measurable. Their values change with the head and opening of the scroll casing. To determine $\alpha = \sqrt{2g} A_1^{-1} A_2$ and $\mu = A_1^{-1}$, the two cross-sectional areas A_1 and A_2 must be estimated from observation for tuning the model.

3.4.1 Estimation of Effective Surface Area (A1) of the Reservoir

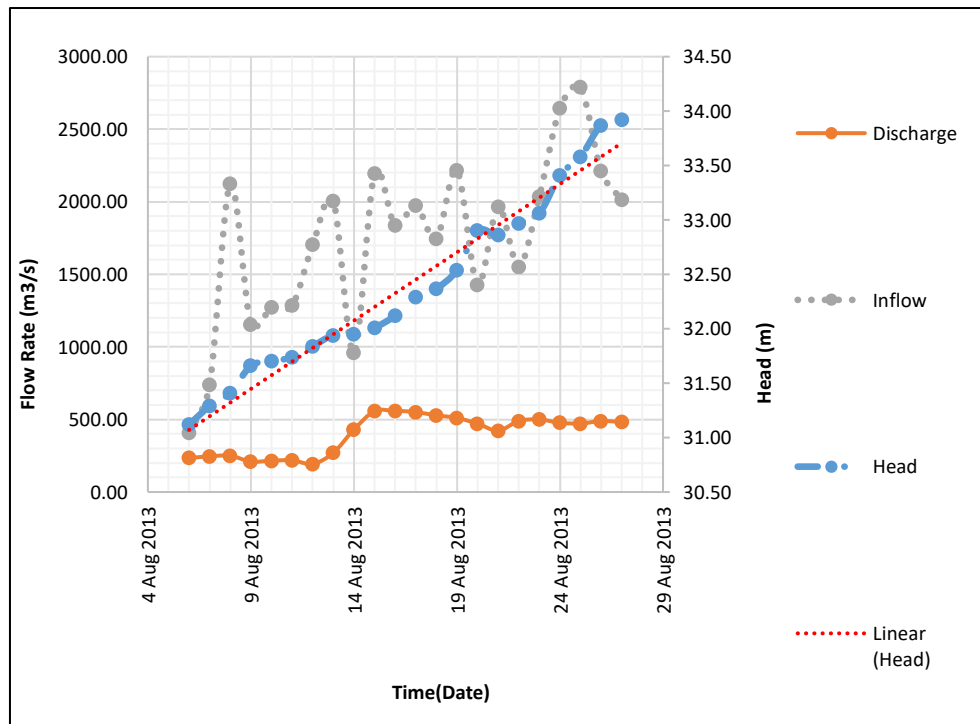
If observations of the inflow and head are studied such that a section of time where the behavior of the system is almost linear is selected, then;

$$Volume = A_1 * \Delta h = \int_{t_1}^{t_n} (Q_{(t)} - q_{(t)}) dt \quad (3.24)$$

$$A_1 = \frac{1}{\Delta h} \int_{t_1}^{t_n} (Q_{(t)} - q_{(t)}) dt \quad (3.25)$$

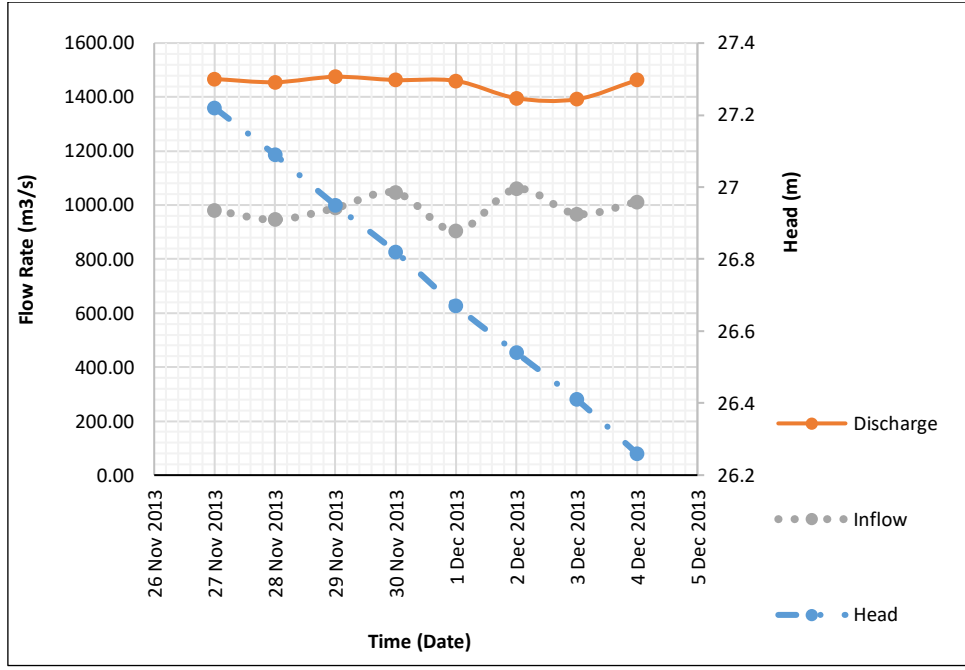
Where is $Q_{(t)}$ is the net inflow in (m^3/s) , $q_{(t)}$ is the discharge in (m^3/s) and Δh is the change in head between time t_1 and t_n .

Observations from KHEPS and JHEPS were studied and a section where the reservoirs head were almost linear for a given time was selected. The inflow, discharge and the head within the time frame were plotted in Figure 3.8 and 3.9. Equation 3.25 was solved to determine the effective surface area for KHEPS and JHEPS. It should be noted that the surface area varies with the head but the variation from the calculated effective value will be very minimal around the operating range of each station. The effective surface area of KHEPS was estimated to be $A_{1k} = 883,208,571.43 m^2$ and that of JHEPS was estimated to be $A_{1j} = 287,775,000 m^2$.



$$A_{1k} = 883,208,571.43 m^2$$

Figure 3.8: Estimation of Effective Surface Area for KHEPS



$$A_{1j} = 287,775,000 \text{ m}^2$$

Figure 3.9: Estimation of the Effective Surface Area for JHEPS

3.4.2 Estimation of Effective Area of the Scroll Casing

The effective area of the scroll casing can be estimated from the observation. This is motivated by the fact that

$$q_i = A_{2i} n_i \sqrt{2gh_i}, \quad (3.26)$$

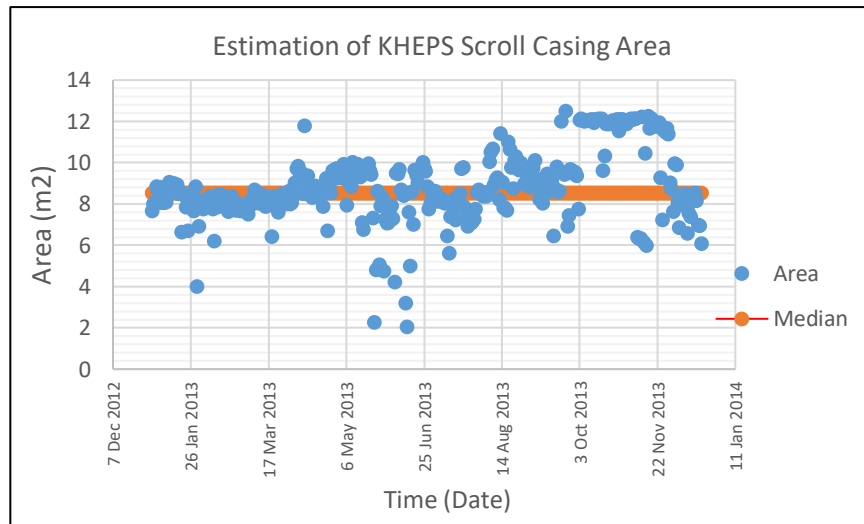
for $i = 1, 2, 3, \dots, 365$

Hence given the observation for a whole year, the median value of A_{21} can be used as A_2 in the model.

$$A_{2i} = \frac{1}{n_i \sqrt{2gh_i}} q_i, \quad (3.27)$$

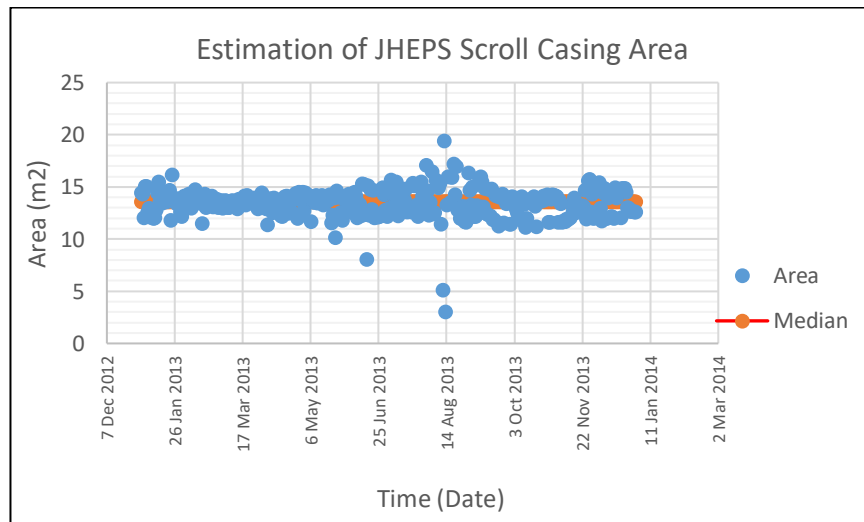
Where n_i represents the number of operating units on day i , q_i is the total station discharge on day i .

In estimating the effective area of the scroll casing (A_2) for KHEPS and JHEPS, the area calculated per day (using equation 3.27) for from 1st of Jan. to 31st of Dec. 2013. The calculated values were plotted against time as shown in Figure 3.10 and 3.11. The median in each case was taken as A_{2k} and A_{2j} .



$$A_{2k} = 8.55005 \text{ m}^2$$

Figure 3.10: Estimation of Effective Area of the Scroll Casing for KHEPS

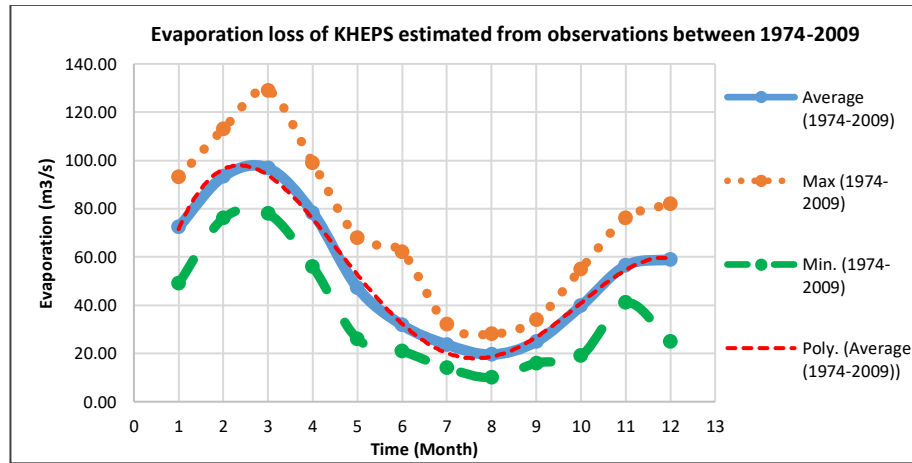


$$A_{2j} = 13.53289 \text{ m}^2$$

Figure 3.11: Estimation of Effective Area of the Scroll Casing for JHEPS

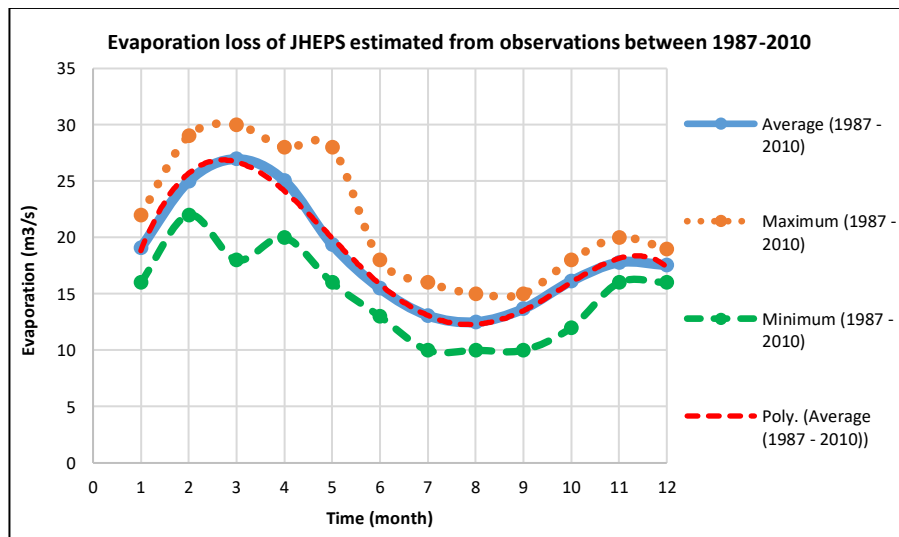
3.4.3 Estimation of Effective Evaporation Loss for KHEPS and JHEPS

As mentioned in Section 3.4, evaporation loss used in this model was estimated from observations between 1974 to 2009 for KHEPS and 1985-2010 for JHEPS. Monthly maximum, minimum and average evaporation loss were plotted in each case. The average value was mathematically modelled as presented in equation 4. 1 and 4.2.



$$Q_{evp,K} = 0.0003t^6 - 0.0126t^5 + 0.0845t^4 + 1.8323t^3 - 25.699t^2 + 88.123t + 7.267 \quad (4.1)$$

Figure 3.12: Estimation of Evaporation Loss for KHEPS



$$Q_{evp,J} = 0.0003t^6 - 0.0126t^5 + 0.0845t^4 + 1.8323t^3 - 25.699t^2 + 88.123t + 7.267 \quad (4.2)$$

Figure 3.13: Estimation of Evaporation Loss for JHEPS

3.5 Optimal Control of the Cascaded KHEPS and JHEPS

The control problems associated with the hydropower stations involve a spectrum of options. In this work, the static problems that are closely related to the design of the systems have already been solved, so attention will be focused on the operational problems that can make the overall performance better.

Operational problems may be formulated in different ways. As indicated above, the classical control problem with the feedback loop may be adopted with techniques that can cope with nonlinearities as is the case in this problem. A whole family of problems can however be formulated to directly address performance related criteria, and these may be cast in an optimal control setting. In this work, the single most difficult problem encountered by the plant operators occurs in the management of water flows between the two reservoirs. Experience from years of operations seems to support this approach given the fact that a necessary and sufficient condition for the proper operation of the turbo-alternators is that the operating net head must satisfy the requirements for acceptable energy conversion by the turbine.

For example, in KHEPS the performance of the Kaplan turbines provided with elaborate governors in the runners has resulted in catastrophic failures whilst the two fixed vane Francis turbines that are not equipped with variable head accessories have suffered one catastrophic loss and the surviving plant operates well only when the head is compatible with its configuration.

At JHEPS, the turbines were selected to be fixed vane Kaplan that operate well within a narrow head range. This requires that the reservoir water levels must be maintained

within relatively tight values. This is the main problem that will be addressed in this work – i.e. to determine the appropriate control so that the head waters at the Jebba reservoir remain within a very tight range of the specified desirable head.

Having developed a proper model for the dynamical system and a means of solving the resulting equations, this stage presents the optimal control. Optimal control is the determination of the best control vector (out of possible vector) and resulting state trajectories which minimize certain performance index, subject to system constraints. Numerous optimal control problems can exist on the cascaded system depending on the type of system (linear or nonlinear), performance index, time domain (discrete or continuous), nature of constraints and the boundary condition on states.

3.5.1 Optimal Control Problems Solved in this Research

The optimal control of the cascaded KHEPS and JHEPS devolves into the determination of best control vector (optimal control) $u(t) \in U(t)$ from KHEPS, which compels the dynamical system $\dot{h}(t) = f(h(t), u(t), t)$ at JHEPS to follow optimal trajectories $h^*(t)$ that minimize specified performance indices. The system is nonlinear and exists in the continuous time domain. The optimal control problem to be solved is the Lagrange problem. The block diagram of the proposed control system is as shown in Figure 3.14.

In solving most optimal control problem, the solutions cannot be found by analytical methods, the engineers depend on numerical procedures. The numerical solution can be by direct or indirect methods. In this research two direct and two indirect methods have been used.

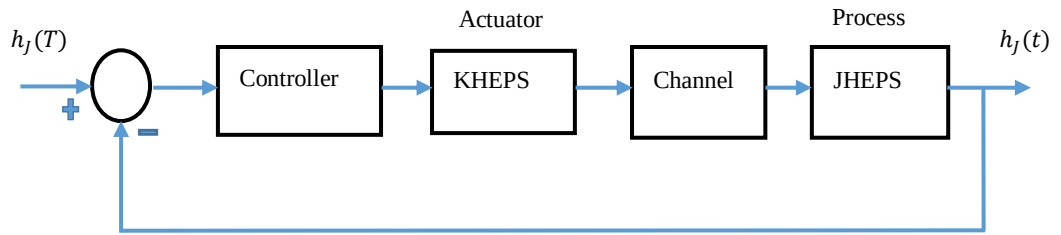


Figure 3.14: Block Diagram of the Control System for the Cascaded KHEPS and JHEPS

3.5.2 Numerical Solution of the Optimal Control

The numerical optimal control methods that were used in this research are presented in Figure 3.17. The steepest descent and conjugate gradient approach were used as direct methods of solutions while the innovated progressive domain extension and conjugate gradient methods were used to indirectly obtain the optimal control.

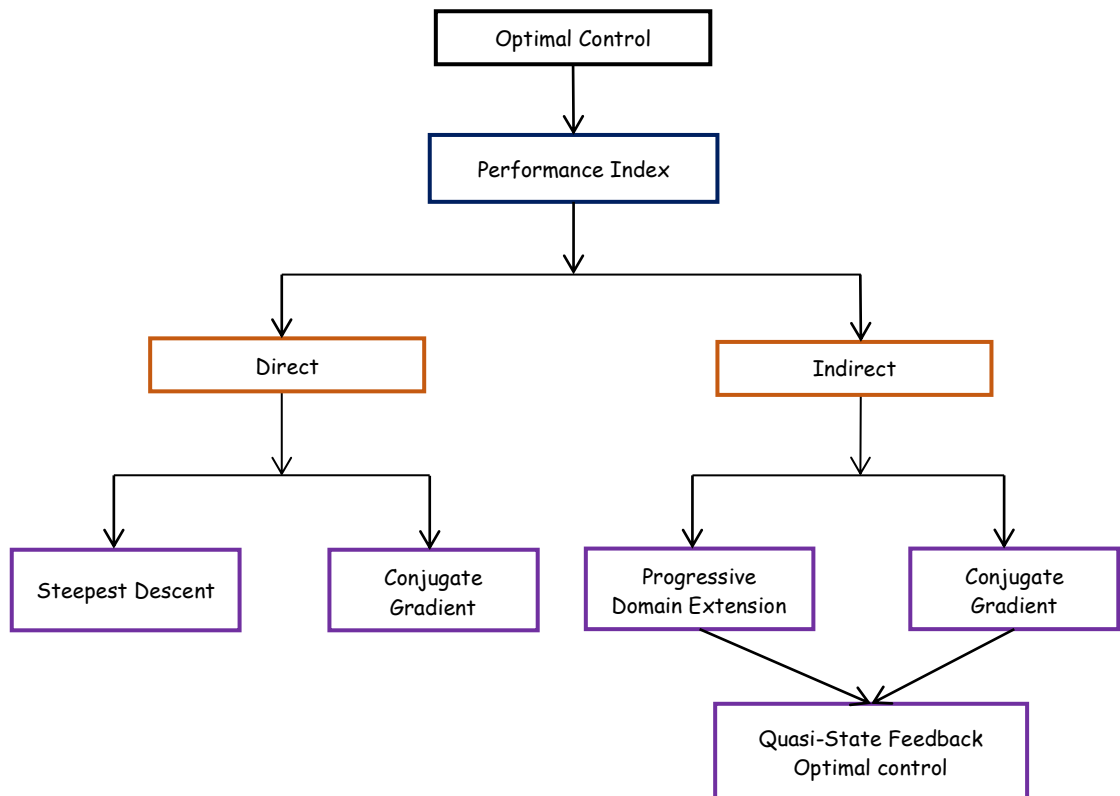


Figure 3.15: Numerical Solution of the Optimal Control for the Cascaded KHEPS and JHEPS

3.5.3 Direct Optimal Control of the Cascaded KHEPS and JHEPS

In general let $u \in \mathcal{L}^2(0,T)$ in the optimal control process to be implemented. This implies that the integral of admissible control squared must be bounded. Numerous possibilities can be adopted as forms for the representation of the control. While this may be attractive, it was decided to determine an approximation to the optimal control using a finite dimensional approximation derived as follows.

Let a control vector $U^{(k)}(t)$ be defined by introducing a finite partition of the time interval $[t_0, t_f]$ such that a family of controls can be associated with each corresponding interval in the partition. This structure can be used to define approximations to the actual controls.

$$\pi[t_0, t_f] = t_0 < t_1 < t_2 < t_3 \dots < t_N = t_f \quad (3.28)$$

$$U(t) = [u_1(t), u_2(t), u_3(t), \dots, u_f(t)] \quad (3.29)$$

$$= \sum_{k=0}^{N-1} u_k [w(t - t_k) - w(t - t_{k+1})] \quad (3.30)$$

Where $w(t)$ is the unit step function starting at $t = 0$.

Under the above provision, the partitioned control will converge to the actual control as $N \rightarrow \infty$. This approximation is what makes it feasible and tractable to determine an optimal control by direct optimization which otherwise poses nearly computationally unsurmountable difficulties in deriving numerical values for derivatives and second derivatives without which optimization may not be possible.

Given the nature of the HEPS problem, various values of N can be selected. However, the massive nature of the systems involved seemed to suggest that values of N such that $1 < N < 24$ will yield more realizable answers to the problem. As a result, it was decided to choose a time frame T of 24 hrs. and a 4-partition representation where the controls in each interval will be assumed to be a constant. This implies that the functional problem can then be treated as a 4-dimensional control problem with a performance index that is functional that reflects deviations from the desired optimum and the system's dynamical equations as its constraints.

Performance Index

Performance indices can be selected to reflect the aspect of the system's behaviour that is considered as vital to the performance. In the most general form, the JHEPS problem might have been formulated as a minimum time problem to reach the desired head constrained so that the flow is constrained to fall within the maximum allowable discharge from KHEPS. In that case, the optimal control problem can be expressed as follows:

$$\min J(h, u, t) = T \quad (3.31)$$

$$\dot{h}(t) = f(h(t), u(t), t) ; t_0 \leq t$$

$$h(t_0) = h_0$$

$$u(t) \leq u_{max} \quad (3.32)$$

While this formulation can be useful for start-up and may be used to determine the minimum time for the reservoir in JHEPS to reach operational conditions, it is not very

useful for continuous operations. As a result, a performance index which accommodates and appropriately penalizes deviation from a specified head and ensures that the control does not require values outside the capability of KHEPS was selected. A quadratic performance index was selected consisting of the integral of the square error from the desired operational head and the square deviation from the maximum discharge possible.

$$J = \min \int_{t_0}^{t_f} \left(K_h (h(t) - h(T))^2 \right) dt \quad (3.33)$$

Subject to the system constraints:

$$\dot{h}(t) = \mathbf{f}(h(t), u(t), t) ; t_0 \leq t \leq t_f$$

$$h(t_0) = h_0$$

$$h(t_f) = h(T)$$

Where $h(T)$ represents the desired final value for the state and K_h is a positive weighing scalar constant.

3.5.3.1 Numerical Solution of the Optimal Control using Steepest Descent Algorithm

Solution of the optimal control using steepest descent approach was discussed in chapter 2. It was established that

$$\mathbf{z}(U_{(t)}^k) = \frac{\left[\frac{\partial J^k}{\partial U^k(t)} \right]^T}{\left\| \frac{\partial J^k}{\partial U^k(t)} \right\|} \quad (3.34)$$

$$U_{\psi}^{k+1}(t) = U^k(t) - \psi^k \mathbf{z}(U^k(t)) \quad (3.35)$$

An innovative approach to finding the optimum ψ^k is by using the quadratic approach.

$$J_{(\psi)} = \mathbf{a} + \mathbf{b}\psi + \mathbf{c}\psi^2 \quad (3.36)$$

$$\frac{dJ}{d\psi_{(min)}^k} = 2\mathbf{c}\psi + \mathbf{b} = 0 \quad (3.37)$$

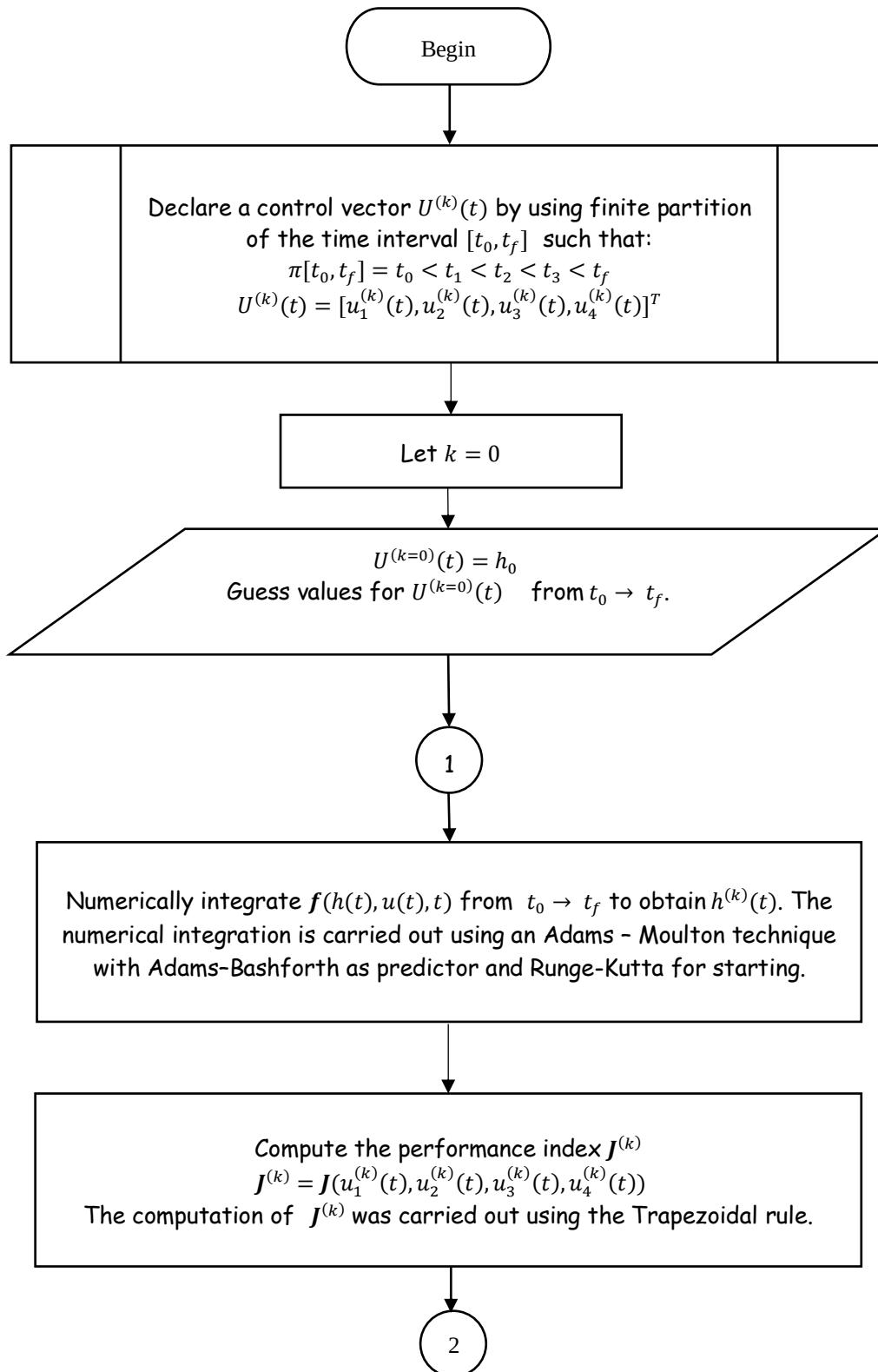
$$\psi_{(min)}^k = -\frac{\mathbf{b}}{2\mathbf{c}} \quad (3.38)$$

$$\begin{bmatrix} \mathbf{b} \\ \mathbf{c} \end{bmatrix} = \begin{bmatrix} (\psi_1 - \psi_2) & (\psi_1^2 - \psi_2^2) \\ (\psi_2 - \psi_3) & (\psi_2^2 - \psi_3^2) \end{bmatrix}^{-1} \begin{bmatrix} J_{(\psi_1)} - J_{(\psi_2)} \\ J_{(\psi_2)} - J_{(\psi_3)} \end{bmatrix} \quad (3.39)$$

Hence,

$$[U^{k+1}(t)]^T = [U^k(t)]^T - \psi_{(min)}^k \frac{\left[\frac{\partial J^k}{\partial u_1^k(t)}, \frac{\partial J^k}{\partial u_2^k(t)}, \frac{\partial J^k}{\partial u_3^k(t)}, \frac{\partial J^k}{\partial u_4^k(t)} \right]^T}{\sqrt{\left(\frac{\partial J^k}{\partial u_1^k(t)} \right)^2 + \left(\frac{\partial J^k}{\partial u_2^k(t)} \right)^2 + \left(\frac{\partial J^k}{\partial u_3^k(t)} \right)^2 + \left(\frac{\partial J^k}{\partial u_4^k(t)} \right)^2}} \quad (3.40)$$

3.5.3.2 Flow Chart of the Numerical Solution of the Optimal Control using Steepest Descent Algorithm



2

Select a perturbation value Δu and compute $J_m^{(k)}$, $m=1, 2, 3, 4$

$$J_1^{(k)} = J(u_1^{(k)}(t) + \Delta u, u_2^{(k)}(t), u_3^{(k)}(t), u_4^{(k)}(t))$$

$$J_2^{(k)} = J(u_1^{(k)}(t), u_2^{(k)}(t) + \Delta u, u_3^{(k)}(t), u_4^{(k)}(t))$$

$$J_3^{(k)} = J(u_1^{(k)}(t), u_2^{(k)}(t), u_3^{(k)}(t) + \Delta u, u_4^{(k)}(t))$$

$$J_4^{(k)} = J(u_1^{(k)}(t), u_2^{(k)}(t), u_3^{(k)}(t), u_4^{(k)}(t) + \Delta u)$$

Compute the approximate gradient vector ∇J .

$$\nabla J = \frac{\partial J_m^k}{\partial u_m} \approx \frac{J^k - J_m^k}{\Delta u}; m = 1, 2, 3, 4$$

Compute the norm of the gradient $\|\nabla J\|$.

$$\left\| \frac{\partial J^{(k)}}{\partial U^{(k)}(t)} \right\| = \sqrt{\left(\frac{\partial J^{(k)}}{\partial u_1^{(k)}(t)} \right)^2 + \left(\frac{\partial J^{(k)}}{\partial u_2^{(k)}(t)} \right)^2 + \left(\frac{\partial J^{(k)}}{\partial u_3^{(k)}(t)} \right)^2 + \left(\frac{\partial J^{(k)}}{\partial u_4^{(k)}(t)} \right)^2}$$

Compute the unit vector $z(U^{(k)}(t))$

$$z(U^{(k)}(t)) = \frac{\left[\frac{\partial J^{(k)}}{\partial U^{(k)}(t)} \right]^T}{\left\| \frac{\partial J^{(k)}}{\partial U^{(k)}(t)} \right\|}$$

3

3

Determine the optimum step $\psi_{min}^{(k)}$

Select a set of three Fibonacci numbers ψ_1, ψ_2 and ψ_3

$$U_{\psi_1}^{(k+1)}(t) = U^{(k)}(t) - \psi_1^{(k)} \mathbf{z} \left(U^{(k)}(t) \right) \text{ and } J_{\psi_1}^{(k)}$$

$$U_{\psi_2}^{(k+1)}(t) = U^{(k)}(t) - \psi_2^{(k)} \mathbf{z} \left(U^{(k)}(t) \right) \text{ and } J_{\psi_2}^{(k)}$$

$$U_{\psi_3}^{(k+1)}(t) = U^{(k)}(t) - \psi_3^{(k)} \mathbf{z} \left(U^{(k)}(t) \right) \text{ and } J_{\psi_3}^{(k)}$$

Solve for the constants b and c

$$\begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} (\psi_1 - \psi_2) & (\psi_1^2 - \psi_2^2) \\ (\psi_2 - \psi_3) & (\psi_2^2 - \psi_3^2) \end{bmatrix}^{-1} \begin{bmatrix} (J_{(\psi_1)} - J_{(\psi_2)}) \\ (J_{(\psi_2)} - J_{(\psi_3)}) \end{bmatrix}$$

Compute $\psi_{(min)}$

$$\psi_{(mn)}^{(k)} = -\frac{b}{2c}$$

$$[U^{(k+1)}(t)]^T = [U^{(k)}(t)]^T - \psi_{(mn)}^{(k)} \frac{\left[\frac{\partial J^{(k)}}{\partial u_1^{(k)}(t)}, \frac{\partial J^{(k)}}{\partial u_2^{(k)}(t)}, \frac{\partial J^{(k)}}{\partial u_3^{(k)}(t)}, \frac{\partial J^{(k)}}{\partial u_4^{(k)}(t)} \right]^T}{\sqrt{\left(\frac{\partial J^{(k)}}{\partial u_1^{(k)}(t)} \right)^2 + \left(\frac{\partial J^{(k)}}{\partial u_2^{(k)}(t)} \right)^2 + \left(\frac{\partial J^{(k)}}{\partial u_3^{(k)}(t)} \right)^2 + \left(\frac{\partial J^{(k)}}{\partial u_4^{(k)}(t)} \right)^2}}$$

4

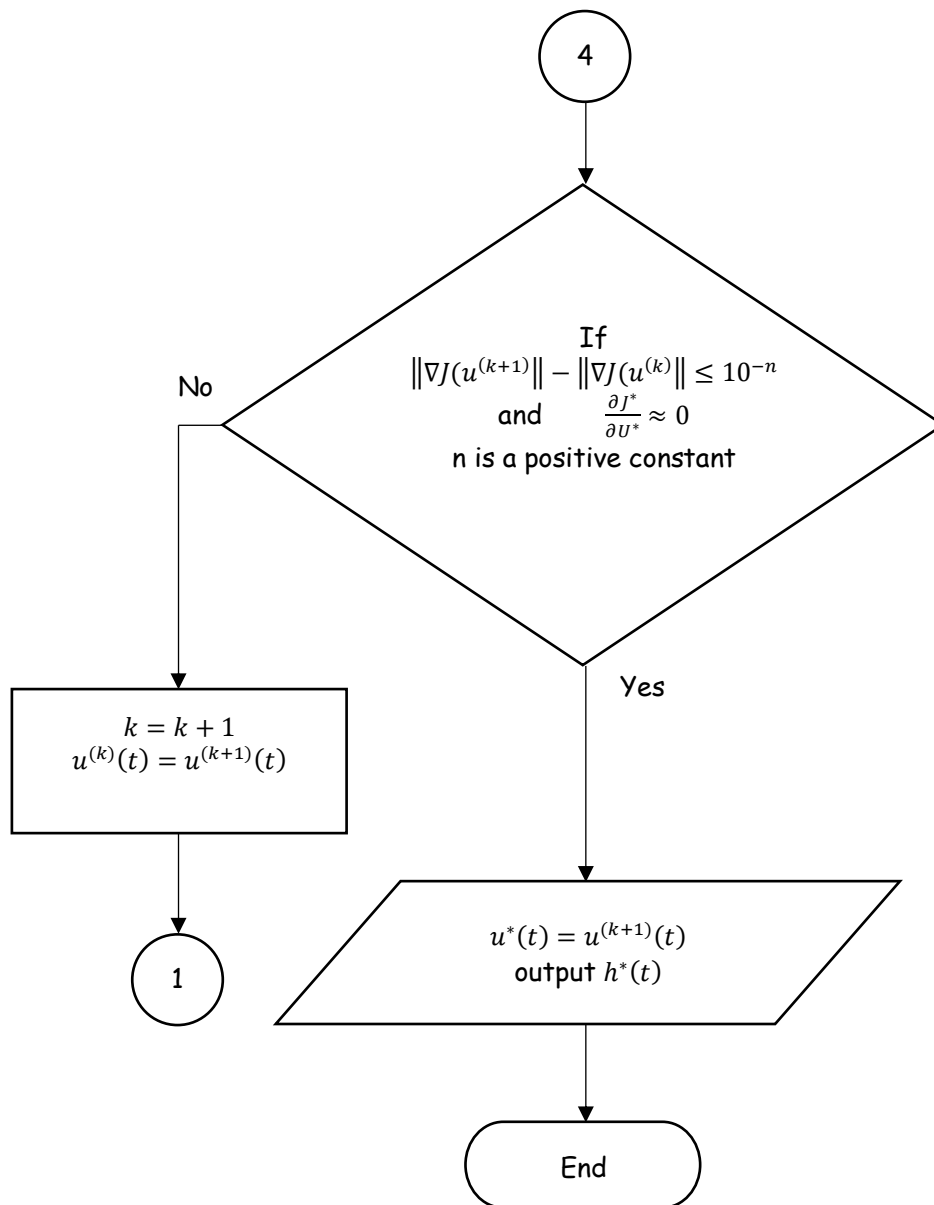


Figure 3.16: Flow Chart for the Numerical Solution of the Optimal Control using Steepest Descent Method

3.5.3.3 Numerical Solution of Direct Optimal Control Problem using a Conjugate

Gradient Algorithm

The procedure starts by guessing an initial control vector $U^{(k)}(t)$ and determine the performance index $J(U^{(k)})$ with the iteration number $k = 0$. Calculate the gradient vector $g^{(k)} = \nabla J$ at $U^{(k)}$. Determine the conjugate gradient parameter $\beta^{(k)}$:

$$\beta^{(k)} = \frac{\int_{t_0}^{t_f} (g^{(k)})^2 dt}{\int_{t_0}^{t_f} (g^{(k-1)})^2 dt} \quad (3.41)$$

$$g^{(k-1)} = 1 \text{ when } k = 0$$

Calculate the direction of search vector s^k .

$$s^k = -g^k + \beta^k s^{k-1} \quad (3.42)$$

$$s^{(k-1)} = 0 \text{ when } k = 0$$

Obtain the optimum step size ψ_{min} by performing a minimization search on $J(U^{(k)} + \psi s^k)$. This procedure remains the same as presented in section 3.5.3.1

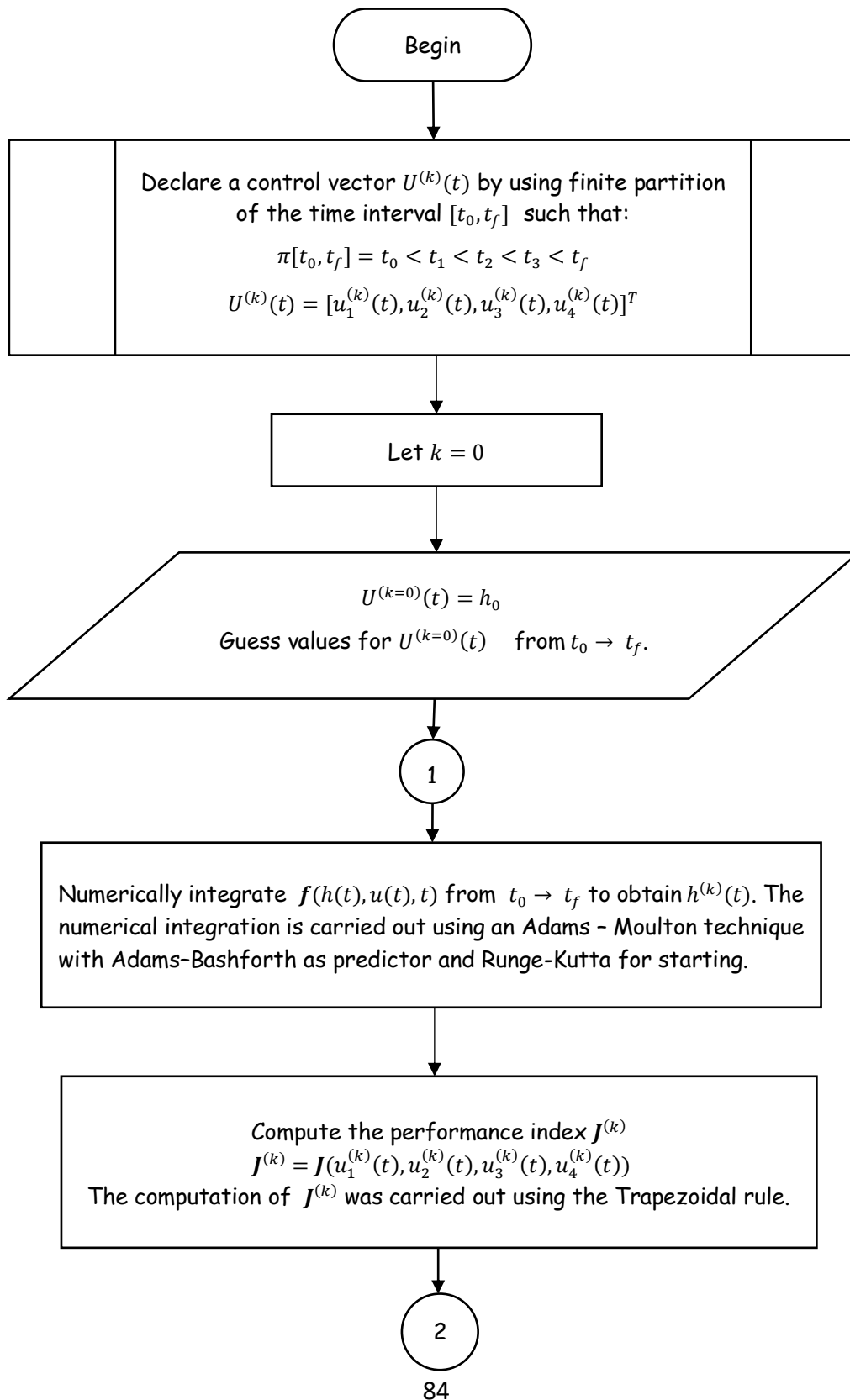
Determine the next control vector $U^{(k+1)}(t)$.

$$[U^{(k+1)}(t)]^T = [U^{(k)}(t)]^T + \psi_{min} [s^k]^T \quad (3.43)$$

Check whether $|J(U^{k+1}) - J(U^k)| \leq 10^{-n}$ to stop the iteration, where n is a constant.

If the stopping criterion is not met, increment k by one and repeat the procedure using $U^{(k+1)}$ as the new guess.

3.5.3.4 Flow Chart for the Numerical Solution of the Direct Optimal Control using Conjugate Gradient Algorithm



2

Select a perturbation value Δu and compute $J_m^{(k)}$, $m=1, 2, 3, 4$

$$J_1^{(k)} = J(u_1^{(k)}(t) + \Delta u, u_2^{(k)}(t), u_3^{(k)}(t), u_4^{(k)}(t))$$

$$J_2^{(k)} = J(u_1^{(k)}(t), u_2^{(k)}(t) + \Delta u, u_3^{(k)}(t), u_4^{(k)}(t))$$

$$J_3^{(k)} = J(u_1^{(k)}(t), u_2^{(k)}(t), u_3^{(k)}(t) + \Delta u, u_4^{(k)}(t))$$

$$J_4^{(k)} = J(u_1^{(k)}(t), u_2^{(k)}(t), u_3^{(k)}(t), u_4^{(k)}(t) + \Delta u)$$

Compute the gradient vector $\mathbf{g}_m^{(k)} = \nabla J_m^{(k)}$

$$\mathbf{g}^{(k)} = [g_1^{(k)}, g_2^{(k)}, g_3^{(k)}, g_4^{(k)}]^T$$

$$g_m^{(k)} = \frac{J^{(k)} - J_m^{(k)}}{\Delta u}; m = 1, 2, 3, 4$$

Compute the conjugate gradient parameter; $\beta^{(k)}$

$$\beta^{(k)} = \frac{\int_{t_0}^{t_f} (g^{(k)})^2 dt}{\int_{t_0}^{t_f} (g^{(k-1)})^2 dt}$$

Where $\int_{t_0}^{t_f} (g^{(0)})^2 dt = 1$ and numerical integration was employed

Compute the direction of search; $s^{(k)}(t)$

$$s^{(k)} = -g^{(k)} + \beta^{(k)} s^{(k-1)}; s^{(-1)} = 0$$

3

3

Determine the optimum step $\psi_{min}^{(k)}$

Select a set of three Fibonacci numbers ψ_1, ψ_2 and ψ_3

$$U_{\psi_1}^{(k+1)}(t) = U^{(k)}(t) + \psi_1^{(k)} s^{(k)}(t) \text{ and } J_{\psi_1}^{(k)}$$

$$U_{\psi_2}^{(k+1)}(t) = U^{(k)}(t) + \psi_2^{(k)} s^{(k)}(t) \text{ and } J_{\psi_2}^{(k)}$$

$$U_{\psi_3}^{(k+1)}(t) = U^{(k)}(t) + \psi_3^{(k)} s^{(k)}(t) \text{ and } J_{\psi_3}^{(k)}$$

Solve for the constants b and c

$$c = \frac{1}{(\psi_1 - \psi_3)} \left[\frac{(J_{(\psi_1)} - J_{(\psi_2)})}{(\psi_1 - \psi_2)} - \frac{(J_{(\psi_2)} - J_{(\psi_3)})}{(\psi_2 - \psi_3)} \right]$$

$$b = \frac{(J_{(\psi_1)} - J_{(\psi_2)})}{(\psi_1 - \psi_3)} - \frac{(\psi_1 + \psi_2)}{(\psi_1 - \psi_3)} \left[\frac{(J_{(\psi_1)} - J_{(\psi_2)})}{(\psi_1 - \psi_2)} - \frac{(J_{(\psi_2)} - J_{(\psi_3)})}{(\psi_2 - \psi_3)} \right]$$

$$\psi_{(min)}^{(k)} = \psi^{(k)} = -\frac{b}{2c}$$

$$[U^{(k+1)}(t)]^T = [U^{(k)}(t)]^T + \psi_{(mn)}^{(k)} [s^{(k)}]^T$$

4

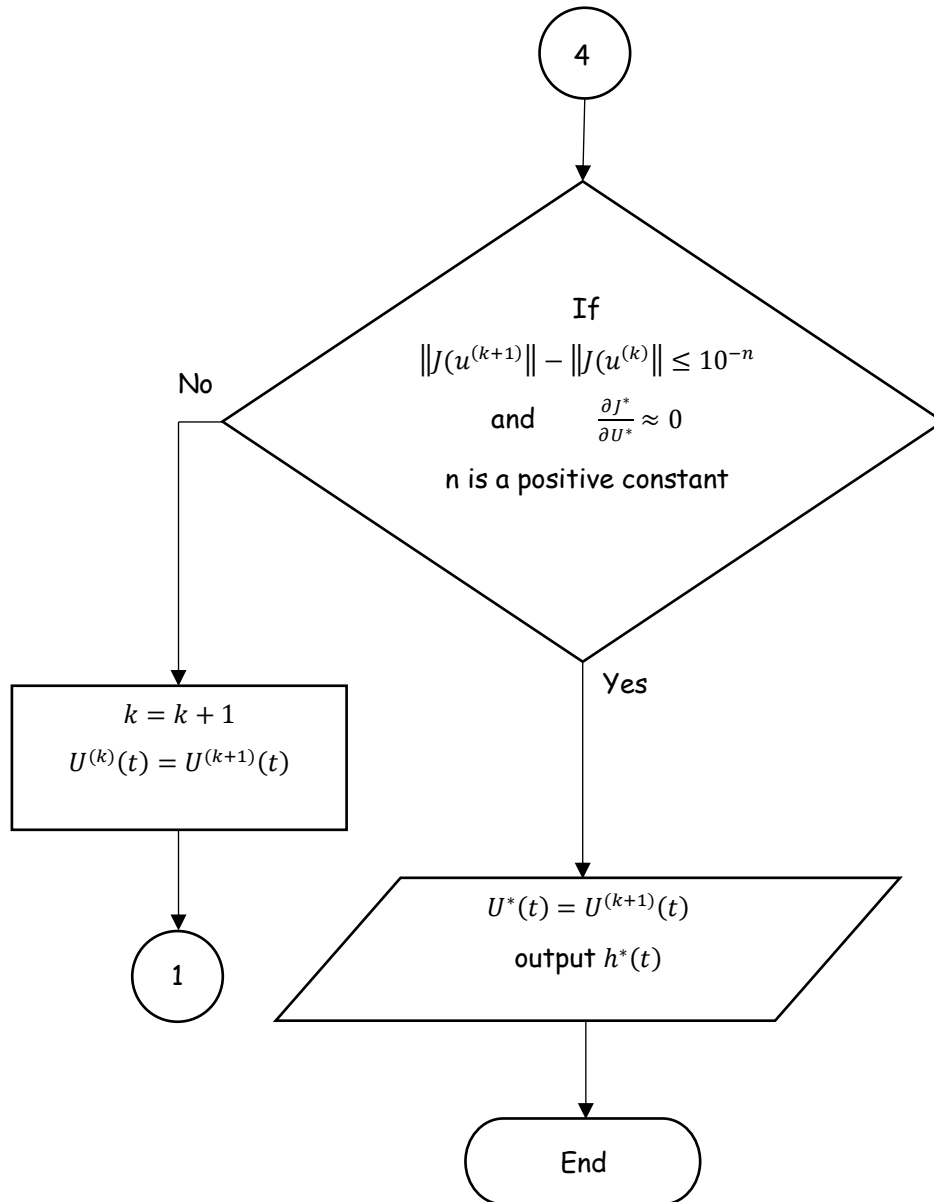


Figure 3.17: Flow Chart for the Numerical Solution of the Optimal Control using Conjugate Gradient Method

3.5.4 Indirect Optimal Control of the Cascaded KHEPS and JHEPS

The indirect approach to optimal control used in this research is based on the use Pontryagin's minimum principle that augments the traditional calculus of variations approach which includes an optimality condition for the control that is usually not part of the basic problem.

The indirect method has an additional requirement that the performance index must include the control variable which is usually not related to the state equations by a dynamical relationship if it is not to be singular. Consequently, the problem to be addressed here adopts a performance index that includes penalties for deviation from a specified state and for deviations from some predefined value of the control variable.

$$\text{Min } J = \min \int_{t_0}^{t_f} \left(K_h (h(t) - h(T))^2 + K_u (u(t) - u(T))^2 \right) dt \quad (3.44)$$

$$\text{Min } J = \min \int_{t_0}^{t_f} \varphi(h(t), u(t), t) dt; \quad (3.45)$$

Subject to:

$$\dot{h}(t) = f(h(t), u(t), t) \quad ; \quad t_0 \leq t \leq t_f$$

$$h(t_0) = h_0$$

$$h(t_f) = h(T)$$

Where φ is a function, K_h and K_u are non-zero positive constants.

Equation (3.45) can be reformulated as the augmented unconstrained optimization problem by introducing a Lagrange multiplier $\lambda(t)$ as follows:

$$J = \int_{t_0}^{t_f} \left(\varphi(h(t), u(t), t) + \lambda(t) \left(f(h(t), u(t), t) - \dot{h}(t) \right) \right) dt \quad (3.46)$$

Applying the method of Pontryagin such a Hamiltonian function $\mathbf{H}$ is defined as:

$$\mathbf{H}(h(t), u(t), \lambda(t), t) = \boldsymbol{\varphi}(h(t), u(t), t) + \lambda(t)\mathbf{f}(h(t), u(t), t) \quad (3.47)$$

$$J = \int_{t_0}^{t_f} \left(\mathbf{H}(h(t), u(t), \lambda(t), t) - \lambda(t)\dot{h}(t) \right) dt \quad (3.48)$$

Applying the Euler-Lagrange Equation to the Hamiltonian gives:

$$\frac{\partial \mathbf{H}(h(t), u(t), \lambda(t), t)}{\partial h(t)} - \frac{d}{dt}[\lambda(t)] = 0$$

$$\dot{\lambda}(t) = - \frac{\partial \mathbf{H}(h(t), u(t), \lambda(t), t)}{\partial h(t)} \quad (3.49)$$

$$\frac{\partial \mathbf{H}(h(t), u(t), \lambda(t), t)}{\partial \lambda(t)} = \mathbf{f}(h(t), u(t), t) - \dot{h}(t) = 0$$

$$\dot{h}(t) = \mathbf{f}(h(t), u(t), t) \quad (3.50)$$

$$\frac{\partial \mathbf{H}(h(t), u(t), \lambda(t), t)}{\partial u(t)} = 0 \quad (3.51)$$

Therefore,

$$\begin{aligned} \mathbf{H}(h(t), u(t), \lambda(t), t) = & K_h(h(t) - h(T))^2 + K_u(u(t) - u(T))^2 \\ & + \lambda(t) \left(-n_j \alpha h^{(1/2)}(t) + \mu u(t) \right) \end{aligned}$$

$$\dot{h}(t) = -n_j \alpha h^{(1/2)}(t) + \mu u(t) \quad (3.52a)$$

$$\dot{\lambda}(t) = -[2K_h(h(t) - h(T)) - \frac{1}{2}\lambda(t) \left(n_j \alpha h^{(-1/2)}(t) \right)] \quad (3.52b)$$

$$0 = 2K_u(u(t) - u(T)) + \mu \lambda(t) \quad (3.52c)$$

$$\left. \begin{aligned} h(t_0) &= h_0 \\ \lambda(t_f) &= 1 \end{aligned} \right\} \text{Boundary condition} \quad (3.52d)$$

From the result above, the determination of the optimal control requires the solution of a two-point boundary value problem (TPBVP) since the initial conditions of the system are specified at the initial time and the value of the Lagrange multiplier or co-state is specified the terminal point. Meanwhile the unknown control is related to the state and co-state through the optimality condition

From equation (3.52c), $u(t)$ can be expressed as;

$$u(t) = -\frac{\mu\lambda(t)}{2K_u} + u(T) \quad (3.53)$$

Substituting equation (3.52c) into equation (3.52a) leads to two sets of first order differential equation with split boundary conditions.

$$\dot{h}(t) = -n_j\alpha h^{(1/2)}(t) - \mu\left(\frac{\mu\lambda(t)}{2K_u} + u(T)\right) \quad (3.54a)$$

$$\dot{\lambda}(t) = -2K_h(h(t) - h(T)) + \frac{1}{2}\lambda(t)\left(n_j\alpha h^{(-1/2)}(t)\right) \quad (3.54b)$$

$$h(t_0) = h_0 \quad \text{and} \quad h(t_f) = \text{unknown}$$

$$\lambda(t_0) = \text{unknown} \quad \text{and} \quad \lambda(t_f) = 1$$

3.5.4.1 Numerical Solution Solution of Optimal Control Canonical Equations Using a Progressive Domain Expansion Method (PDEM)

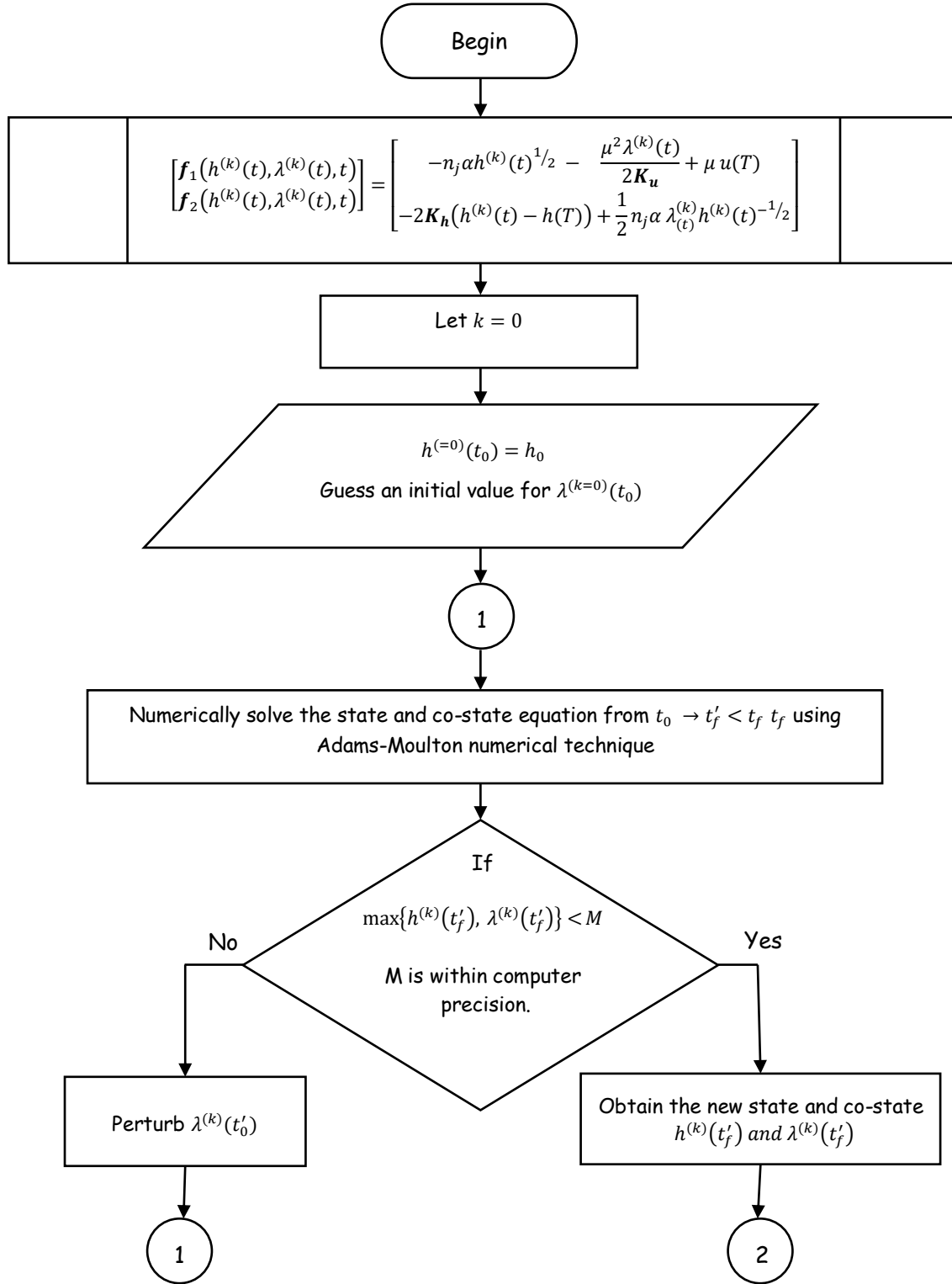
Equations (3.54) provide the basis for the computation of an optimal control that consists of two nonlinear differential equations with split conditions. The state equation being specified at the initial time while the co-state equation has its condition specified at the final time. TPBVP that has attracted considerable attention in the past century. While many methods are outlined in chapter 2, one method that was often proposed but seldom used because of computational difficulties is the shooting technique. While it is attractive because it involves guessing values for the missing initial conditions which in principle ought to be determined by one of several possibilities, the nature of the co-state equations lead to a rapid growth of the initial value problem that the computed values soon lose relationship with the problem since errors in computation are exponentially amplified by the system.

Olufeagba , Almquist, & Flake (1975) and Olufeagba & Flake (1976) explored and applied an approach based on parallel shooting method by resetting the problem and increasing the single initial value problem to a family of initial value problems configured so as to limit the effect of the growth of computational errors. The outcome resulted in a method that increased the number of guesses which were much fewer than the methods that depended on variational and other approximation methods that for the same accuracy may involve the solution of large linear or nonlinear equations that have dimensionality several orders of magnitude when compared with the corresponding shooting method. The method was applied successfully in the modelling of distributed parameter systems and proved to be very efficient, accurate and fast.

The progressive Domain Expansion method is another modification proposed and pre-tested by Olufeagba (1978) whereby instead of partitioning the domain $[0, T]$, the final domain boundary is adjusted in such a manner that the initial guess results still retain a semblance with the original problem and the growing equations are bounded so that the correct initial value problem is solved assuming the pseudo-domain $[0, T_k]$ where T_k is determined on the fly. In the next iteration the initial guess for the missing boundary condition assumes the value that would have solved the problem for the pseudo-domain. Meanwhile the problem is solved beyond T_k again until the growth begins to cause concern. The new T_{k+1} is thus defined and a new correction made so that the corresponding problem is solved. This process is repeated until $T_k = T_f$ in which case the value of the initial guess converges to the correct initial guess for the problem.

The proof of convergence depends on the existence of the solution to the original TPBVP which when it exists implies that solutions exist for pseudo-domains that are subsets of the original problem.

3.5.4.2 Flow Chart for the Numerical Solution Solution of Optimal Control Canonical Equations Computed Using PDEM



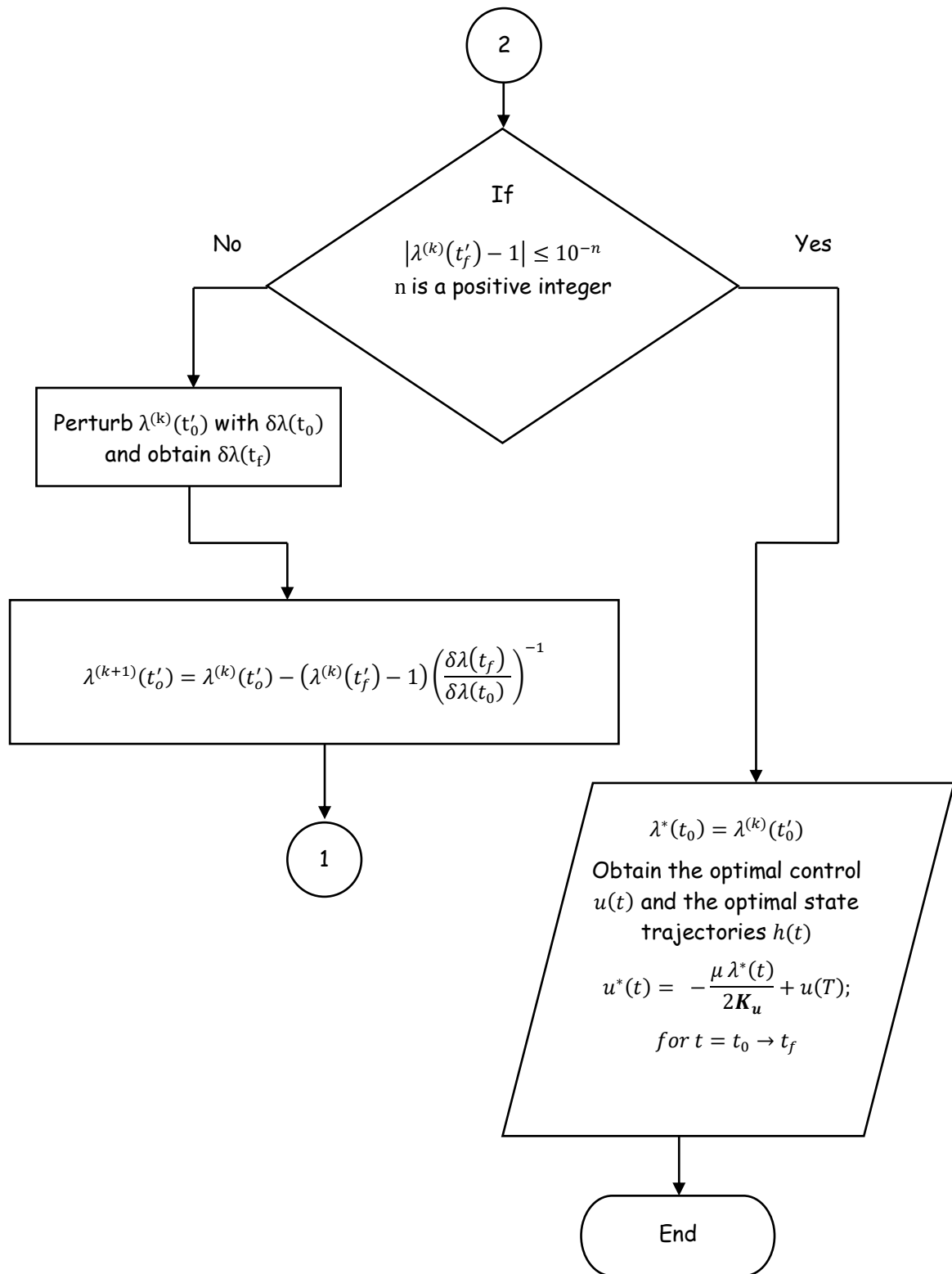


Figure 3.18: Flow Chart of the PDEM Solution of the Canonical Equations

3.5.4.3 Solution of Optimal Control Canonical Equations by a Conjugate Gradient

Method (CGM)

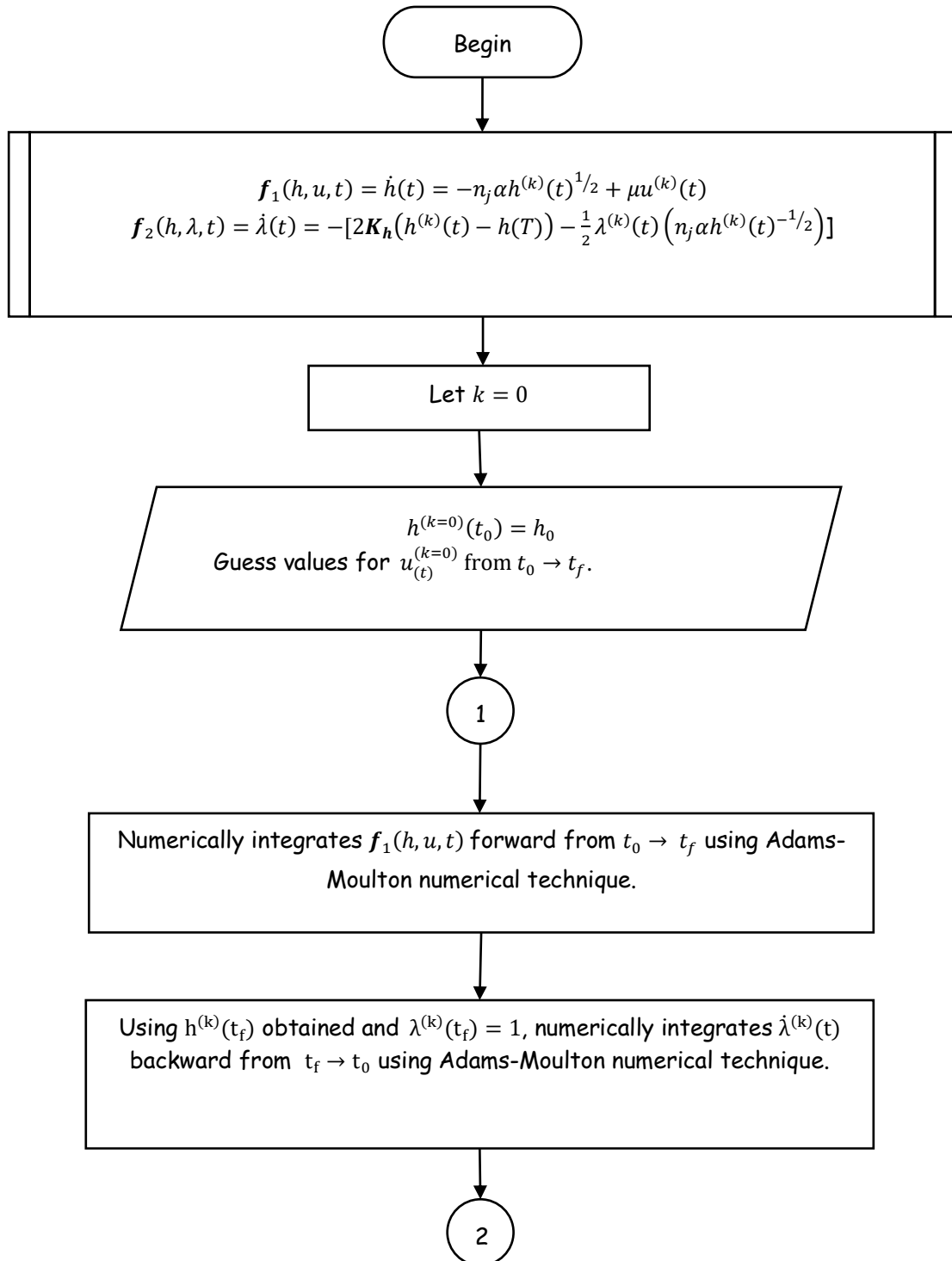
Many iterative methods try to exploit the fact that solving the state equations forwards in time and the co-state backwards eliminates the ill-conditioning that discouraged many users from the shooting methods. An examination of the canonical equations, 3.54a and 3.54b, however reveals that for a given function $u(t)$, it is possible to decouple the state from the co-state although a procedure must be put in place to improve the initial guess for the control function.

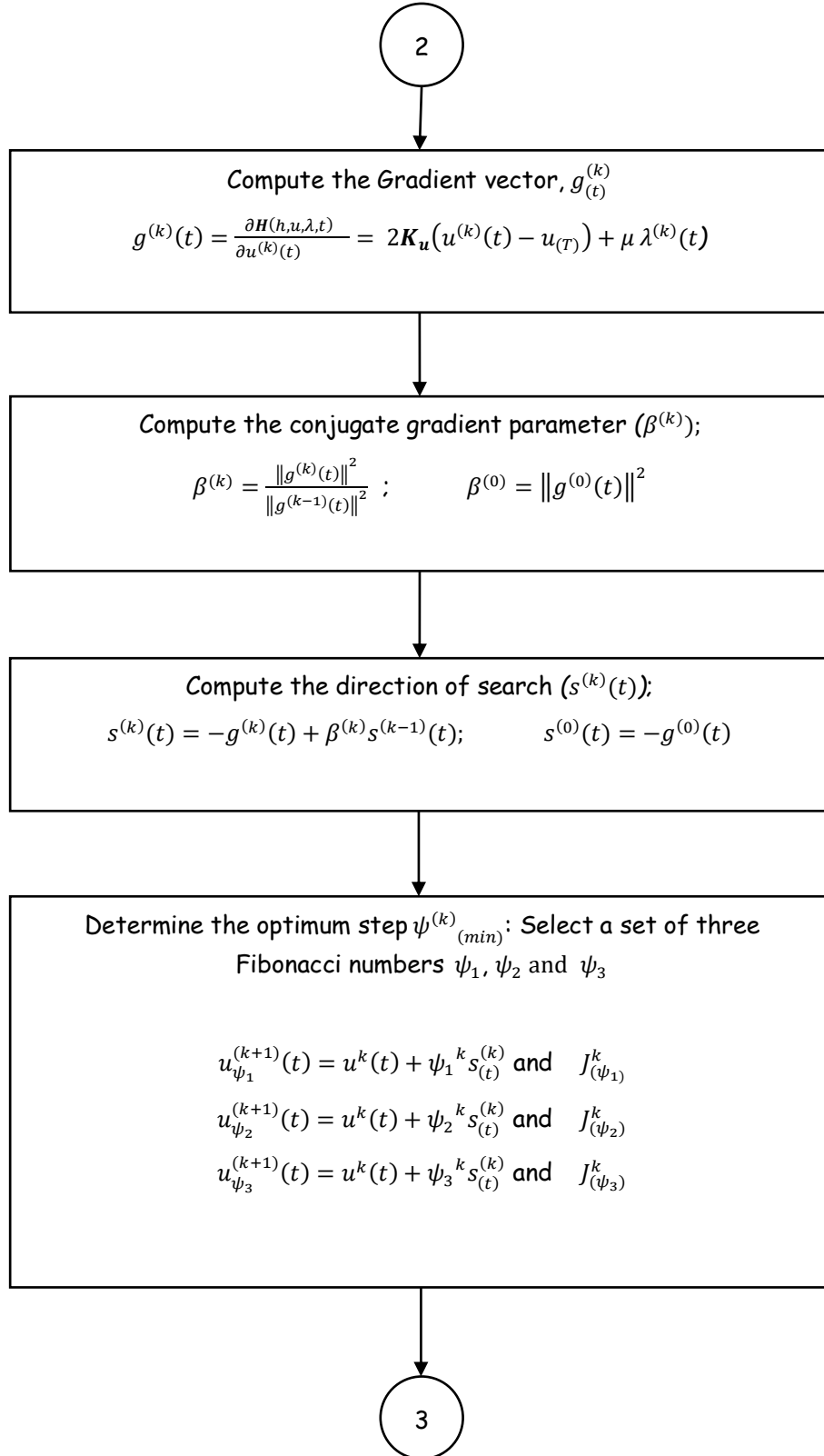
During this work, a method dependent on the conjugate gradient procedure was adopted. As stated earlier, the conjugate gradient method provides a method of essentially approaching the same convergence rate as the second order Newton methods without the added complication of determining complex second derivatives that for functional variables become almost intractable.

Unlike the PDEM, CGM is an iterative technique that starts with any initial guess for the optimal control and no need of a guess for the co-state as it would be solved from backward (from $t_f \rightarrow t_0$). The iteration continues until the convergence is reached.

The procedure for solving the optimal control canonical equation using CGM is as presented in the flow chart of Figure 3.19.

3.5.4.4 Flow Chart for the Numerical Solution of Optimal Control Canonical Equations by a CGM





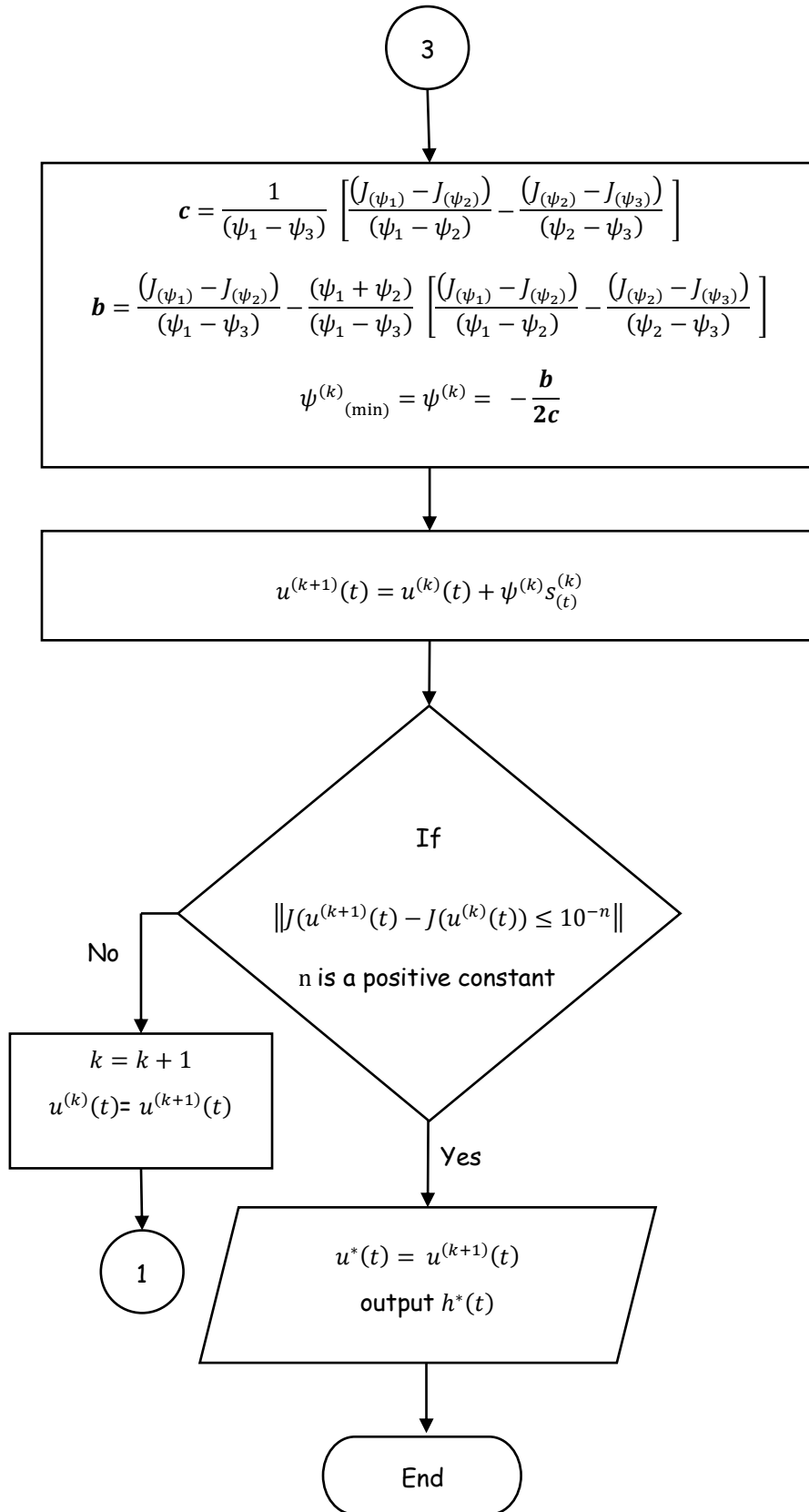


Figure 3.19: Flow Chart for the Solution of Optimal Control Canonical Equations by a CGM

3.6 Chapter Summary

The methodologies for achieving the objectives were discussed in this chapter. The actual formulation of the mathematical models describing the dynamics of the Kainji and Jebba reservoirs was implemented using consistent energy and material conservation principles and including various system physical parameters as well as essential inputs and outputs that sustain the dynamics. The result was a set of nonlinear differential equations, one for each reservoir, which were then integrated numerically using the Adams-Moulton technique with a Runge-Kutta starter. A procedure for estimating model parameters using observations of inputs was briefly outlined while the actual process will be fully discussed in chapter 4.

An optimal control problem that addresses one of the most difficult problems confronting operators was then formulated. Solutions were then developed for both direct and indirect methods for the optimal control. Basically, four methods are presented reflecting a basic steepest descent method and its more versatile counterpart and the indirect methods that depend on solving the canonical equations of the problem. The results will be presented in the next chapter.

CHAPTER FOUR

Results and Discussion

4.1 Preamble

The operators of the HEPS face serious challenges that involve balancing conflicting needs involving the operational safety of the stations and the demand requirements from an energy starved electricity grid. Practical observation of operations reveals the serious challenges confronting the JHEPS operators as they try to perform the functions of a regulator that was omitted in the fixed vane designs of the JHEPS turbo alternators.

These operational problems are present in each HEPS in one form or other and it's only because of the robust nature of the turbo-alternators that major catastrophes have not occurred. Nonetheless there is evidence of such problems lurking in the possibilities when one observes that one turbo alternator catastrophically failed at JHEPS and another at KHEPS. Both cases share the feature that no local regulator was included in the initial design. For example, the use of Francis Turbines at a very low head scheme as the KHEPS posed especial problems that meant poor performance and ultimately catastrophic failure. In the KHEPS case, the narrow operating head demanded by the fixed vane Kaplan turbines posed a very serious challenge to the operators. The latter problem however lends itself to the application of optimal control methods.

Indeed, this is the problem addressed in this research. It is posed as an optimal control problem to determine the inflow to the KHEPS reservoir so that the head falls within a specified range. Applying the method developed in Chapter 3, the control problem was easily formulated as the following optimal control problem:

$\min J(x) = \text{Integral of square of the deviation from the preferred head and subject to the dynamical equations of the reservoir.}$

The next sections report the results of the controls determined by the different methods. It starts with the model validation, a brief study of the potential performance of the two stations and prospect for future hydropower project on river Niger. The results from the computation of controls comprises of solutions of optimal control using steepest descent method, direct conjugate gradient method, progressive domain expansion method, indirect conjugate gradient method and quasi-feedback optimal control method. In all, routines written for Microsoft EXCEL VBA[®] environment were used for computation and analysis.

4.2 Model Validation for KHEPS and JHEPS using observations

The essence of computing a numerical solution for the resulting model and calibration of the parameters was to validate the model using measured data. This section presents the comparison between computed head from the model and the observed head from the data obtained. Other data passed into the model include the inflows and the number of operating machines per day. Figure 4.1(a) presents the inflow into KHEPS in the year 2013 and Figure 4.1(b) shows the number of units operating per day. These two parameters, together with the evaporation losses were passed into the calibrated model to a compute the operating head. Model validation for year 2013 and 2014 for both KHEPS and JHEP are plotted in Figure 4.2 (a) to (d).

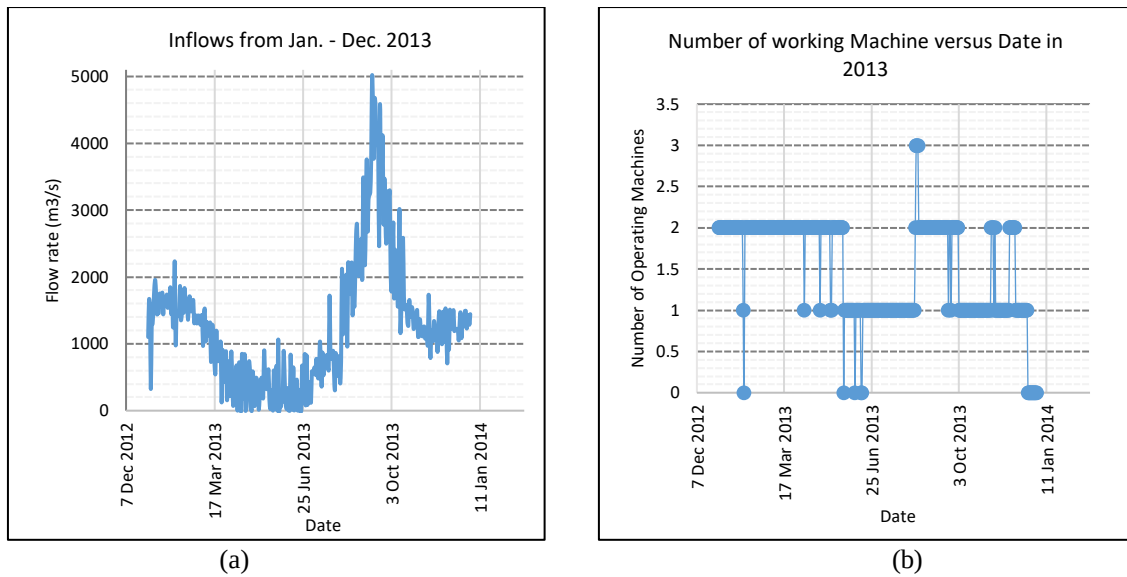
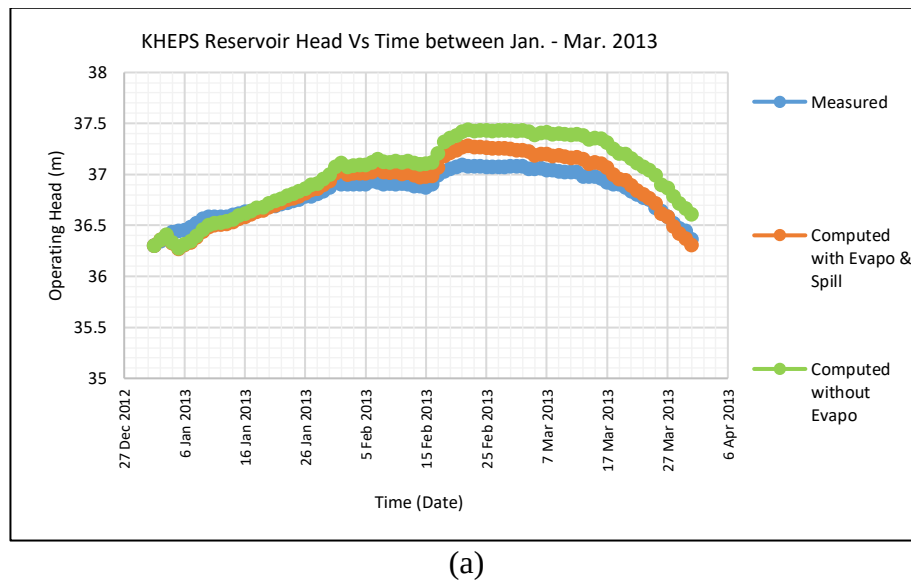
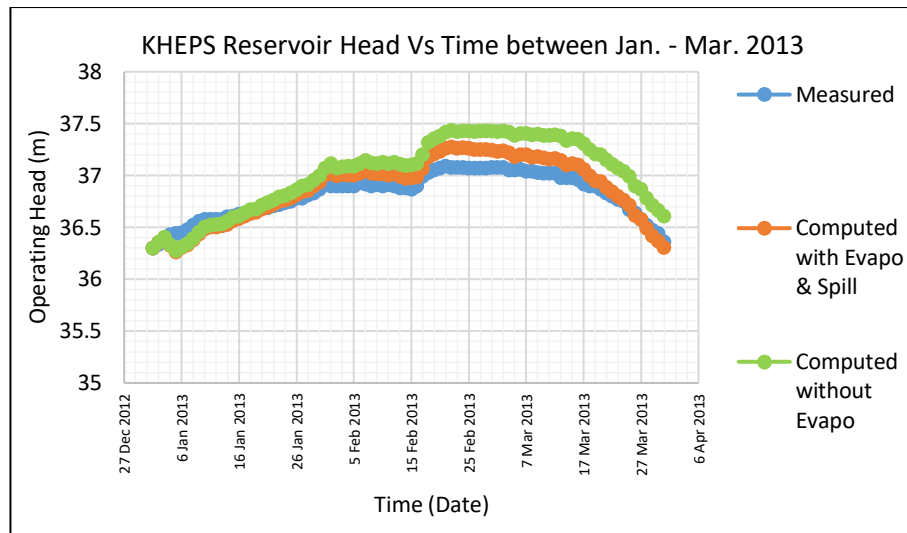


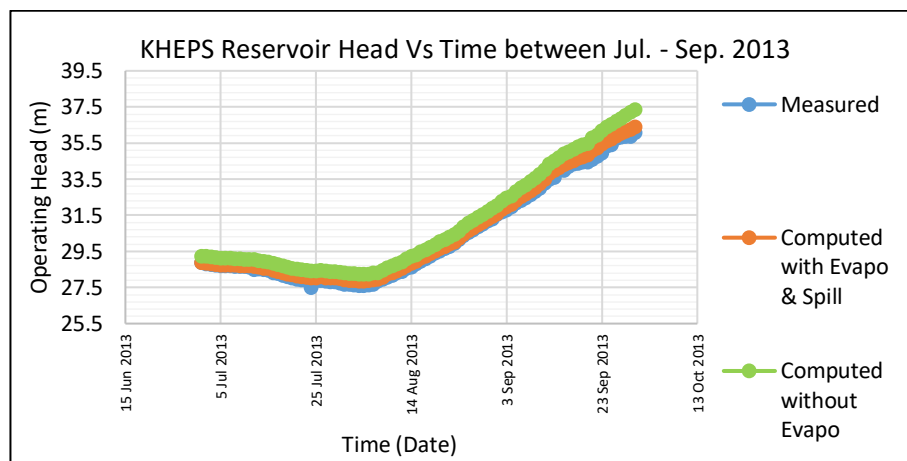
Figure 4.1: KHEPS Inflows and Machine State in 2013
(a) Inflows (b) Machine State

Comparison between the measured and computed head for KHEPS in year 2013 are shown in Figure 4.2 (a) to (d). The plots show that the model is highly affected by the evaporation losses on the reservoir. The evaporation loss varies per month, hence the curve for a whole year is divided into four. The curves show that the model relatively predict the performance of KHEPS with minimal error.

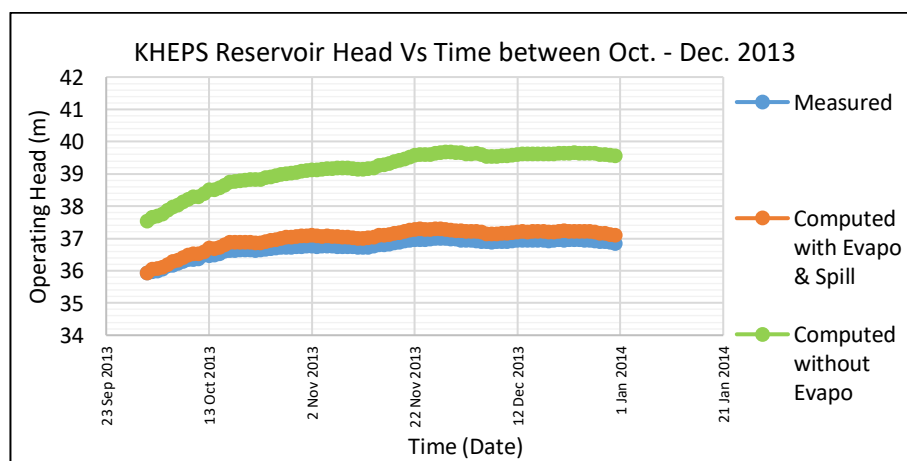




(b)



(c)



(d)

Figure 4.2: Comparison of Observed Head with Computed Head for KHEPS in 2013

Figure 4.3 shows the difference between the measured head and the computed head for KHEPS in year 2013. The curve shows a deviation of less than $\pm 2\%$, meaning that the model is dependable and accurately predict the performance of the system.

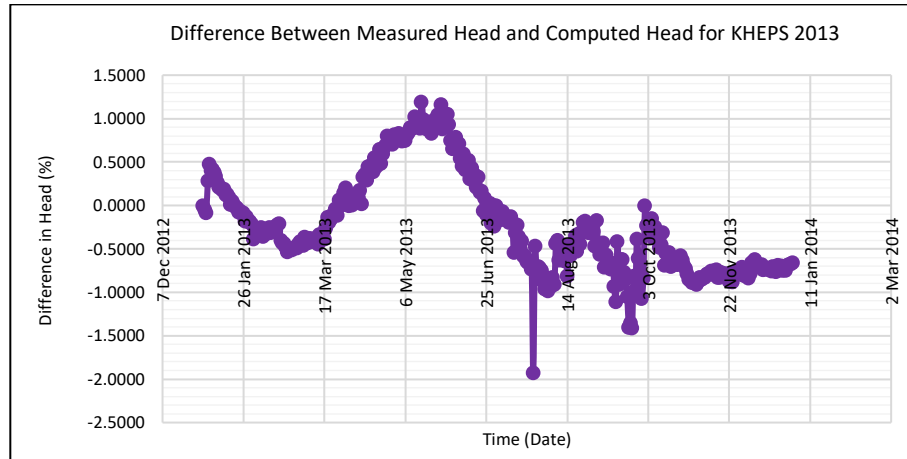
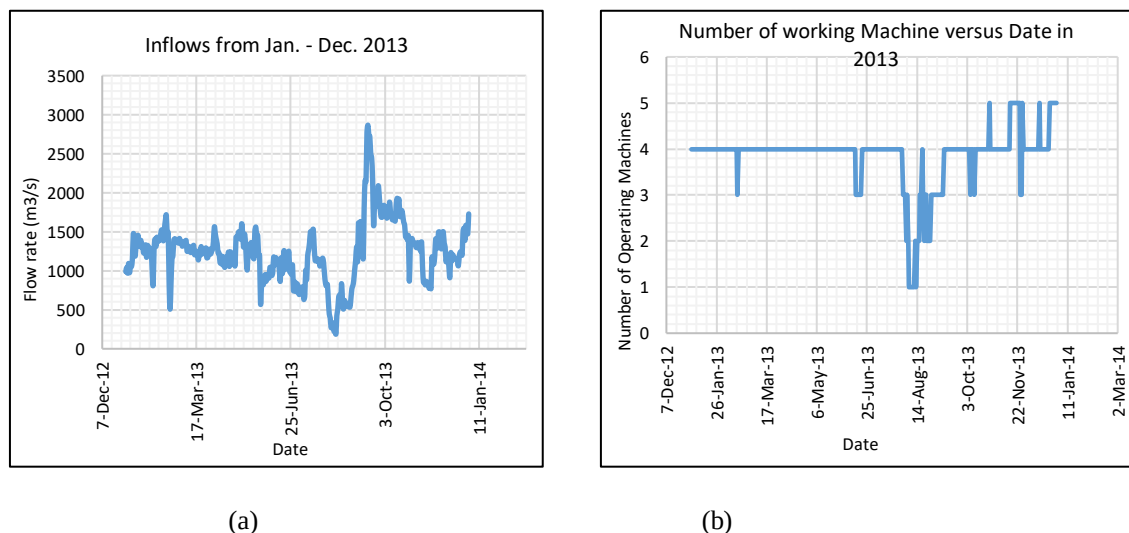


Figure 4.3: Difference between Measured Head and Computed Head for KHEPS 2013

Figure 4.4(a) presents the inflow into JHEPS in the year 2013 and Figure 4.4(b) presents the number of units operating per day. In a manner like the validation of KHEPS case, these two parameters, together with the evaporation losses were passed into the calibrated model to a compute the operating head of JHEPS in year 2013.

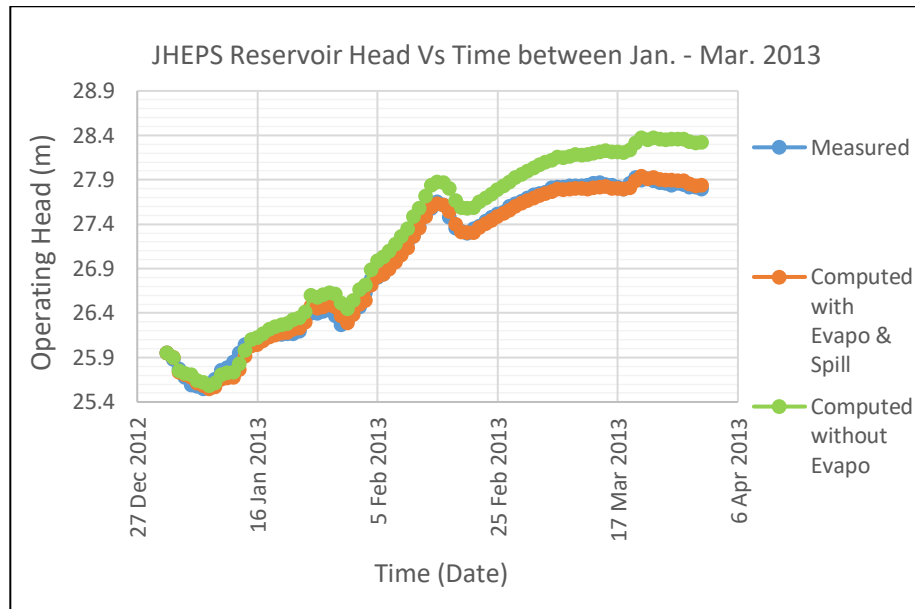


(a)

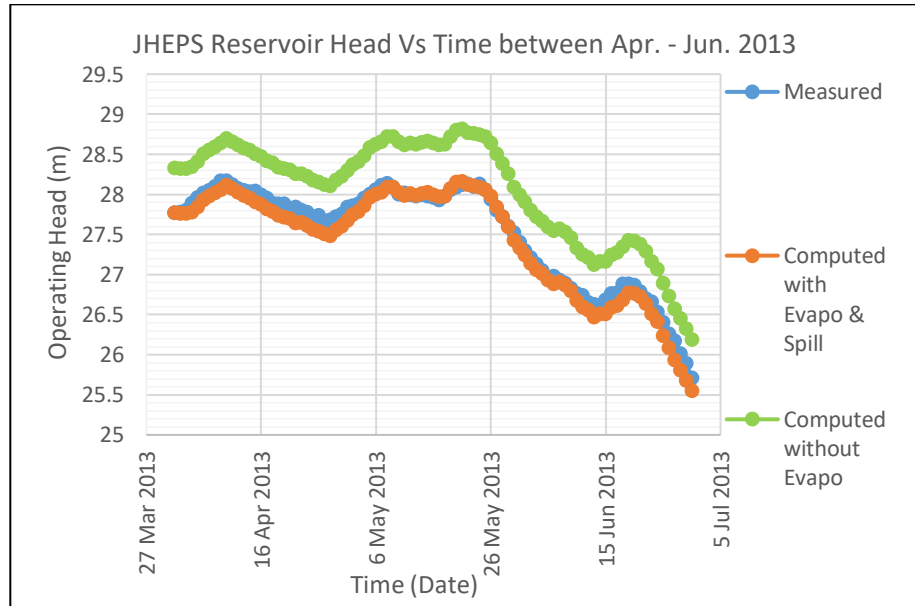
(b)

Figure 4.4: JHEPS Inflows and Machine States in 2013
(a) JHEPS Inflows (b) JHEPS Machine State

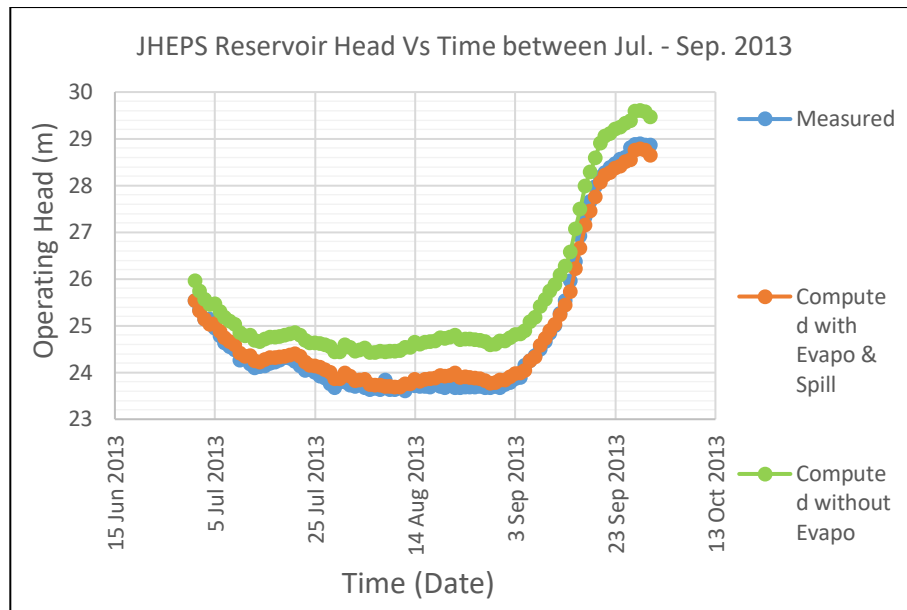
Comparisons of observed head with computed head for JHEPS in 2013 are presented in Figure 4.5 (a) to (d). Similarly, the computed model is in two parts, one computed without evaporation losses while the other is computed with losses due to evaporation.



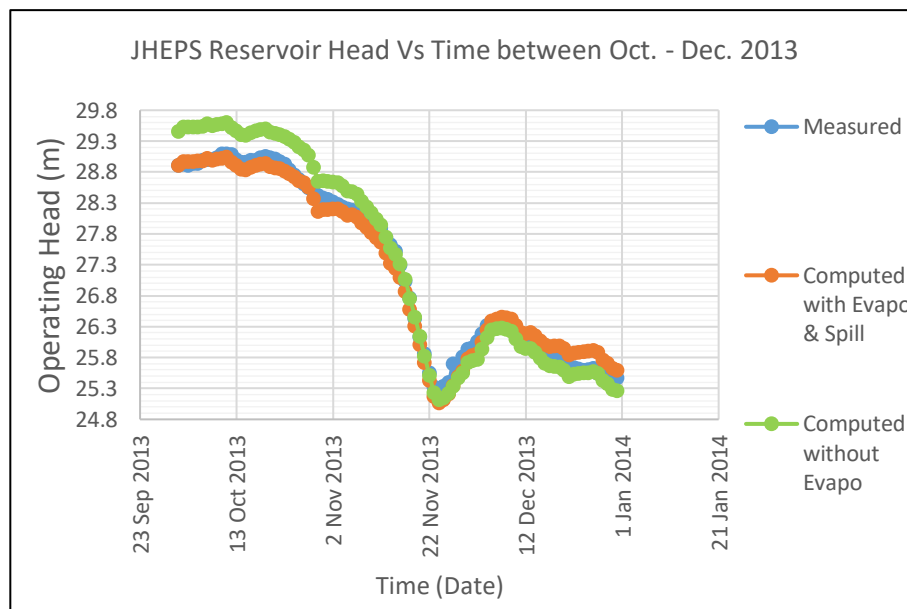
(a)



(b)



(c)



(d)

Figure 4.5: Comparison of Observed Head with Computed Head for JHEPS in 2013

The difference between the observed head and computed head for JHEPS in 2013 has been estimated in Figure 4.6 to be less than $\pm 1.5\%$. The accuracy is slightly better than that of KHEPS with a maximum deviation of slightly above $\pm 1.5\%$ but less than $\pm 2\%$

Numerous factors may be responsible for the deviation but a significant one is that the estimated evaporation losses may vary from the actual evaporation losses in 2013. Also, the actual number of units operating per day may not reflect the value used in the model. A machine may not work for larger part of the day but in as much as it was operating as at the time the reading was taken, it will be assumed that it was operating for a whole day. These among others are responsible for the slight deviation between the measured value and computed value.

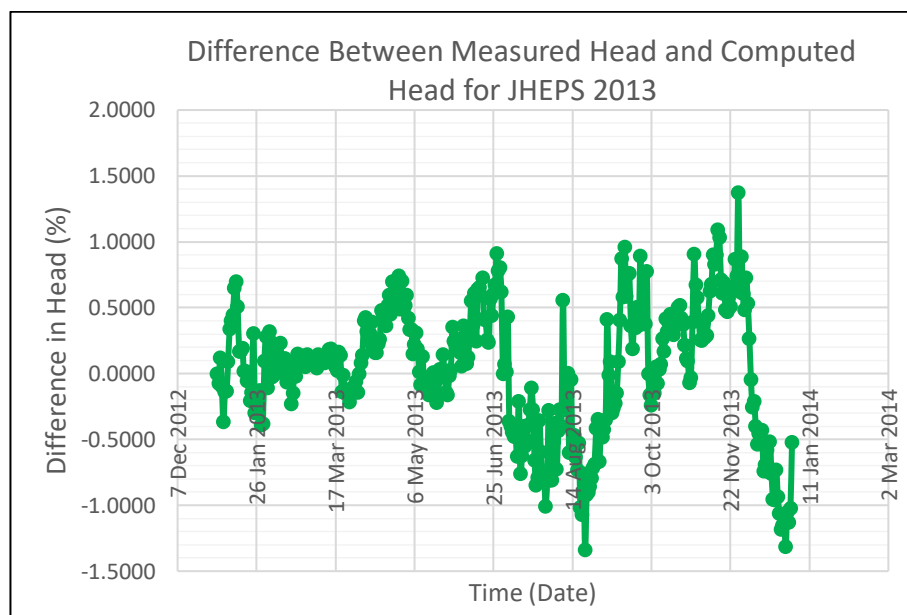
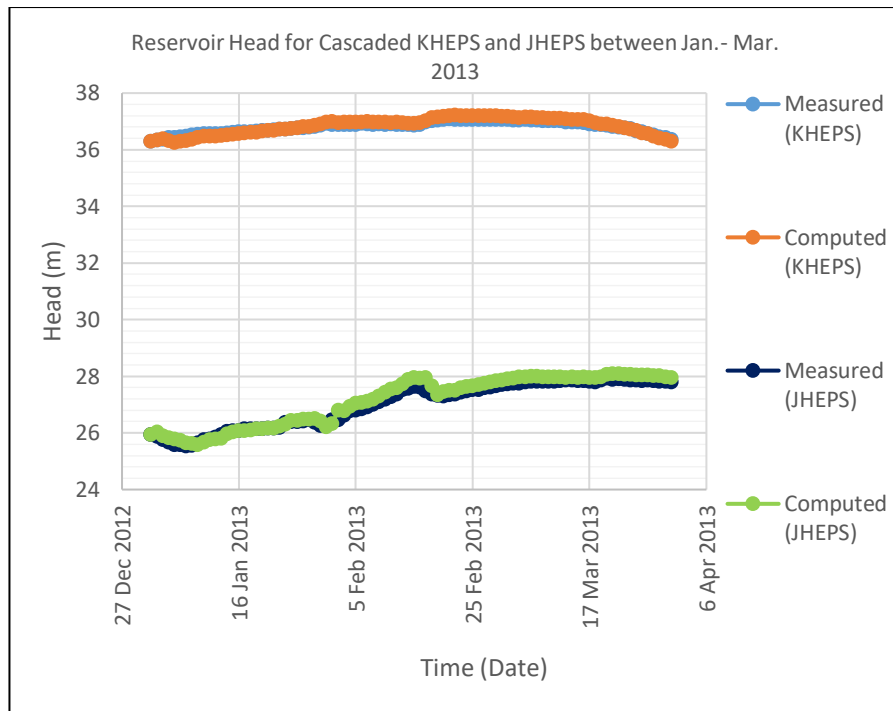


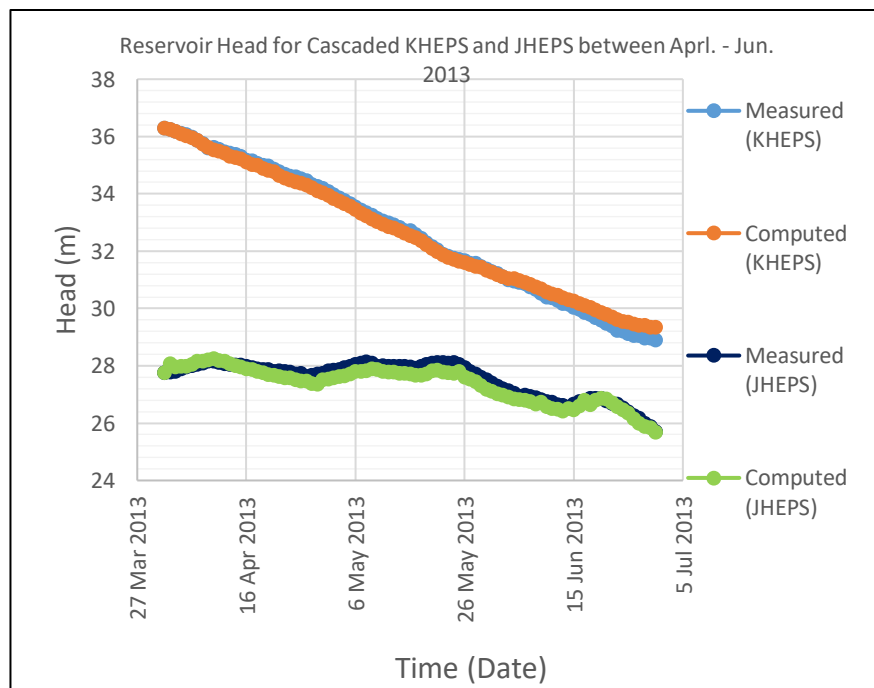
Figure 4.6: Difference between Measured Head and Computed Head for JHEPS 2013

4.3 Model Validation of the cascaded System

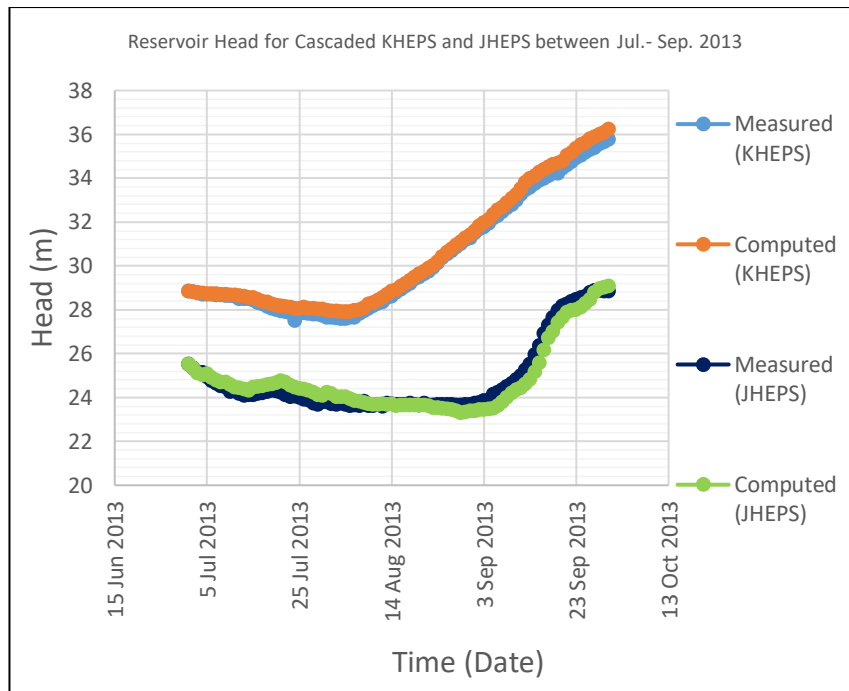
Consider equation 3.23, if the two stations are connected through the channel such that the inflow into JHEPS is equal to the sum of the discharge from KHEPS, spill from KHEPS and the flow from tributaries along the connected channel, the comparison between the measured head and the computed head for year 2013 is as presented in Figure 4.7 (a) to (d). The results show a good compliance between the measured and computed head in each case.



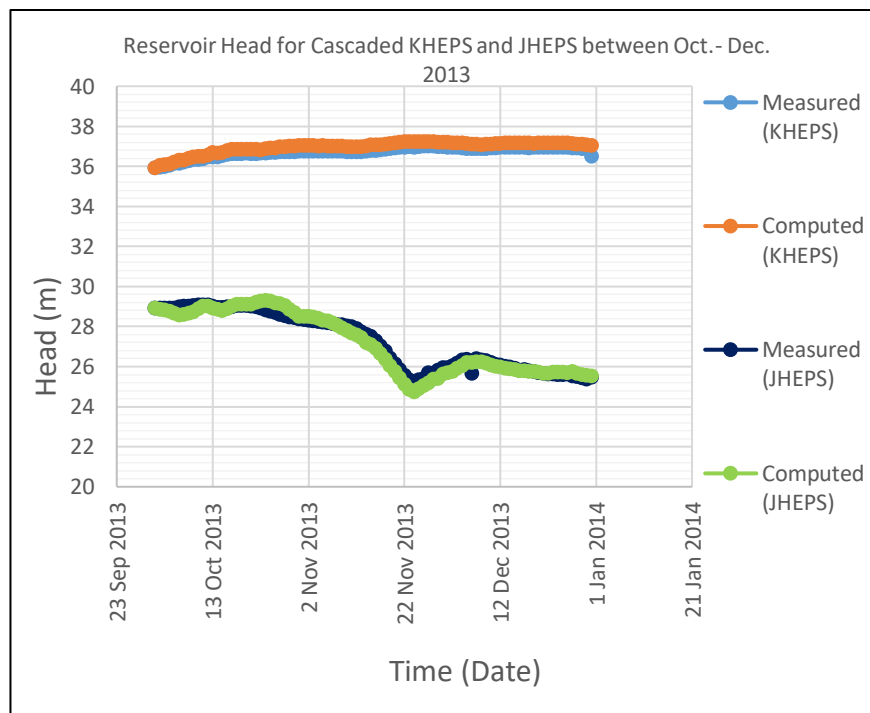
(a)



(b)



(c)



(d)

Figure 4.7: Comparison of Observed Head with Computed Head for the Cascade KHEPS and JHEPS in 2013

4.4 Potential Performance of KHEPS and JHEPS under Different Conditions.

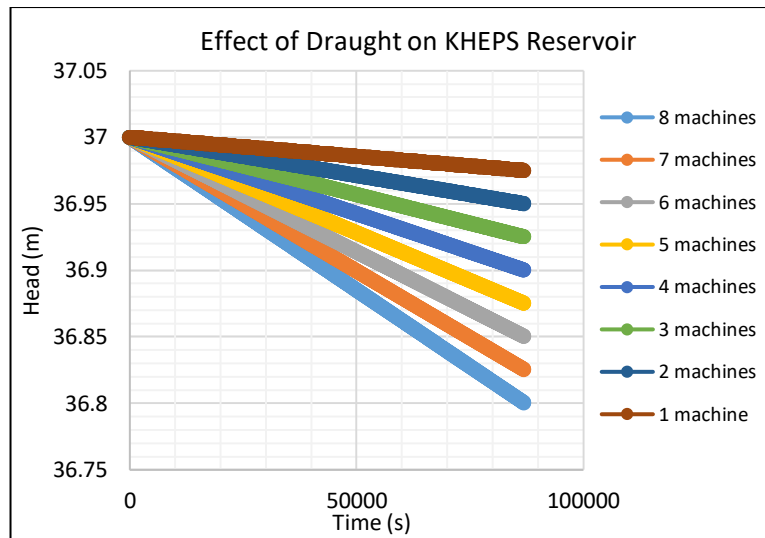
A library of macro routines using Microsoft EXCEL-VBA was developed in this work to compute a numerical solution for the developed models, calibrate and validate the models. The macros serve as a computer model for extensive investigation of the potential performance of hydropower station under different conditions, it can be used for training hydropower station operators and planning tool managers. For control, it was used to study the stability of the stations, performance under different flows and machine availability.

4.4.1 Effect of Drought and Machine State on KHEPS and JHEPS

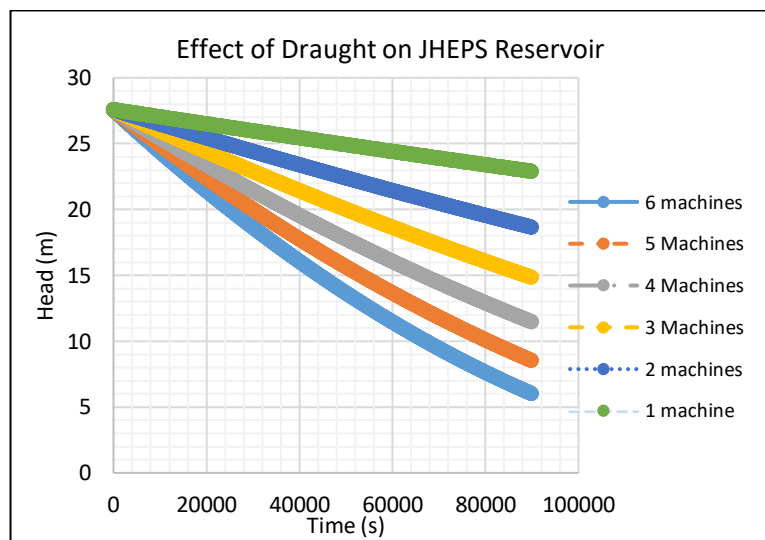
The plots in Figure 4.8 show the effect of drought on KHEPS and JHEPS. The curves show the time with which several machines can be operated when there is no inflow.

Figure 4.8 (a) shows that if there is low or no inflow into KHEPS reservoir for one day (86400 s), the head will fall faster with all the 8 units operating than just one unit. If the reservoir starts at maximum head of 37m, in one day the head will fall to 36.8 m if 8 units are working, 36.90 m if 4 units are working and 36.97m with only one machine.

Similarly, observation from Figure 4.8 (b) shows that if JHEPS reservoir starts at maximum head of 27.6 m, in one day the head will fall to 6.62 m if 6 units are working, 12.0 m if 4 units are working and 26.3m with only one machine. JHEPS cannot generate for a whole day with little discharge from KHEPS.



(a)



(b)

Figure 4.8: Effect of Drought on KHEPS and JHEPS Reservoir

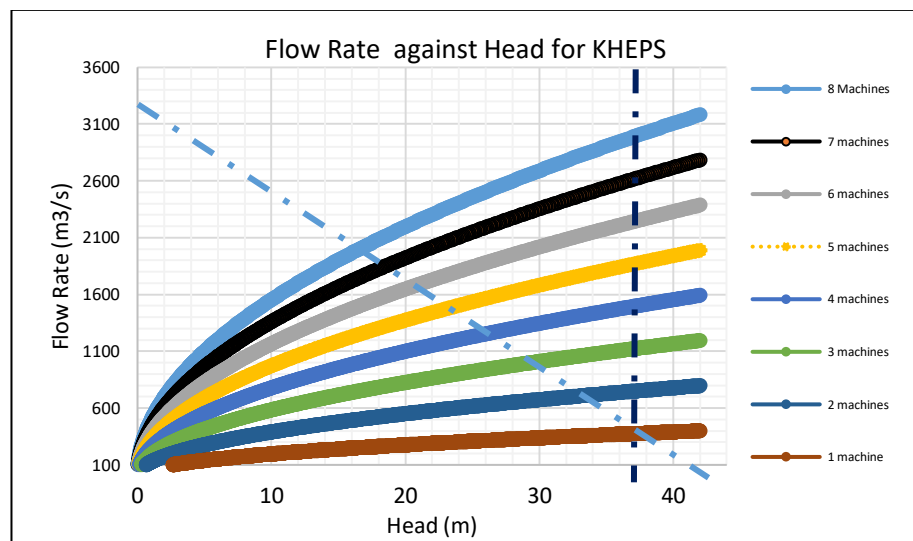
(a) KHEPS (b) JHEPS

4.4.2 Effect of Inflow and Operating Units on the Head of KHEPS and JHEPS

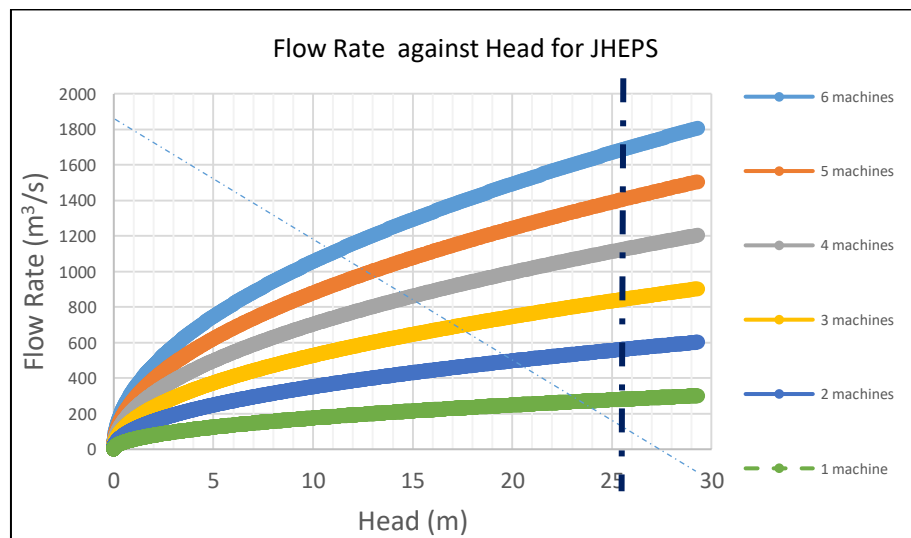
An important information in the operation of KHEPS and JHEPS is that for a given inflow, how many machine units can be in operation such that the head does not fall below a level in one day. This information can be obtained from Figure 4.9 (a) and (b).

They are plots of the inflow against the operating head at different number of operating units.

If a load line is drawn on the graphs, the point of intersection of the curve with the line gives the flow that will keep the operating head constant at the intersecting value when certain number of units are in operation.



(a)



(b)

Figure 4.9: Flow Rate versus Head for KHEPS and JHEPS

(a) KHEPS (b) JHEPS

Consider a situation where the net inflow into KHEPS is $1600 \text{ m}^3/\text{s}$ and the head is at 42 m. Figure 4.10 (a) shows the system response with 4 units operating, a unit was restored after some time to make it 5 units and lastly a unit failed to restore the operating units to 4 in number. The graph predicts the system response in such a scenario.

Another scenario is shown in Figure 4.10 (b), the operating head of JHEPS is allows to decrease from 29.3 m under a net inflow of $1400 \text{ m}^3/\text{s}$ and 5 operating units. The head remain relatively constant 25.3 m. This confirm the prediction of Figure 4. 9 (b).

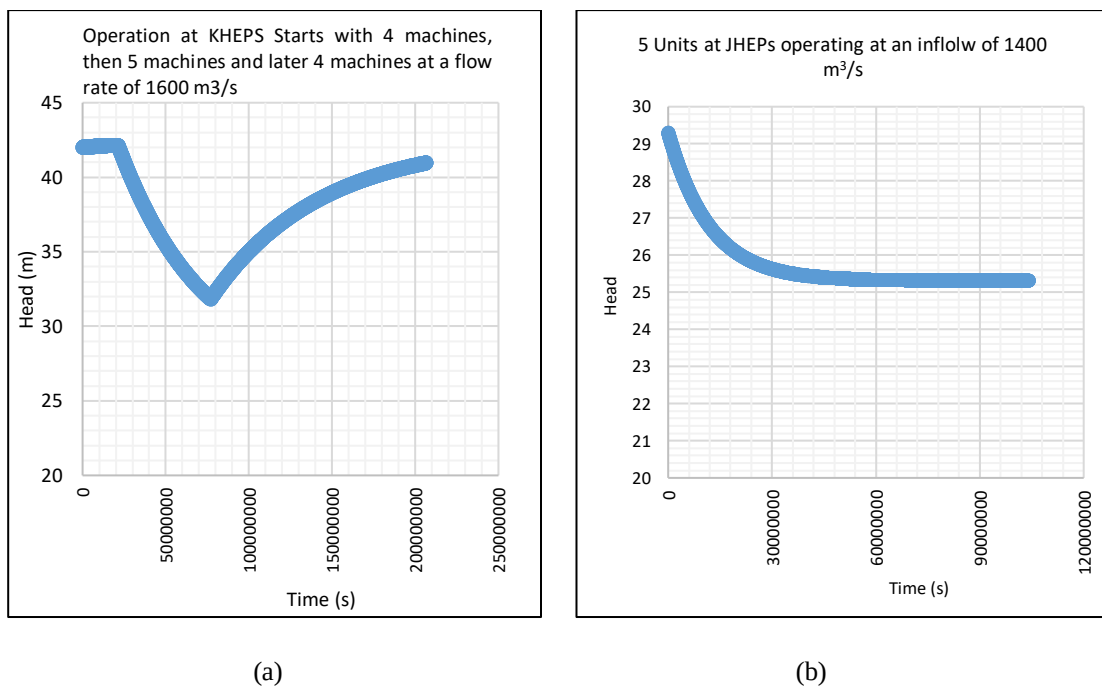


Figure 4.10: System Response under different Scenario

(a) KHEPS and (b) JHEPS

4.4.3 Prospect of Further Hydroelectric Power Project along the River Niger

Figure 4.11 is a plot of the net inflow into KHEPS, net inflow into JHEPS and the outflow from JHEPS. A comparison between the three graphs reveals that the outflow is large enough for possibility of another hydropower project downstream of Jebba reservoir. Hence the generation potential for River Niger has not been maximised, the country should invest in cascading hydropower project of gas fired thermal stations that keep increasing yearly.

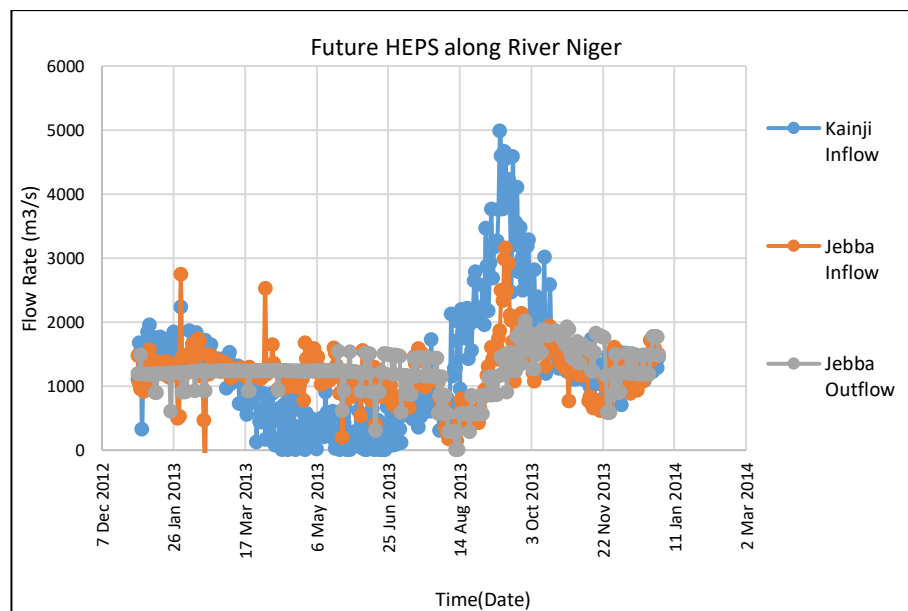


Figure 4.11: Prospect of Hydroelectric Power Project along River Niger

4.5 An Optimal Control Problem for the HEPS and Results

The next few sections focus on the results of optimal controls applying the methodologies developed in chapter 3 where the EXCEL-VBA realizations were also implemented. The optimal control solved in this work is the determination of the control signal $u(t)$ (generated by the control System) to be released from KHEPS (actuator) that will force the operating head $h(t)$ of JHEPS (Controlled Plan) to follow a predefined trajectory within a given time ($t_0 \rightarrow t_f$).

Optimal control laws based on five methods of solution were discussed in Chapter 3, results from the simulation of these methodologies are presented. In each case, the following parameters should be specified before computation:

Table 4:1: Required Parameters for Computation of Optimal Control

Parameters	Definitions
$h(0)$ (m)	Reservoir head at time t_0 .
$h(T)$ (m)	Desired reservoir head at time t_f
$U(0)$ (m^3/s)	Control input at time t_0
No of Machines	Number of operating units between $t_0 \rightarrow t_f$
Day	Required time for the head to move from $h(t_0)$ to $h(t_f)$
Δt (s)	Numerical time step
$u(min)$ (m^3/s)	Minimum allowable control signal
$u(max)$ (m^3/s)	Maximum allowable control signal

4.5.1 Computation of an Optimal Control by a Steepest Descent Algorithm

The problem addressed here can for convenience be formulated as follows:

$$J = \min \int_{t_0}^{t_f} \left(K_h (h(t) - h(T))^2 \right) dt$$

Subject to the system constraints:

$$\dot{h}(t) = f(h(t), u(t), t) ; t_0 \leq t \leq t_f$$

$$h(t_0) = h_0$$

$$h(t_f) = h(T)$$

Due to the high computational load of the direct method, the approximation to the control sought may be represented as follows where the control interval t_f is 24 hrs; $t_k = 6(k - 1)$ hrs and the corresponding control in the interval is T_k for $k = 1, 2, \dots, 4$.

A coordinate format is used to simplify the case being considered, the format is as follows: (Number of Machines, Starting head, Number of Days, Constraints). Also note that $h(T)$

4.5.1.1 Case 1: (5, 25.8, 1, *Unconstrained*)

Applying these specifications into the steepest descent algorithm of chapter 3 and assuming a convergence criterion for the gradient led to convergence after 15 iterations with trajectories as shown in Figure 4.12.

The final trajectory of course depends on the initial conditions and for the case reported of Figure 4.12 exhibits some overshoot because the interval of 6 hrs used to specify the

controls prevent the adjustment of the control with the precision that would guarantee the desired terminal value nearly precisely.

It is evident that operators using protocols based on this result will be able to more precisely control and manage their plants. The results demonstrate the utility of the optimal control approach and provides dependable methods for operations with a negligible deviation of the optimum head $h^*(t_f)$ from the set value $h(T)$.

Inputs										
$h(0)$	$h(T)$	$u_1(0)$	$u_2(0)$	$u_3(0)$	$u_4(0)$	No of Machines	Day	Δt	$u(min)$	$u(max)$
25.8	26.1	1000	1000	1000	1000	5	1	1800	Unconstrained	
Outputs										
No of Iterations	$u_1^*(t)$	$u_2^*(t)$	$u_3^*(t)$	$u_4^*(t)$	$I \text{ (min } J)$	$\ \nabla J\ $	$\psi(min)$	$h^*(t_f)$		
15	7127.522	470.5617	1626.928	1714.475	501.4565	0.042747	0.0053628	26.10869		

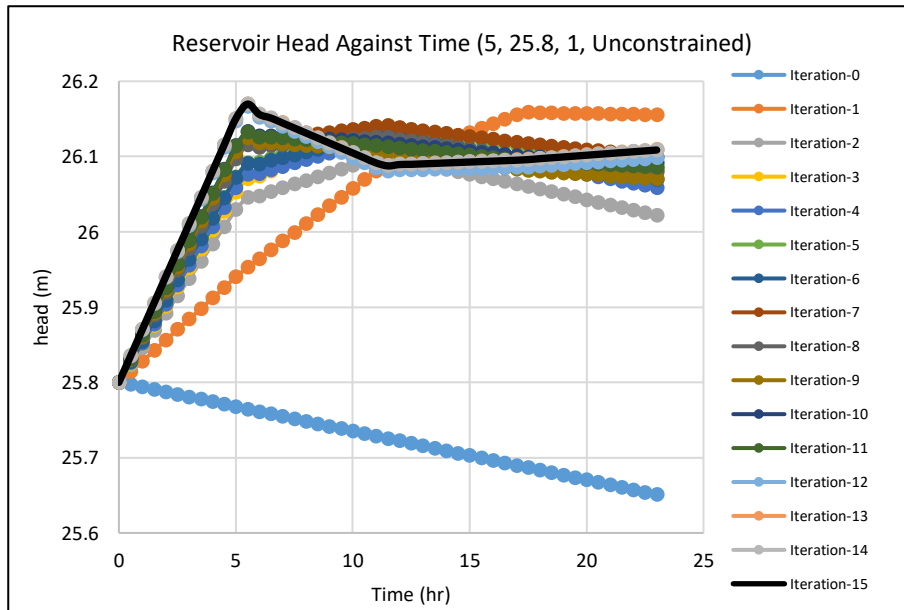


Figure 4.12: Reservoir Head versus Time (5, 25.8, 1, Unconstrained)

Figure 4.13 presents the optimal control that is required to produce the needed head trajectory of Figure 4.12. This is the optimal control at the fifteenth iterations. The control law starts with a high inflow in the first 6 hours of the day and reduces to the

minimum value in the second 6 hours of the day. The control increases lightly in the remaining 12 hours to move the head to the optimum level.

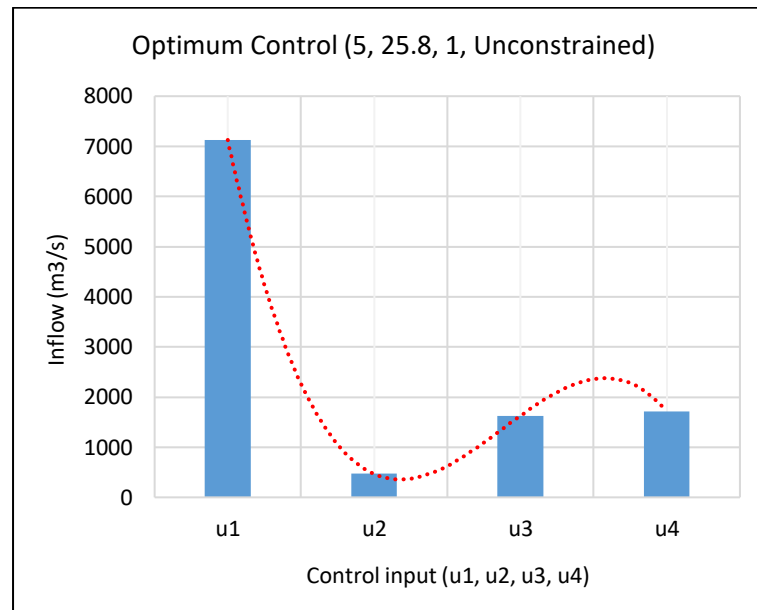


Figure 4.13: Optimum Control (5, 25.8, 1, Unconstrained)

Plots showing the performance and characteristics of the steepest descent algorithm are shown in Figure 4.14 to 4.17. The plot in Figure 4.14 presents the successive iterations determined by using a unidirectional search along the descent on the gradient vector to the local minimum. A peculiar feature of this algorithm can be observed by studying $u_1(t)$ and $u_2(t)$. While $u_1(t)$ kept increasing after every iteration, $u_2(t)$ only increases for the first two iterations and it decreases till the allowable tolerance level.

A necessary condition for a control to be optimum is that the performance index at the last iteration must be minimum. This can be observed in Figure 4.15 that the performance index keeps decreasing till the last iteration. If the curve differs from this expected behaviour, then the control is not optimum, and the trajectories may not be as seen in Figure 4.12. It can be observed that the first three iterations reduce the performance index greatly, this is a unique characteristics of steepest descent method.

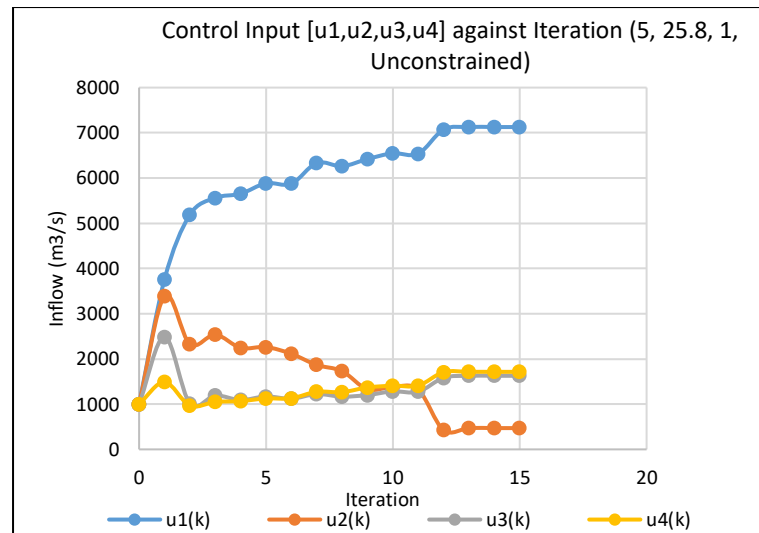


Figure 4.14: Control Input versus Iteration (5, 25.8, 1, *Unconstrained*)

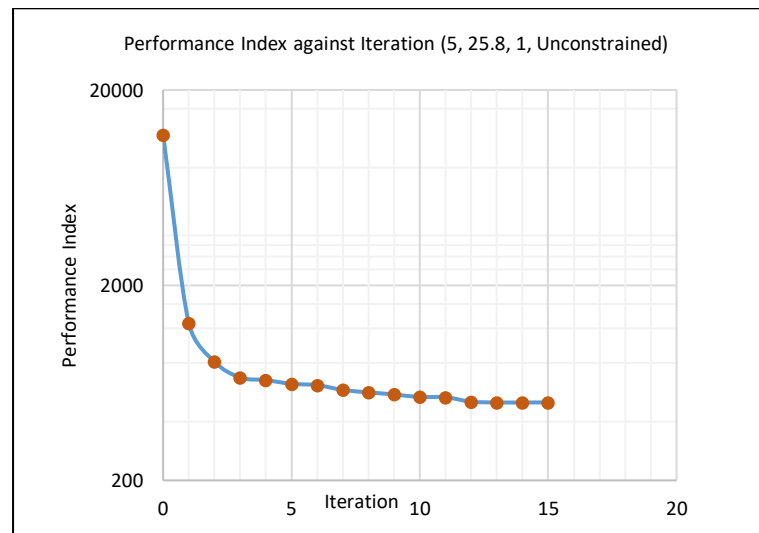


Figure 4.15: Performance Index versus Iteration (5, 25.8, 1, *Unconstrained*)

Figure 4.16 and 4.17 shows the variation of the norm of gradient and the steepest descent steps size. The stopping criteria in the algorithm is that the norm of gradient must be approximately zero and the steepest descent step size should also be very small and insignificant. It is only then that the control can be assumed to be optimum. As could be seen during the implementation, the gradient attains the lowest value and the step size became so small that the changes computed were within the error of

computation. This means that the head in Figure 4.12 and control in Figure 4.13 are optimum.

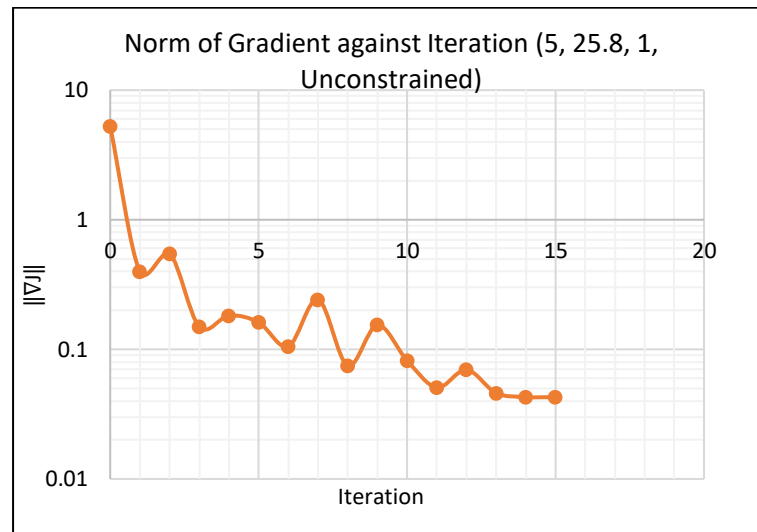


Figure 4.16: Norm of Gradient versus Iteration (5, 25.8, 1, *Unconstrained*)

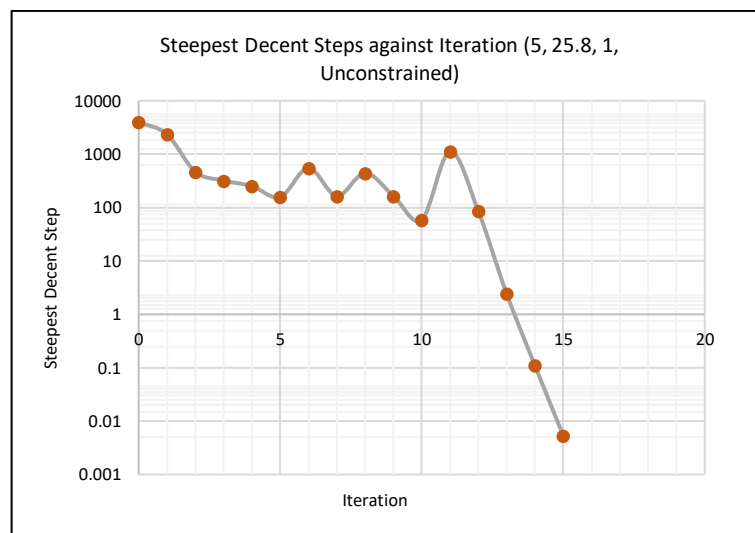


Figure 4.17: Steepest descent Steps versus Iteration (5, 25.8, 1, *Unconstrained*)

4.5.1.2 Case 2: (5, 25.5, 1, *Unconstrained*)

Case 1 considered a situation where the initial head is at 25.8m and the control is unconstrained. The simulation here assumed that the initial head was at 25.5 m. It can be observed from Figure 4.18 that the number of iteration to get to the optimum value

increases from 15 iterations in Case 1 to 22 iterations in Case 2. This reveals that the steepest descent method may take a longer time to converge if the distance between $h(0)$ and $h(T)$ is too wide.

Inputs										
$h(0)$	$h(T)$	$u_1(0)$	$u_2(0)$	$u_3(0)$	$u_4(0)$	No of Machines	Day	Δt	$u(min)$	$u(max)$
25.5	26.1	1000	1000	1000	1000	5	1	1800	Unconstrained	
Outputs										
No of Iterations	$u_1^*(t)$	$u_2^*(t)$	$u_3^*(t)$	$u_4^*(t)$	$I(min J)$	$\ \nabla J\ $	$\psi(min)$	$h^*(t_f)$		
22	12688.488	-553.3125	1730.032	1836.900	2005.7548	0.0369629	0.0261528	26.1137		

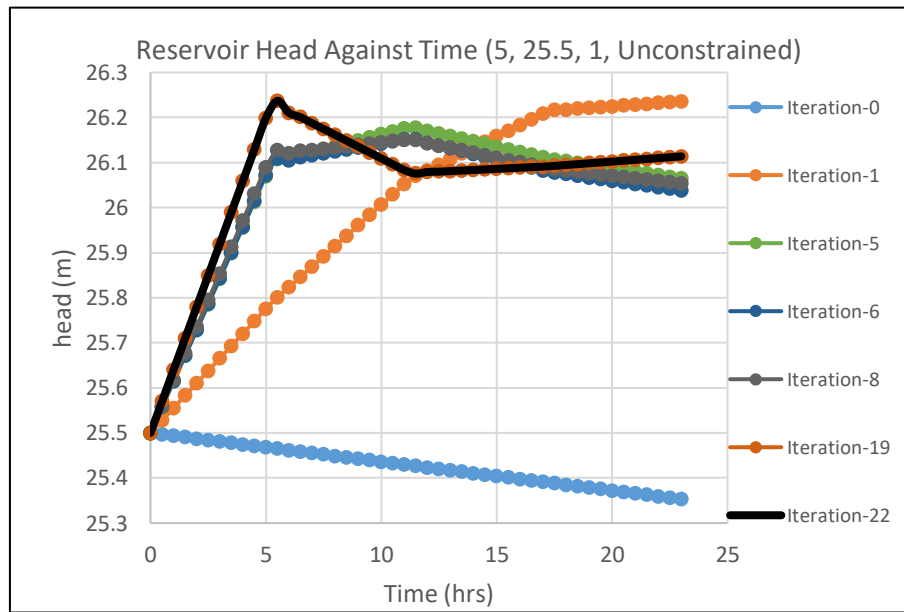


Figure 4.18: Reservoir Head versus Time (5, 25.5, 1, **Unconstrained**)

The results of the optimum control for Case 2 is shown in Figure 4.19. The control in the first 6 hr shot above $12000 \text{ m}^3/\text{s}$ and decreases to a negative value in the second 6 hr. This may not be easy to implement practically because such control may not be available from the actuator and the negative flows during the second period of the time means that JHEPS will have to spill some water to get the head to the optimum level.

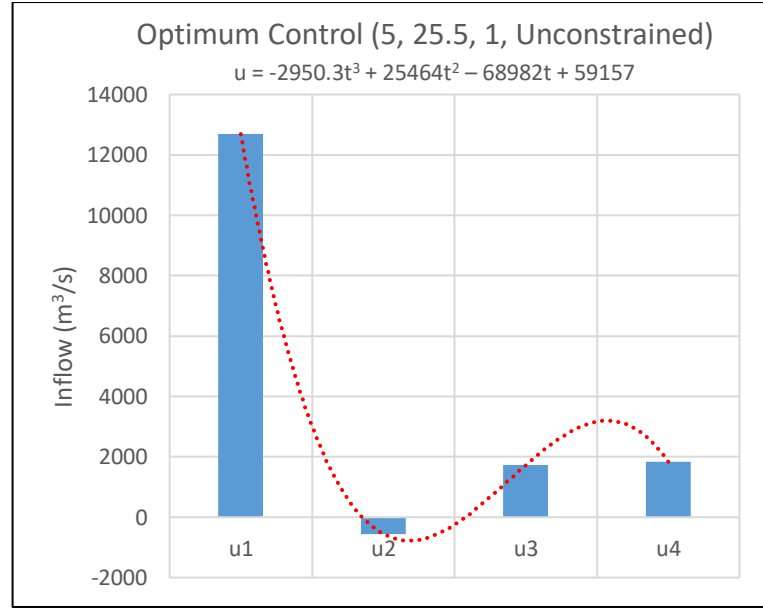


Figure 4.19: Optimum Control (5, 25.5, 1, *Unconstrained*)

4.5.1.3 Case 3: (5, 25.5, 2, *Unconstrained*)

A means of obtaining a better control instead of spilling water (such as in case 2) is to increase the time span for which the head is to increase from $h(0)$ to $h(T)$ from one day to two days. The optimum head trajectory and the optimum control are as shown in Figure 4.20 and 4.21. The control is now positive for the time frame. No need of spilling water at JHEPS. Nevertheless, the control in the first quarter of the time seems high for all the three simulations so far.

Inputs										
$h(0)$	$h(T)$	$u_1(0)$	$u_2(0)$	$u_3(0)$	$u_4(0)$	No of Machines	Day	Δt	$u(\min)$	$u(\max)$
25.5	26.1	1000	1000	1000	1000	5	2	1800	Unconstrained	

Outputs								
No of Iterations	$u_1^*(t)$	$u_2^*(t)$	$u_3^*(t)$	$u_4^*(t)$	$I (\min J)$	$\ \nabla J\ $	$\psi(\min)$	$h^*(t_f)$
24	6774.2572	428.72696	1685.3244	1635.4193	4245.1465	0.3685796	0.0029686	26.112019

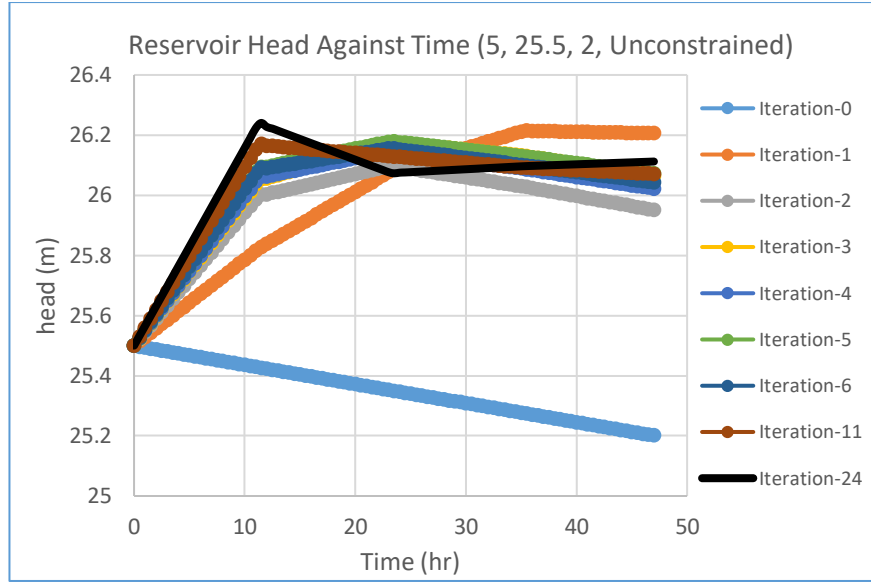


Figure 4.20: Reservoir Head versus Time (5, 25.5, 2, *Unconstrained*)

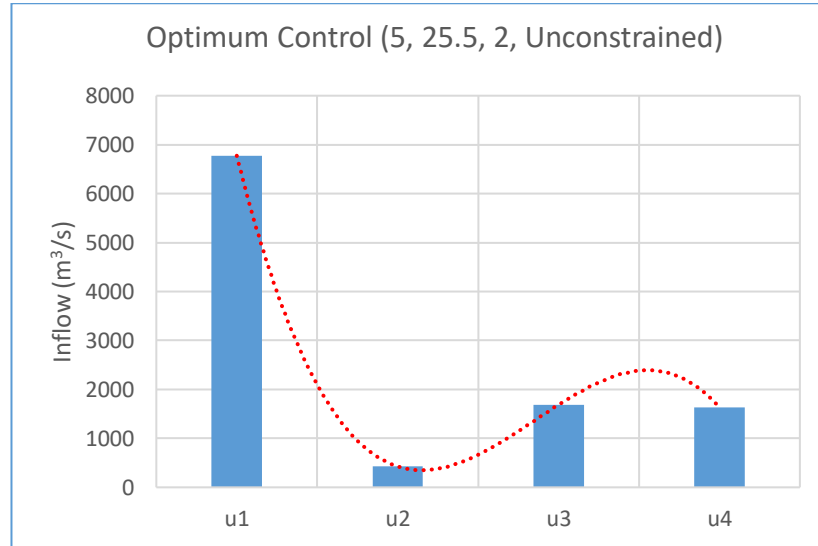


Figure 4.21: Optimum Control (5, 25.5, 2, *Unconstrained*)

4.5.1.4 Case 4: (5, 25.8, 1, *Constrained at u_{min} and u_{max}*)

It is possible to modify the problem such that the control is constrained. The constraint used in this work was to impose a maximum and minimum value on the control. The control is not allowing to be negative or exceed a value. Different simulation for constrained optimal control are presented in cases 4, 5 and 6. Figure 4.22 and 4.23 presents the results of the case (5, 25.8, 1, $u_{min} > 0$ and $u_{max} = 3000 \text{ m}^3/\text{s}$. The

effect is that the overshoot on the state trajectory is removed, and it takes more iterations. The optimum control looks more appropriate than the results produced with cases where the control is unconstrained.

Inputs										
$h(0)$	$h(T)$	$u_1(0)$	$u_2(0)$	$u_3(0)$	$u_4(0)$	No of Machines	Day	Δt	u_{min}	u_{max}
25.8	26.1	1000	1000	1000	1000	5	1	1800	Constrained	

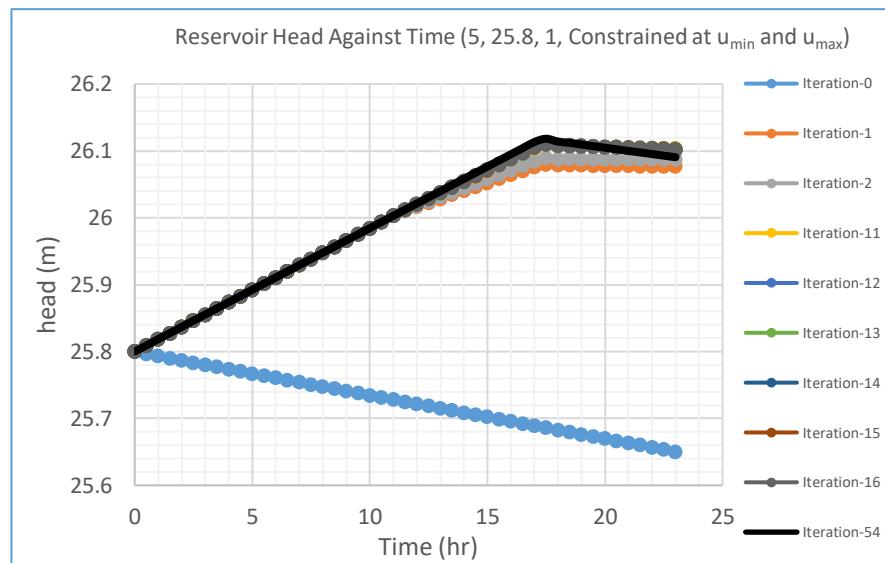


Figure 4.22: Reservoir Head versus Time (5, 25.8, 1, Constrained at u_{min} and u_{max})

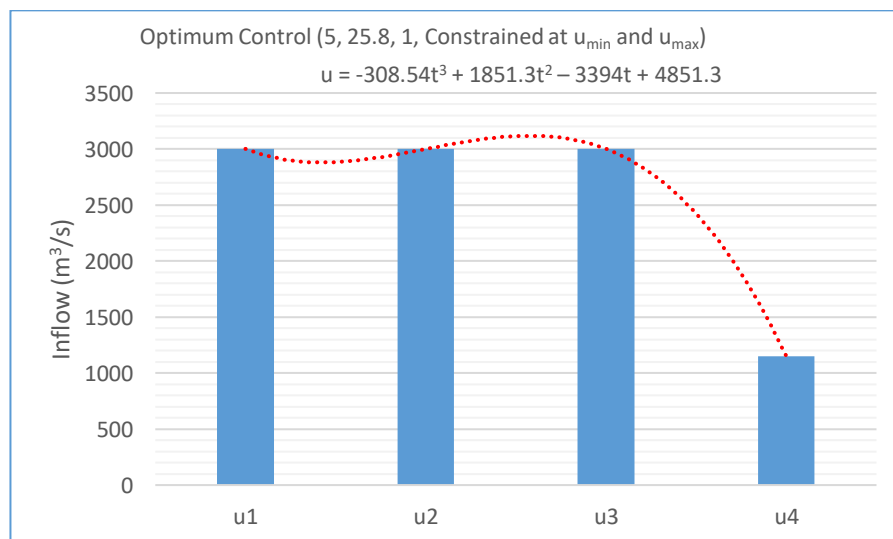


Figure 4.23: Optimum Control (5, 25.8, 1, Constrained at u_{min} and u_{max})

4.5.1.5 Case 5: (3, 25.8, 1, Constrained at u_{min} and u_{max})

The simulation results in Figure 4.24 and 4.25 are for the case with three operating machines while $u_{min} > 0$ and $u_{max} = 3000 \text{ m}^3/\text{s}$. The algorithm converges after one iteration and the optimal control reduces gradually after every quarter of time.

Inputs										
$h(0)$	$h(T)$	$u_1(0)$	$u_2(0)$	$u_3(0)$	$u_4(0)$	No. of Machines	Day	Δt	u_{min}	u_{max}
25.8	26.1	1000	1000	1000	1000	3	1	1800	Constrained	

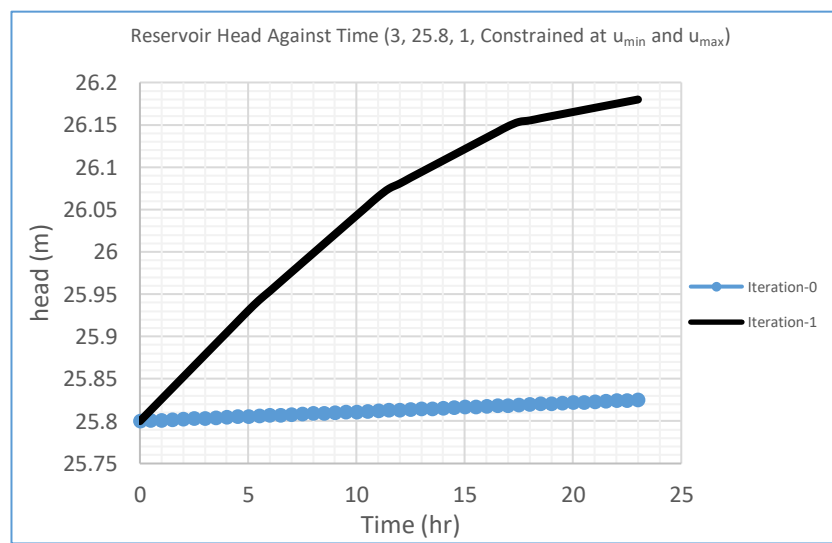


Figure 4.24: Reservoir Head versus Time (3, 25.8, 1, Constrained at u_{min} and u_{max})

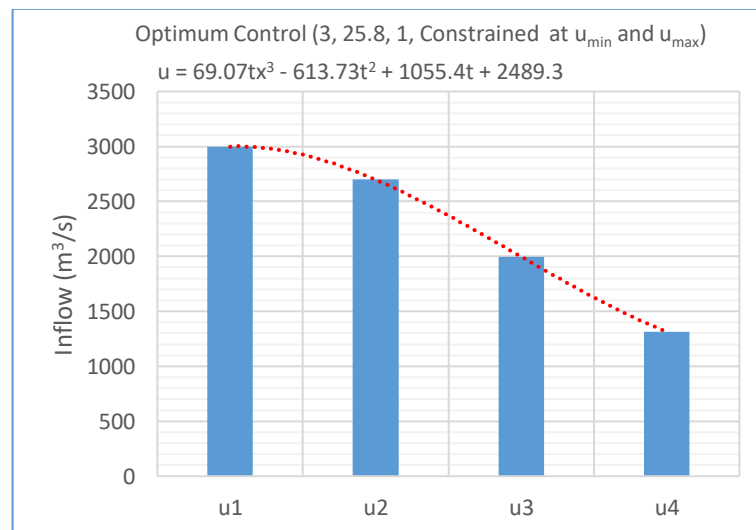


Figure 4.25: Optimum Control (3, 25.8, 1, Constrained at u_{min} and u_{max})

4.5.2 Computation of an Optimal Control by a Conjugate Gradient Method

The results obtained from computation of optimal control by a conjugate gradient method is in every way like that of steepest. Therefore, comparison is made to check if there are improvement in the performance of the algorithm.

The cases below shows the simulation performed to study the performance of conjugate gradient method as compared with the steepest descent method. The coordinate format used in steepest descent method is extended to this section, the format is as follows: (Number of Machines, Starting head, Number of Days, Constraints).

4.5.2.1 Case 1: (5, 25.8, 1, u_{max} unconstrained)

The head trajectories and optimal control of Figure 4.26 shows that the conjugate gradient method yielded a result that is very similar to its steepest descent counterpart except that former takes 10 iterations to converge as against 13 iterations with the latter, this can be seen in Figure 4.27.

In addition to the comparison in terms of number of iterations. The steepest descent method requires more integrations per iteration as compared to conjugate gradient, hence the algorithm is faster.

$h(0)$	$h(T)$	$u_1(0)$	$u_2(0)$	$u_3(0)$	$u_4(0)$	No of Machines	Day	Δt	u_{min}	u_{max}
25.8	26.1	1000	1000	1000	1000	5	1	1800	Constrained	Unconstrained

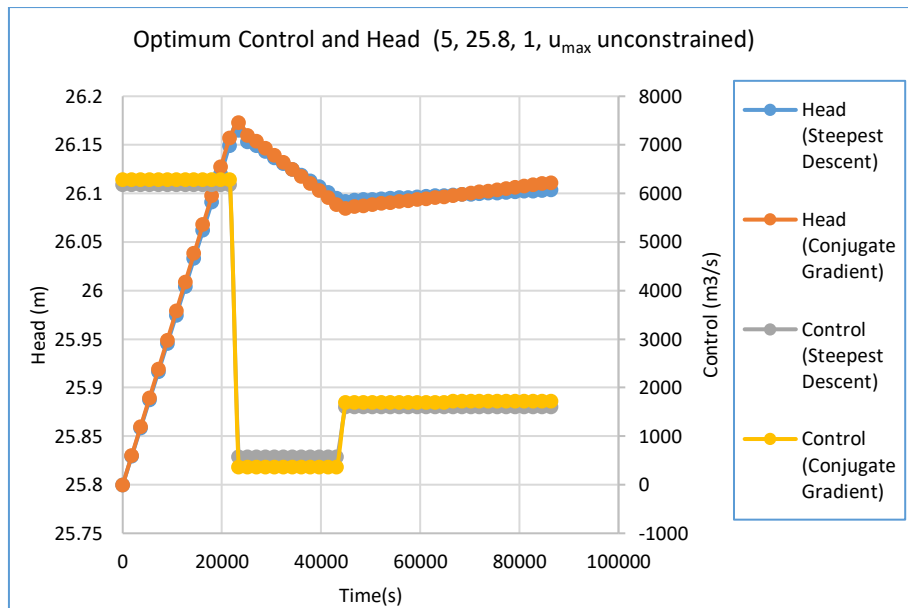


Figure 4.26: Optimum Control and Head for (5, 25.8, 1, u_{max} unconstrained)

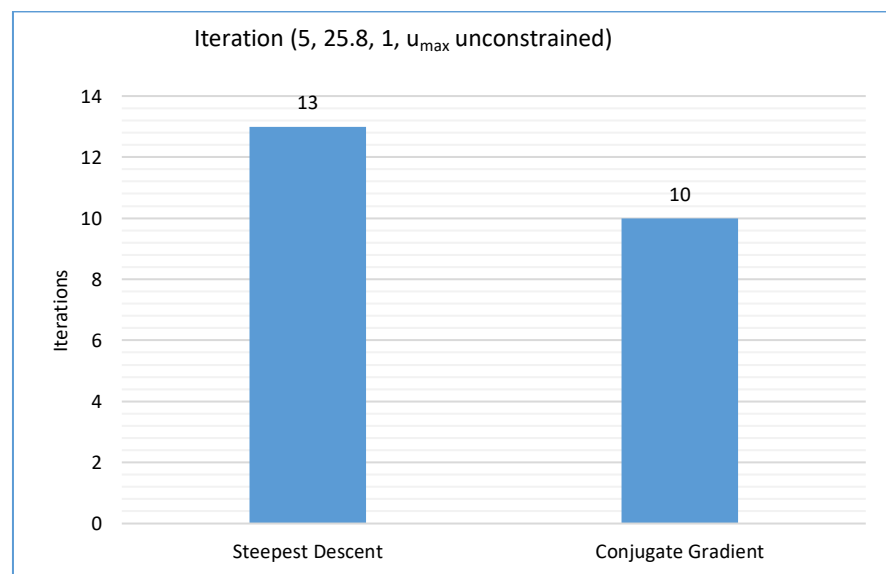


Figure 4.27: Comparison of convergence of Steepest Descent and Conjugate Gradient Technique for (5, 25.8, 1, u_{max} unconstrained)

4.5.2.2 Case 2: (5, 25.5, 1, $u_{max} = 4000 \text{ m}^3/\text{s}$)

It can be observed from Figure 4.28 that the conjugate gradient method will give an accurate result even if the maximum control is constrained. Figure 4.29 reveals that when there is a constraint on the maximum control, the conjugate gradient method will converge faster than the steepest descent method.

$h(0)$	$h(T)$	$u_1(0)$	$u_2(0)$	$u_3(0)$	$u_4(0)$	No of Machines	Day	Δt	u_{min}	u_{max}
25.5	26.1	1000	1000	1000	1000	5	1	1800	Constrained	4000

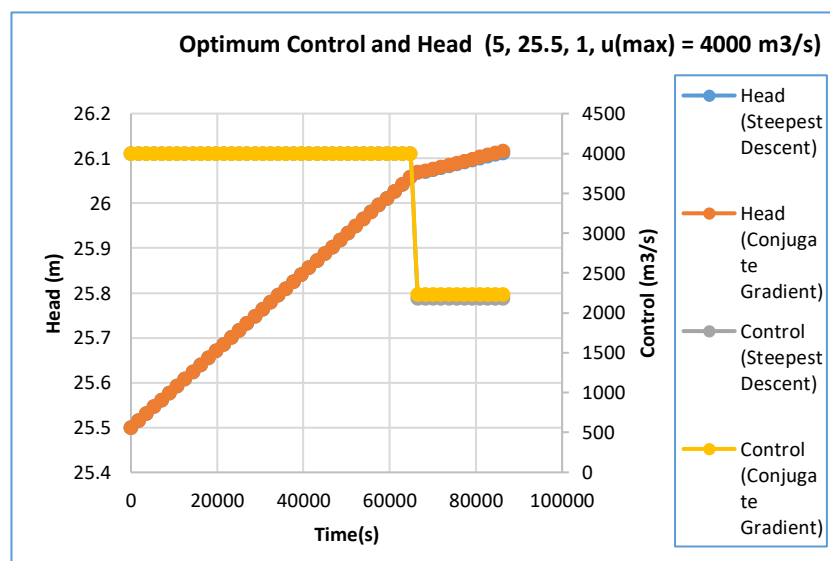


Figure 4.28: Optimum Control and Head for (5, 25.5, 1, $u_{max} = 4000 \text{ m}^3/\text{s}$)

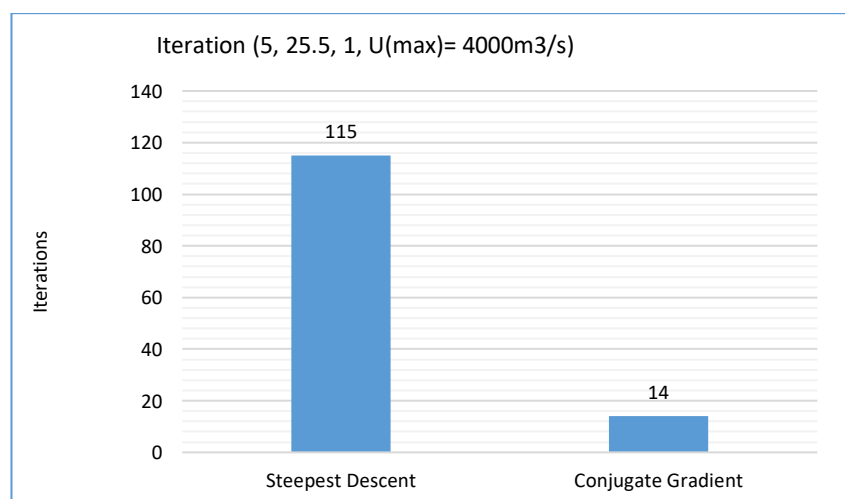


Figure 4.29: Comparison of convergence of Steepest Descent and Conjugate Gradient Technique for (5, 25.5, 1, $u_{max} = 4000 \text{ m}^3/\text{s}$)

4.5.2.3 Case 3: (4, 25.8, 1, $u_{max} = 4000 \text{ m}^3/\text{s}$)

From Figure 4.30 and 4.31, it can be deduced that the accuracy of conjugate gradient method may be slightly less, but the convergence time remain an advantage.

$h(0)$	$h(T)$	$u_1(0)$	$u_2(0)$	$u_3(0)$	$u_4(0)$	No of Machines	Day	Δt	u_{min}	u_{max}
25.8	26.1	1000	1000	1000	1000	4	1	1800	Constrained	4000

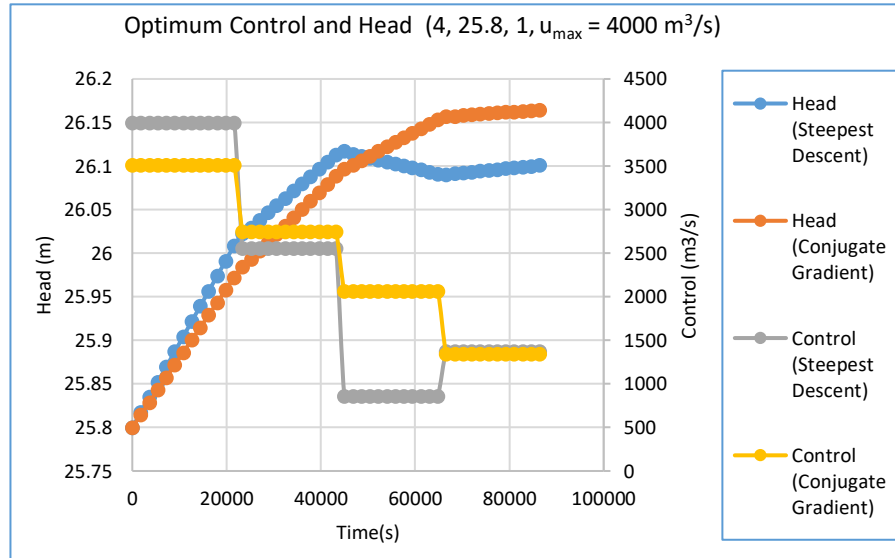


Figure 4.30: Optimum Control and Head for (4, 25.8, 1, $u_{max} = 4000 \text{ m}^3/\text{s}$)

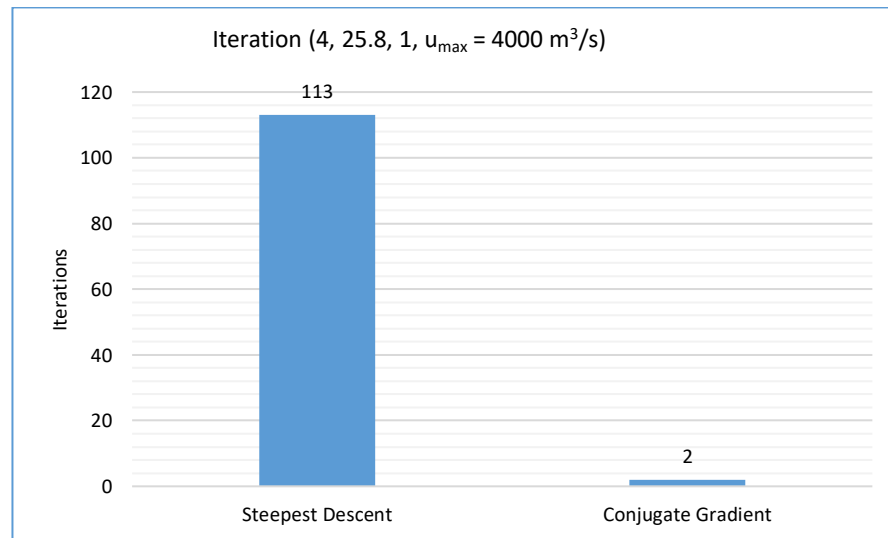


Figure 4.31: Comparison of convergence of Steepest Descent and Conjugate Gradient Technique (4, 25.8, 1, $u_{max} = 4000 \text{ m}^3/\text{s}$)

4.5.3 Computation of the Optimal Control by a Progressive Domain Expansion Method (PDEM)

This section presents the results obtained from the use of PDEM for solving a two-point boundary value problem produced by the optimal control canonical equations. It's better to define a presentation format that reflects important features of the trajectory many of which are deduced but not definitive by themselves in assessing a given trajectory. This leads to a very important matter of how to use the optimal control for operational purposes. As usual, the section nomenclature format remains the same: (Number of Machines, Starting head, Number of Days, Constraints).

To employ the tools developed here for operations, the different possibilities that may arise should form the basis of studies. As a result, one may outline many options: Let us first note that except some catastrophe occurs the head will not be allowed to drift far below the safe band for operations hence this provides a lower limit for studies. The next set of studies will focus on situations where for some reason the head is high and may continue to rise. What should be the control strategy? This can occur say during a rainy season when all plants in KHEPS are running but plants in JHEPS are down. The final case occurs when the head is within the tolerance and events may occur that affect inflows and outflows to JHEPS.

4.5.3.1 Case 1: (5, 25.8, 1, $u(T)$ *Unconstrained*)

Figure 4.32 presents the result for an indirect optimal control with the operational conditions of the machines and the initial value of the head at Jebba as stated above. It

can be observed that the optimal control is infinite dimensional as against the method used in the direct methods.

Time (hr)	No of Machines	Δt	$h(0)$	$h(T)$	$u(T)$	K_Q	$\lambda(0)$	$\lambda(T)$	$h^*(t_f)$
24	5	60	25.8	26.1	0	1E-08	-21904.87	1.00000009	25.9426172

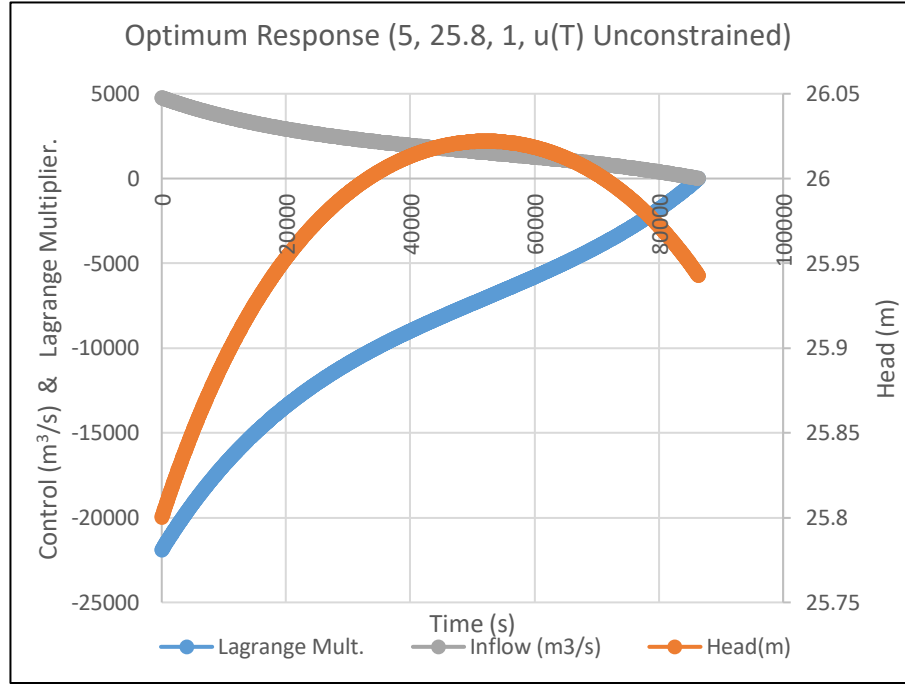


Figure 4.32: Optimum Response (5, 25.8, 1, $u(T)$ Unconstrained)

The control started from a value a little bit less than 5000 m³/s at t_0 and decreases gradually to zero at t_f . Because the control is unconstrained, the trajectory of the head could not reach the terminal value $h(T)$, the head also rose to a peak value and decreases later. Meanwhile the Lagrange multiplier shows the much-expected characteristics by moving from an initial value to 1 at t_f .

4.5.3.2 Case 2: (3, 25.8, 1, $u(T)$ Unconstrained)

The simulation in Section 4.5.3.1 Case 1 was repeated but with 3 units in operation, a similar result to that of Figure 4.32 was observe from Figure 4.33. It can be concluded

that to use PDEM, a constraint in the form of penalty on the terminal control can be incorporated into the algorithm. A proportional integral controller (PI) will also force the controls towards zero.

Time (hr)	No of Machines	Δt	$h(0)$	$h(T)$	$u(T)$	K_Q	$\lambda(0)$	$\lambda(T)$	$h^*(t_f)$
24	3	60	25.8	26.1	0	1E-08	-19309.5	1	25.997203

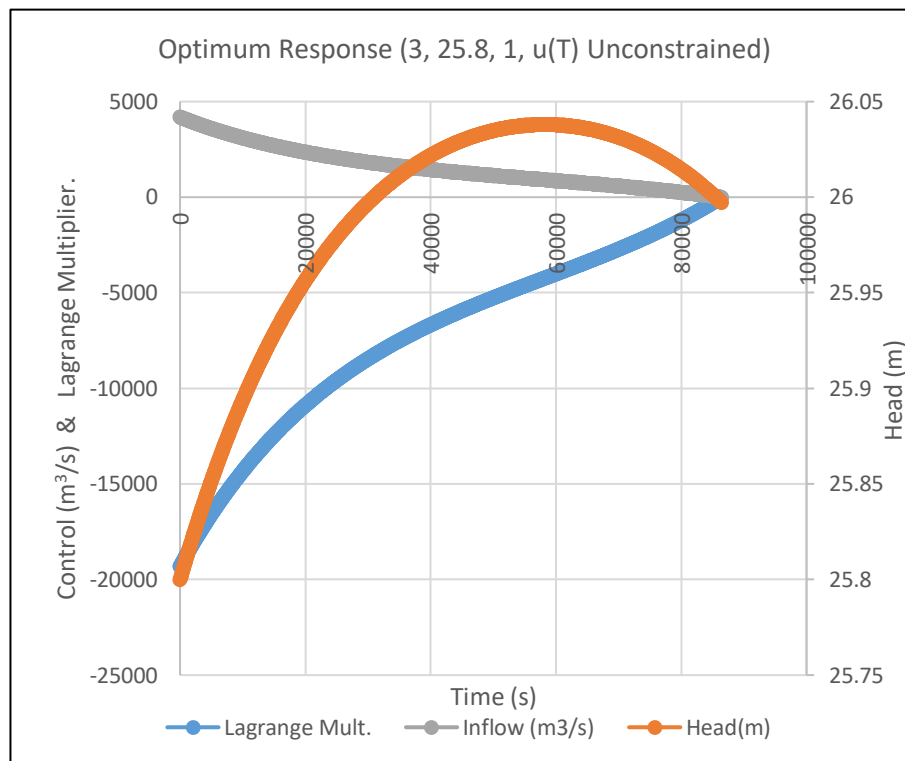


Figure 4.33: Optimum Response (3, 25.8, 1, $u(T)$ Unconstrained)

4.5.3.3 Case 3: (5, 25.8, 1, $u(T) = 1800 \text{ m}^3/\text{s}$)

Case 3 presents a similar condition to that of Section 4.5.3.1 but with constraint on the final control. This implies that the final control cannot decrease to zero, but a finite value specified as $u(T)$, this can be seen in Equation 3.46. The value of $u(T)$ depends on the result presented in Figure 4.9.

Figure 4.34 shows a better result of which the head moves from an initial value $h(0) = 25.8 \text{ m}$, to a final value $h(T) = 26.1 \text{ m}$ without an overshoot, as experienced in the direct optimal control. The optimal control defined as $u(t) = 5 \times 10^{-07}t^2 - 0.0702t + 4327.4$ starts around $4327.4 \text{ m}^3/\text{s}$ and ends at $1800 \text{ m}^3/\text{s}$. The plot of the Lagrange multiplier confirms that the solution is reliable.

Time (hr.)	No of Machines	Δt	$h(0)$	$h(T)$	$u(T)$	K_Q	$\lambda(0)$	$\lambda(T)$	$h^*(t_f)$
24	5	60	25.8	26.1	1800	1E-08	-15737.8	1	26.09715

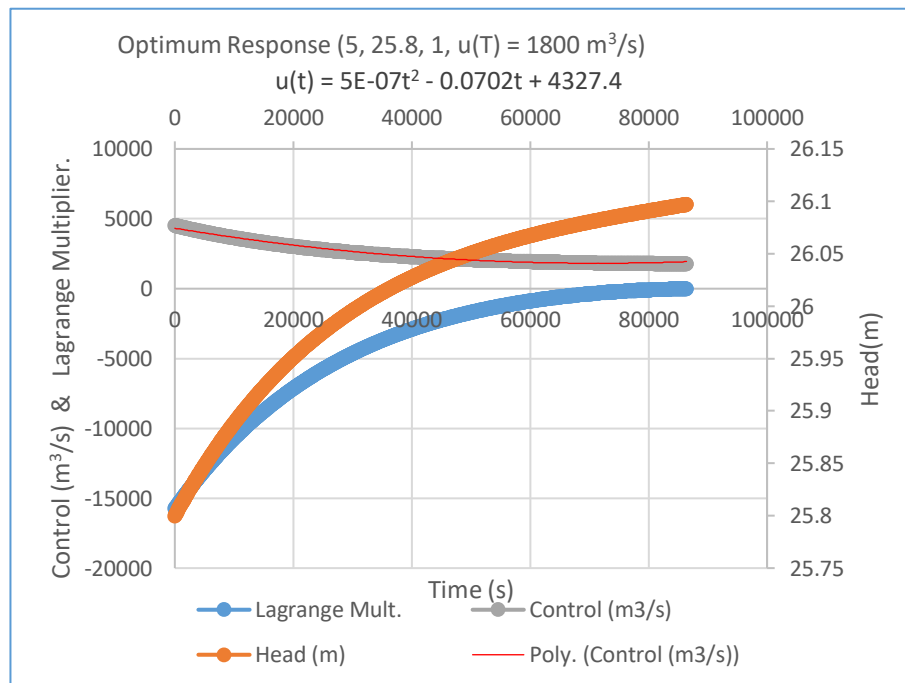


Figure 4.34: Optimum Response (5, 25.8, 1, $u(T) = 1800 \text{ m}^3/\text{s}$)

4.5.3.4 Case 4: (5, 25.8, 2, $u(T) = 1800 \text{ m}^3/\text{s}$)

Consider a situation where the time limit for which optimal control is desired is two days (48 hr.). It can be observed from Figure 4.35 that the PDEM could determine the solution as well except that the time does not have significant effect on the starting control $u(0)$. In Case 3, $u(0)$ started with $4327.4 \text{ m}^3/\text{s}$ but reduces slightly $4270.1 \text{ m}^3/\text{s}$ in Case 4.

Day	No of Machines	Δt	$h(0)$	$h(T)$	$u(T)$	K_Q	$\lambda(0)$	$\lambda(T)$	$h^*(t_f)$
48	5	60	25.8	26.1	1800	1E-08	-15676.5	1	26.12546

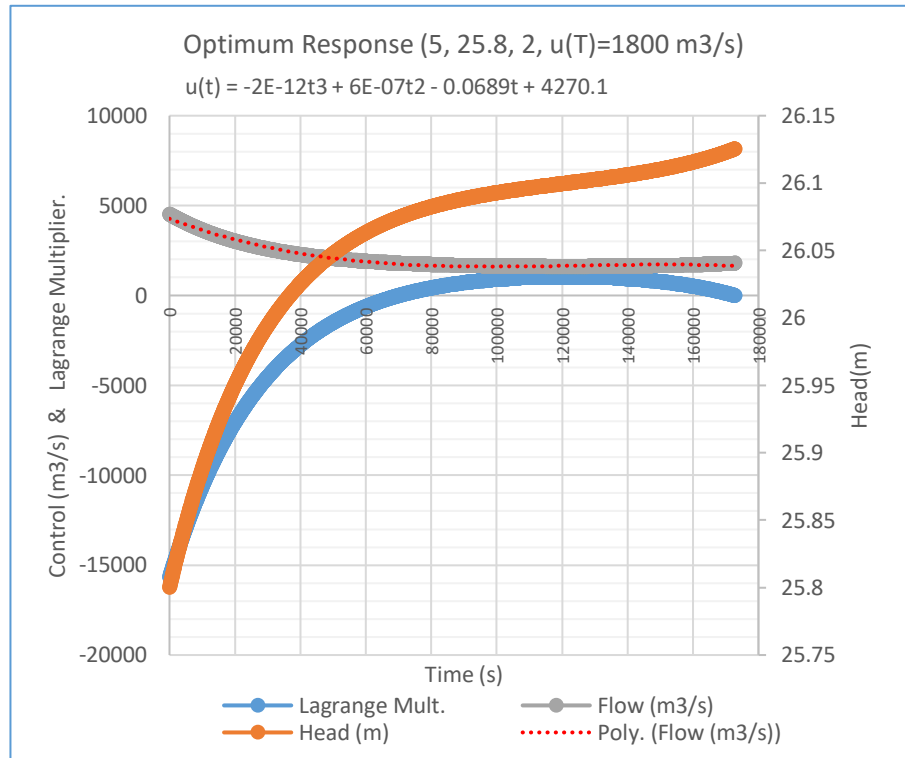


Figure 4.35: Optimum Response (5, 25.8, 2, $u(T) = 1800 \text{ m}^3/\text{s}$)

4.5.4 Computation of the Indirect Optimal Control by a Conjugate Gradient Method (CGM)

The section presents the results obtained by solving a two-point boundary value problem with a CGM. The case nomenclature used is the same for PDEM: (Number of Machines, Starting head, Number of Days, Constraints). Recall that CGM is an iterative technique, the last iteration will produce the optimal control and the optimal head trajectory. This should be noted in all the plots that will be discussed under this section.

4.5.4.1 Case 1: (5, 25.8, 1, $u(T)$ Unconstrained)

Figure 4.36 shows that the head trajectory will not respond as desired if the terminal control $u(T)$ is unconstrained, this is like the observation in section 4.5.3.1. A result like this will be expected because the optimal control as observed from Figure 4.37 starts from $4710.59 \text{ m}^3/\text{s}$ at t_0 and decreases gradually to zero at t_f .

Figure 4.36 to Figure 4.40 shows the performance of the CGM algorithm. If the plots are not as expected theoretically, the control cannot be said to be optimal. The co-state must start from a value that must be determined by the algorithm at t_0 and ends with 1 at t_f , this can be observed from Figure 4.38. In Figure 4.39, the gradient in the first iteration can be observed to be rising and this is contrary to the condition for optimality. Meanwhile, GGM could iterates quickly such that the gradient is observed showing a decreasing trajectory the fifth iteration. Finally, the condition for convergence of the algorithm can be observed in Figure 4.40, the performance index and norm of gradient must be decreasing with iteration until a tolerance value is reached.

Time(hr.)	No of Machines	Δt	$h(0)$	$h(T)$	K_Q	$u(0)$	$u(T)$
24	5	60	25.8	26.1	8.00E-09	3000	Unconstrained

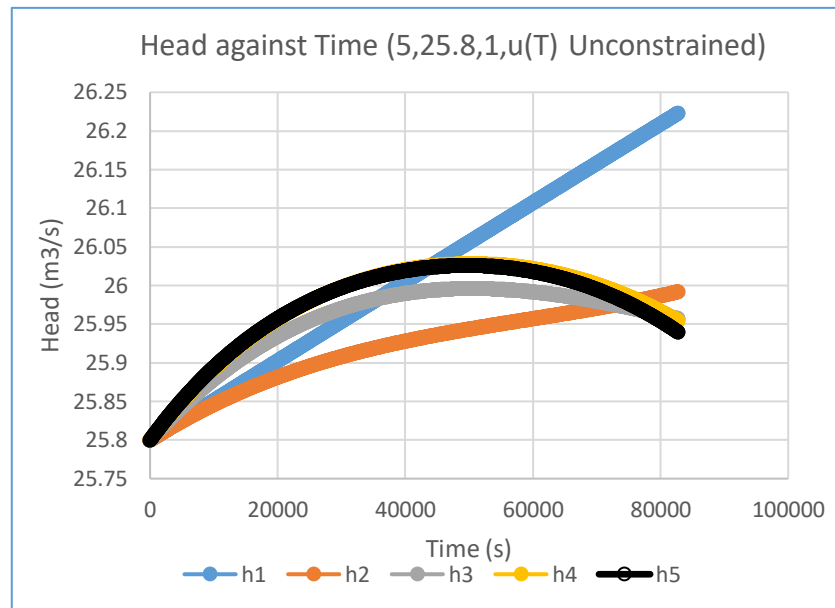


Figure 4.36: Head versus Time (5, 25.8, 1, $u(T)$ Unconstrained)

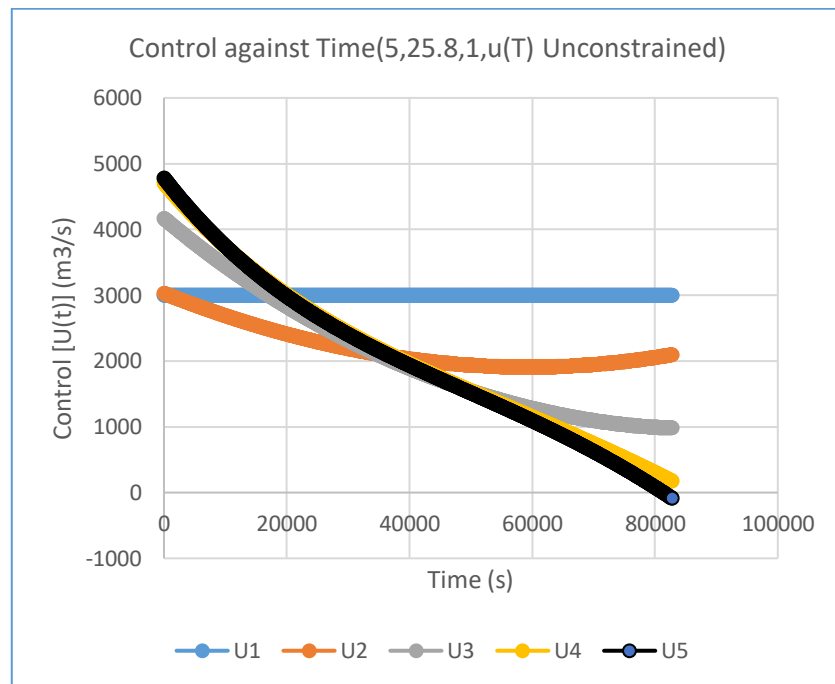


Figure 4.37: Control versus Time (5, 25.8, 1, $u(T)$ Unconstrained)

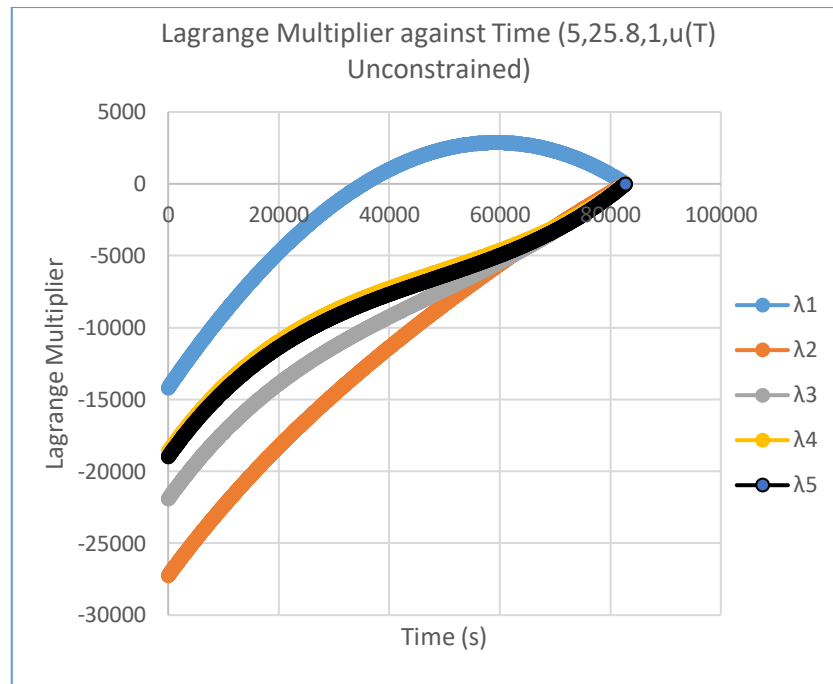


Figure 4.38: Lagrange Multiplier versus Time (5, 25.8, 1, $u(T)$ Unconstrained)

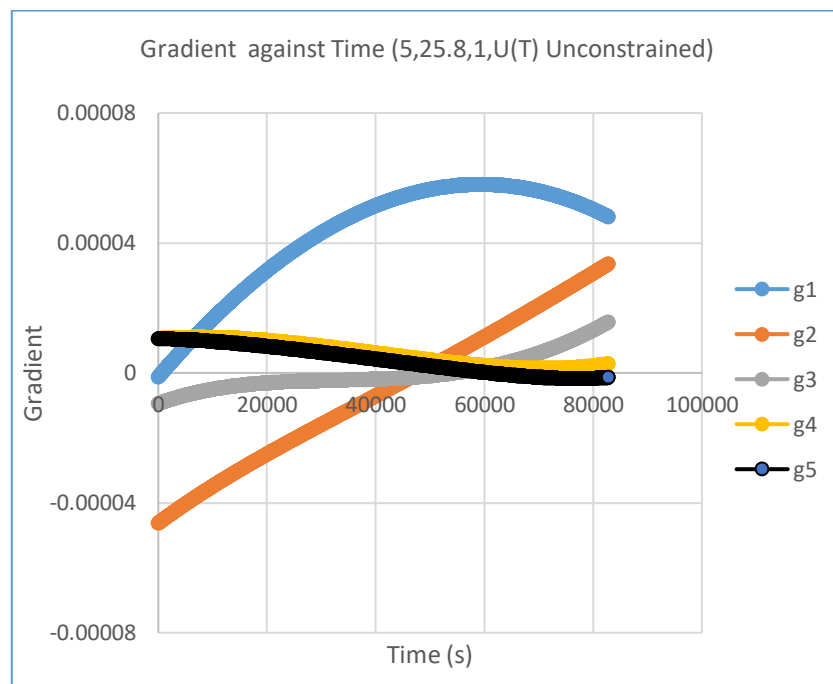


Figure 4.39: Gradient versus Time (5, 25.8, 1, $u(T)$ Unconstrained)

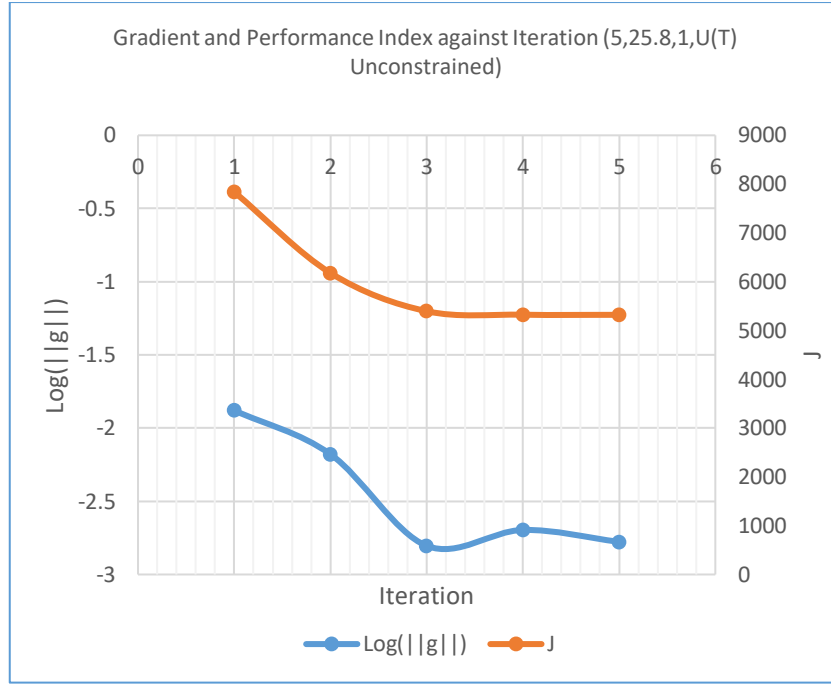


Figure 4.40: Gradient and Performance Index versus Iteration (5,25.8,1, $U(T)$ *Unconstrained*)

4.5.4.2 Case 2: (5,25.8,1, $u(T) = 1800 \text{ m}^3/\text{s}$) and $u_{(t)}^{(k=0)} = 2500 \text{ m}^3/\text{s}$

If a penalty is imposed on the control $u(T)$ in Case 1, CGM will produce the much-desired result. Figure 4.41 shows the head trajectory for the scenario in this Case 2 and the optimal control that produces such a highly desired response can be observed in Figure 4.42.

A comparison between Figure 4.37 and Figure 4.42 shows that a penalty on $u(T)$ increases the number of iteration taken for the algorithm to reach the convergence point.

It can also be said that the indirect solution to the optimal control solved in this work is superior to the direct solution as there is no overshoot in the trajectory of the head.

The head rises smoothly from 25.8m at t_0 to 26.1m at t_f .

Time(hr.)	No of Machines	Δt	$h(0)$	$h(T)$	K_Q	$u(0)$	$u(T)$
24	5	60	25.8	26.1	8.00E-09	2500	1800

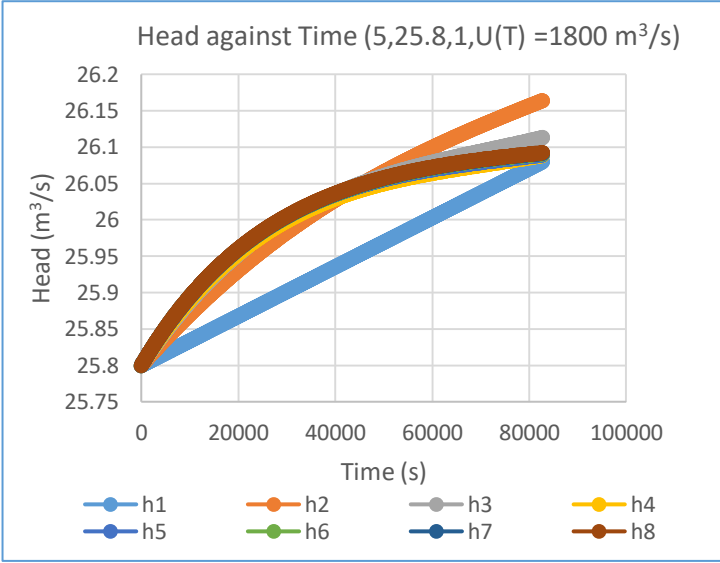


Figure 4.41: Head versus Time (5, 25.8, 1, $u(T) = 1800 \text{ m}^3/\text{s}$)

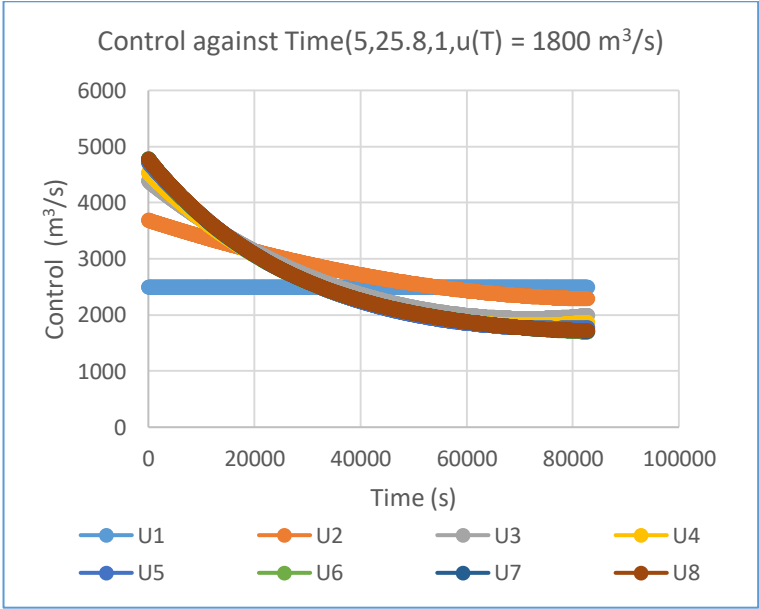


Figure 4.42: Control versus Time (5, 25.8, 1, $u(T) = 1800 \text{ m}^3/\text{s}$)

4.5.4.3 Case 3: (5, 25.8, 1, $u(T) = 1800 \text{ m}^3/\text{s}$) and $u_{(t)}^{(k=0)} = 500 \text{ m}^3/\text{s}$

A unique difference between Case 2 and Case 3 here is the initial guess for $u(t)$. In Case 2, the initial guess was $2500 \text{ m}^3/\text{s}$ as seen in Figure 4.42 but what will happen if this initial guess reduces to $500 \text{ m}^3/\text{s}$. As seen from Figure 4.43, the CGM will yielded the desired trajectory whereby the head rises from $h(0) = 25.8 \text{ m}$ to $h(T) = 26.1 \text{ m}$ in 24hr. The point to note is that this does not have a significant effect on the starting value for the optimal control shown in Figure 4.44. In fact, the number for iteration even decreases from eight to seven. Therefore, the determination of optimal control does not depend on the flow rate at time t_0 but on $h(0)$ and $h(T)$.

Time (hr.)	No of Machines	Δt	$h(0)$	$h(T)$	K_Q	$u(T)$
24	5	60	25.8	26.1	8.00E-09	1800

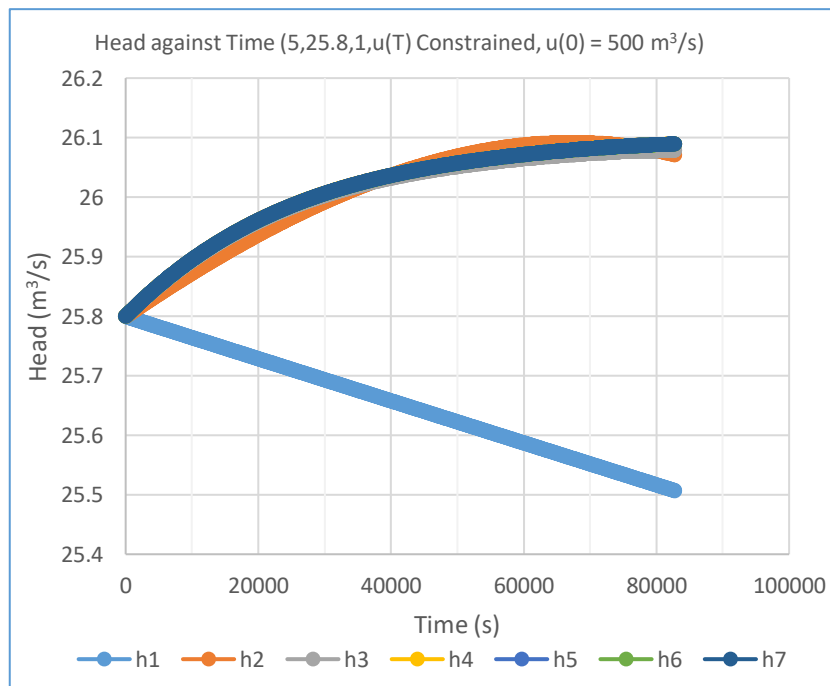


Figure 4.43: Head versus Time (5, 25.8, 1, $u(T)$ Constrained, $u_{(t)}^{(k=0)} = 500 \text{ m}^3/\text{s}$)

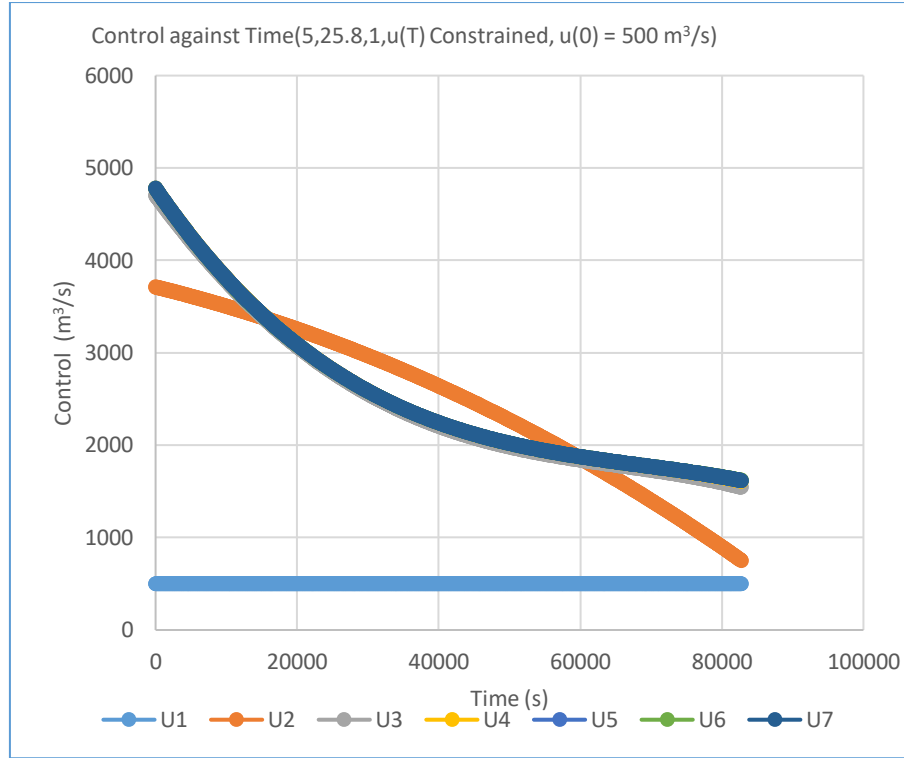


Figure 4.44: Control versus Time (5, 25.8, 1, $u(T)$ *Constrained*, $u(0) = 500 \text{ m}^3/\text{s}$)

4.5.4.4 Case 4: (5, 25.5, 3, $u(T) = 2000 \text{ m}^3/\text{s}$)

Case 4 considered a situation where the initial head is very low ($h(0) = 25.5 \text{ m}$) and the time interval between $t_0 \rightarrow t_f$ is 36 hr. Figure 4.45 and Figure 4.46 shows that the algorithm will not fail in determining the optimal control in such situation. It can be observed that a case like this may produce a trajectory of the head and control that tends to oscillate. Also, the required control may also seem higher and a direct optimal control with constraints may be more appropriate in a case like this.

Time (hr.)	No of Machines	Δt	$h(0)$	$h(T)$	K_Q	$u(T)$
36	5	60	25.5	26.1	8.00E-09	Constrained

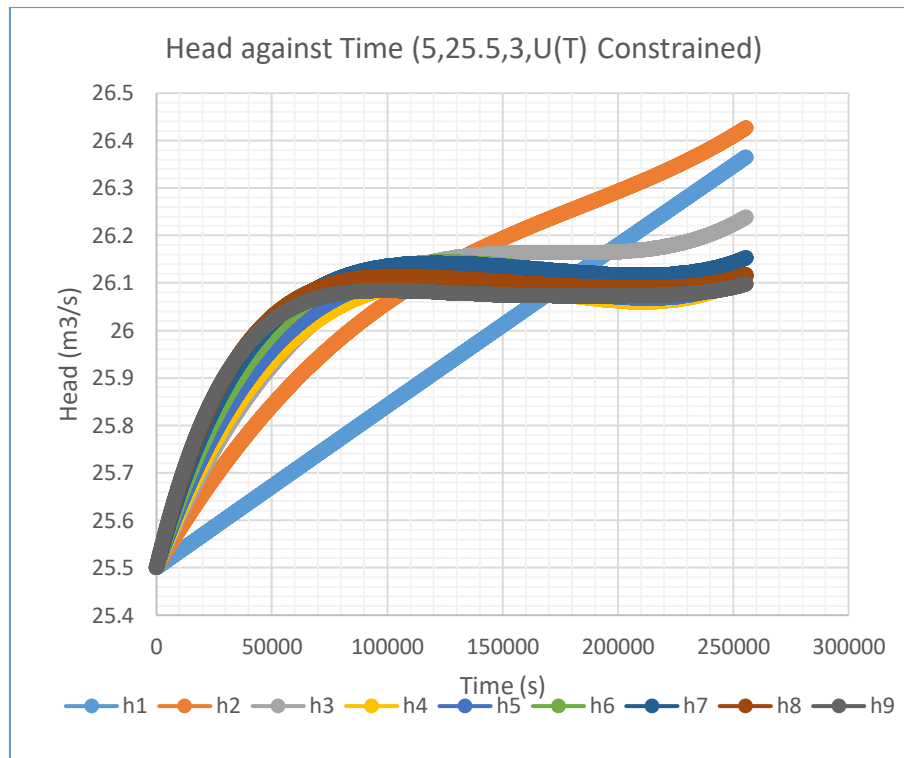


Figure 4.45: Head versus Time (5, 25.5, 3, $U(T)$ Constrained)

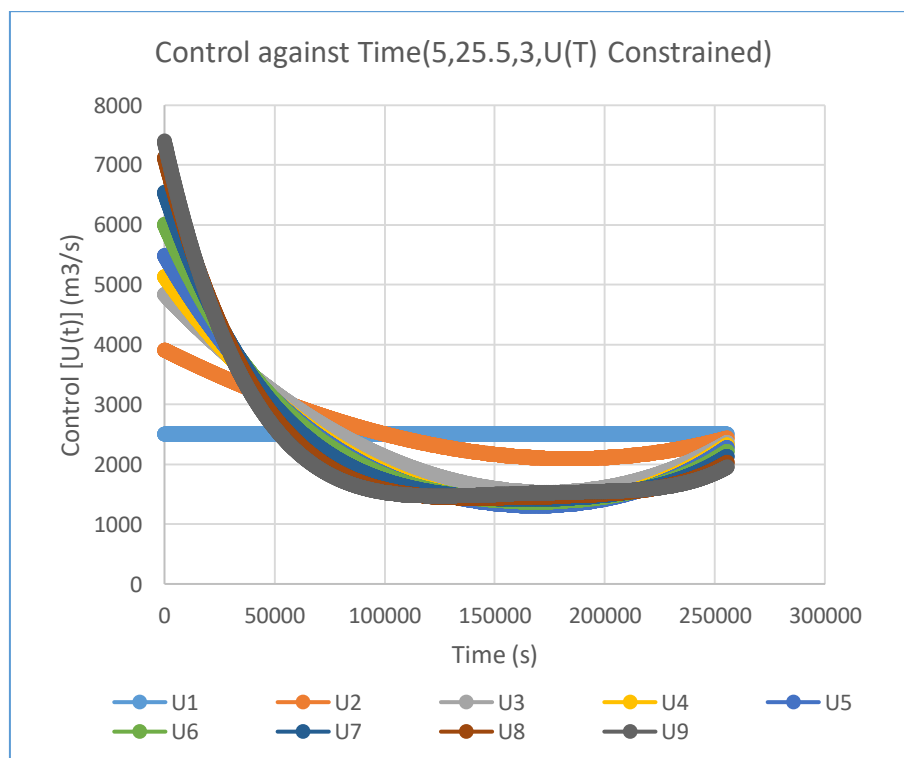


Figure 4.46: Control versus Time (5, 25.5, 3, $U(T)$ Constrained)

4.5.5 Approximate Indirect Optimal Control

The infinite dimensional optimal control resulting from the PDEM and CGM may not be easier to realize in practice since precise spillway control is seldom possible. It implies that there is a distinct control $u(t)$ for every time t . A controller that will release water in such manner must be very sophisticated. Hence an approximate indirect optimal control can be obtained where by the average control after six hours interval of time is determined and used as the optimal control for the section of time.

The cases in sections 4.5.3.3 and 4.5.4.4 were repeated, an approximate optimal control was computed as a discrete optimal control and plotted in Figure 4.47 and Figure 4.48. When the controls were passed into the system, it can be observed from Figure 4.49 and Figure 4.50 that the system responded with the much-desired head trajectory. It can be concluded that the solution of optimal control for a hydropower station can be solved using direct or indirect approach.

Time (hr.)	No of Machines	Δt	$h(0)$	$h(T)$	$u(T)$
24	5	60	25.8	26.1	Constrained

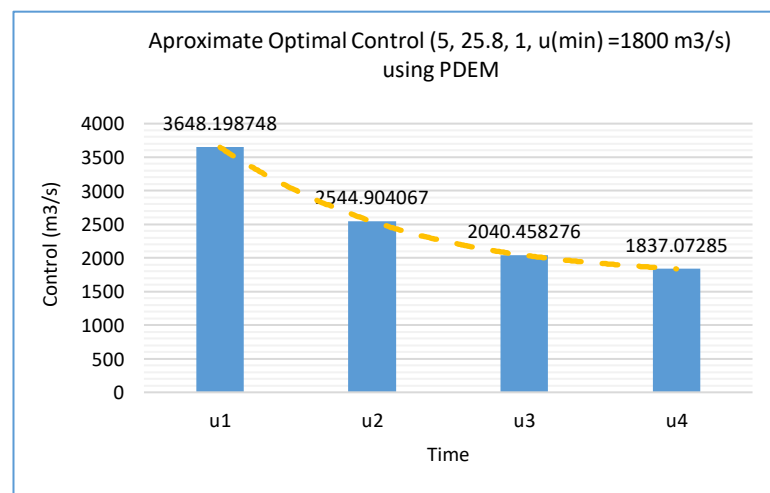


Figure 4.47: Approximate Optimal Control for (5, 25.8, 1, $u(\min) = 1800 \text{ m}^3/\text{s}$) using PDEM

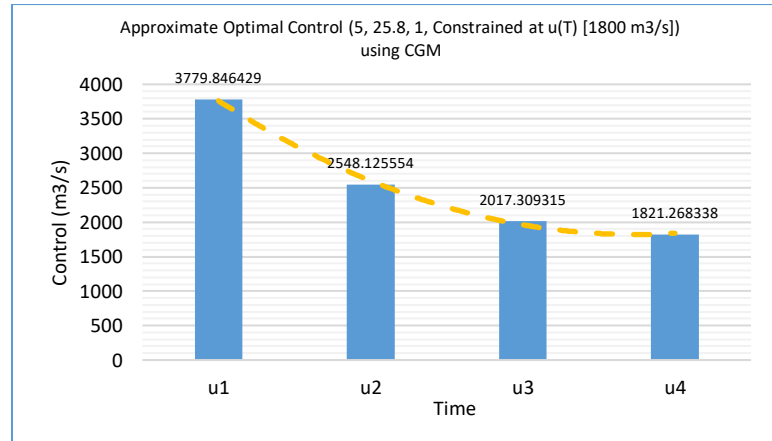


Figure 4.48: Approximate Optimal Control for $(5, 25.8, 1, u(\min) = 1800 \text{ m}^3/\text{s})$ using CGM

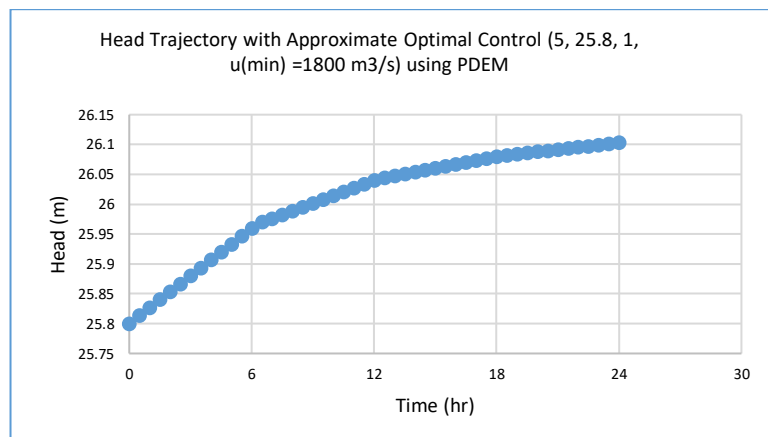


Figure 4.49: Head Trajectory with Approximate Optimal Control for $(5, 25.8, 1, u(\min) = 1800 \text{ m}^3/\text{s})$ using PDEM

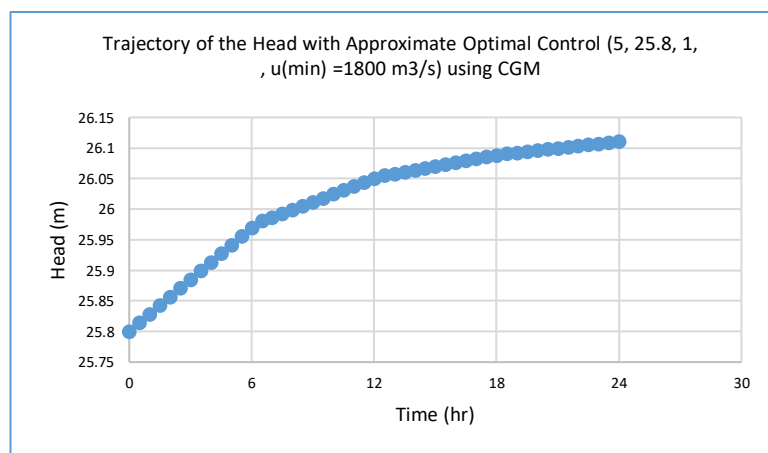


Figure 4.50: Head Trajectory with Approximate Optimal Control for $(5, 25.8, 1, u(\min) = 1800 \text{ m}^3/\text{s})$ using CGM

4.5.6 Comparison of PDEM and CGM of Solving Optimal Control Canonical Equations

Two indirect solution to the determination of optimal control were used in this work, it will be appropriate to compare the results produced by methods; PDEM and CGM.

The optimal head trajectory and optimal control plots of Figure 4.51 and Figure 4.52 looks almost similar. In fact, the difference is in terms of the weighted constant K_Q which are not the same for the two methods of solution. Any of the two methods will produce the needed optimal control for a desired head trajectory.

Meanwhile, if the results are compared in terms of performance, it should be noted that the initial guess of $\lambda(t_0)$ for PDEM must be chosen intelligently and require experience, else the solution diverges. CGM only requires any guess of $u^{(k=0)}(t)$ and the algorithm iterates to the desired solution.

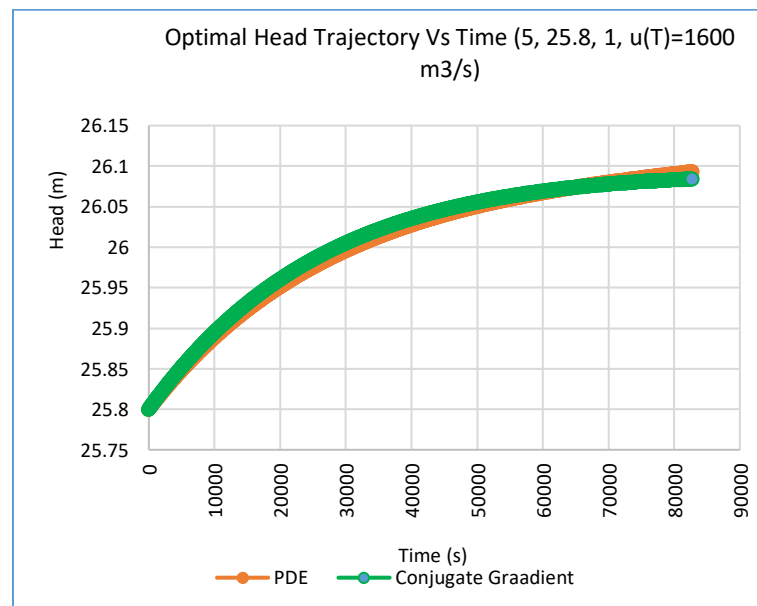


Figure 4.51: Head versus Time (5, 25.8, 1, $u(T) = 1600 \text{ m}^3/\text{s}$)

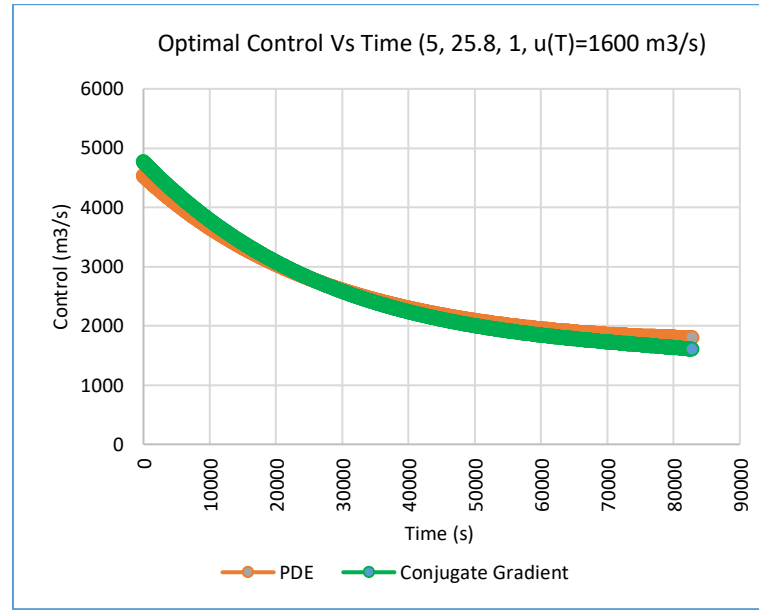


Figure 4.52: Control versus Time (5, 25.8, 1, $u(T) = 1600 \text{ m}^3/\text{s}$)

Nevertheless, once the initial guess of $\lambda(t_0)$ for PDEM is within tolerance limit, the method converges faster and less computationally intensive as compared to CGM and other methods of solutions in literature. Also, PDEM consumes less memory space than steepest descent and CGM (direct and indirect).

4.5.7 Quasi-State Feedback Optimal Control

One of the celebrated solution of optimal control is the determination of a control signal that is a function state feedback. This is popular for linear optimal control and an example is the optimal control of linear regulator.

A similar approach can be solved for a nonlinear optimal control that is the case study in the work. The problem is as follows:

Given the nonlinear system of equation defined as $\dot{h}_{(t)} = \mathbf{f}(h_{(t)}, u_{(t)}, t)$, a zero input $u(t_0) = 0$ and a non-zero initial state $h(t_0) = h_0$, where $h(t_0)$ is the excitation to the system.

The problem is to determine the optimal control signal $u_{(t)} = \mathbf{K}(h(t), t)$ that minimises the performance index $J(h, u, t)$. This can be described by the block diagram of Figure 4.53.

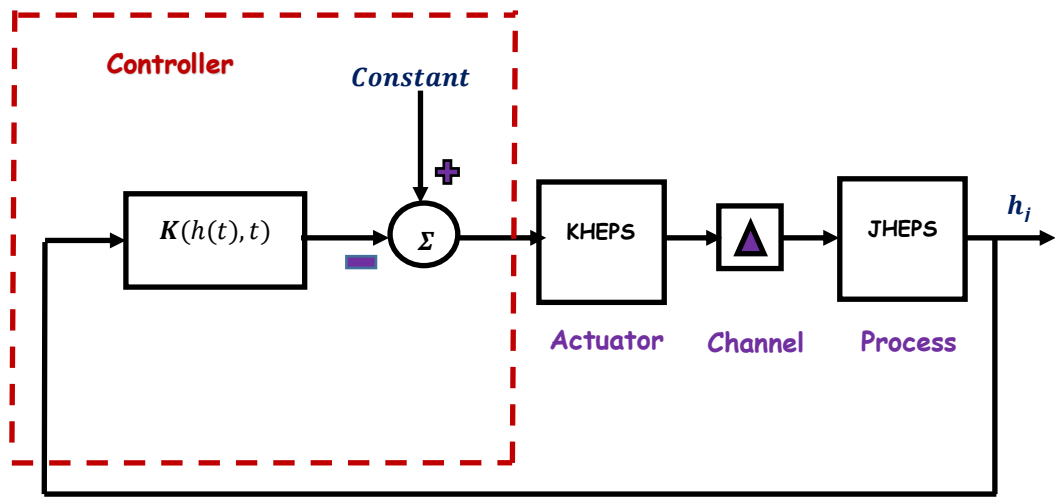
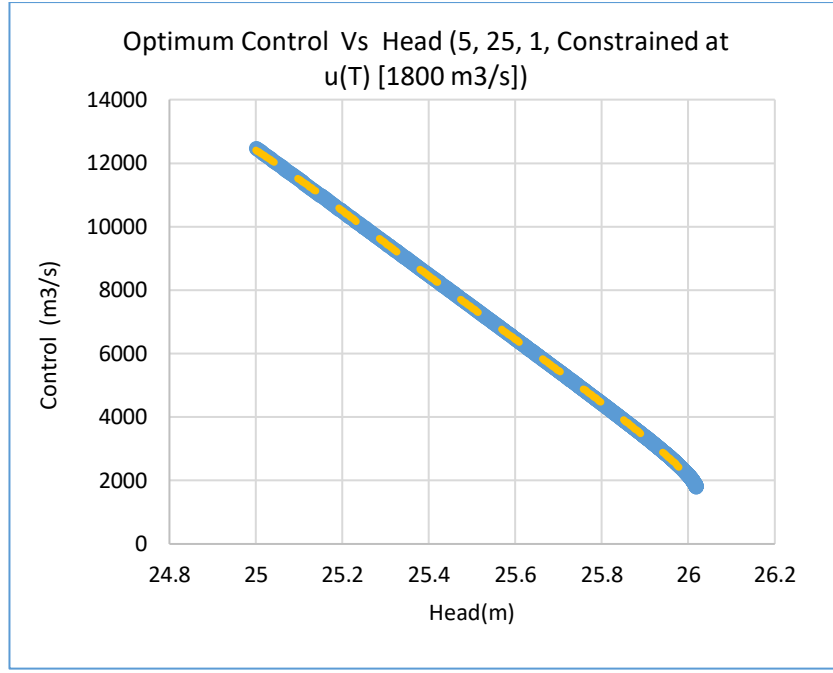


Figure 4.53: Quasi-State Feedback Optimal Control Block Diagram

Consider the determination of the optimal control $u_{(t)} = \mathbf{K}(h(t), t)$ for the problem with $(5, 25, 1, u(T) = 1800 \text{ m}^3/\text{s})$. Firstly, a PDEM or CGM can be used to solve for the nonlinear optimal control and the resulting head trajectory. The Optimal control is then plotted against the head as shown Figure 4.54. It can be observed that the relationship is nonlinear if the whole-time interval $t_0 \rightarrow t_f$ is considered.

Time (hrs.)	No of Machines	Δt	$h(0)$	$h(T)$	$u(T)$	$h^*(t_f)$
24	5	60	25	26.1	1800	26.00367

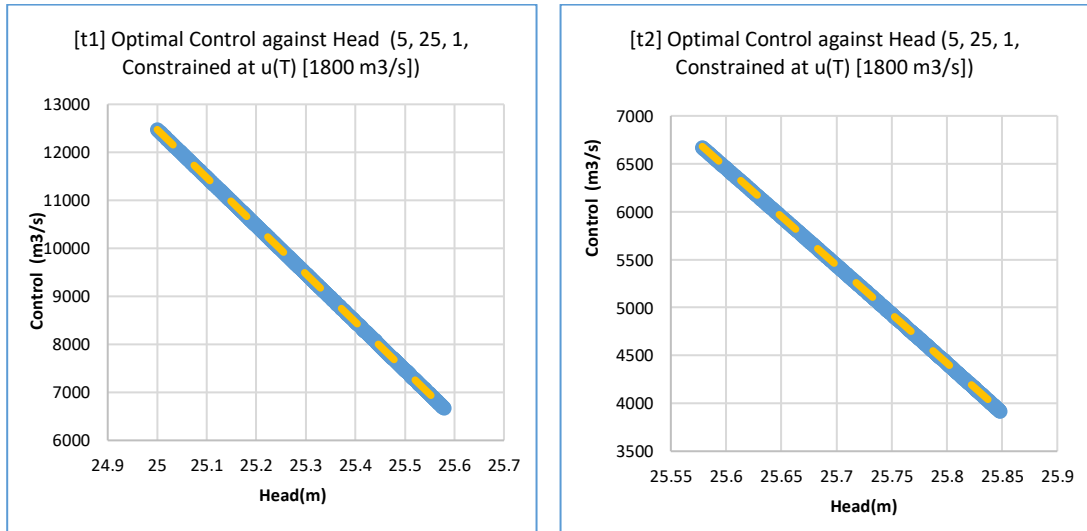


$$u = -6359.3h^4 + 647557h^3 - 2 \times 10^{07}h^2 + 4 \times 10^{08}h - 3 \times 10^{09}$$

Figure 4.54: Quasi-State Feedback Control

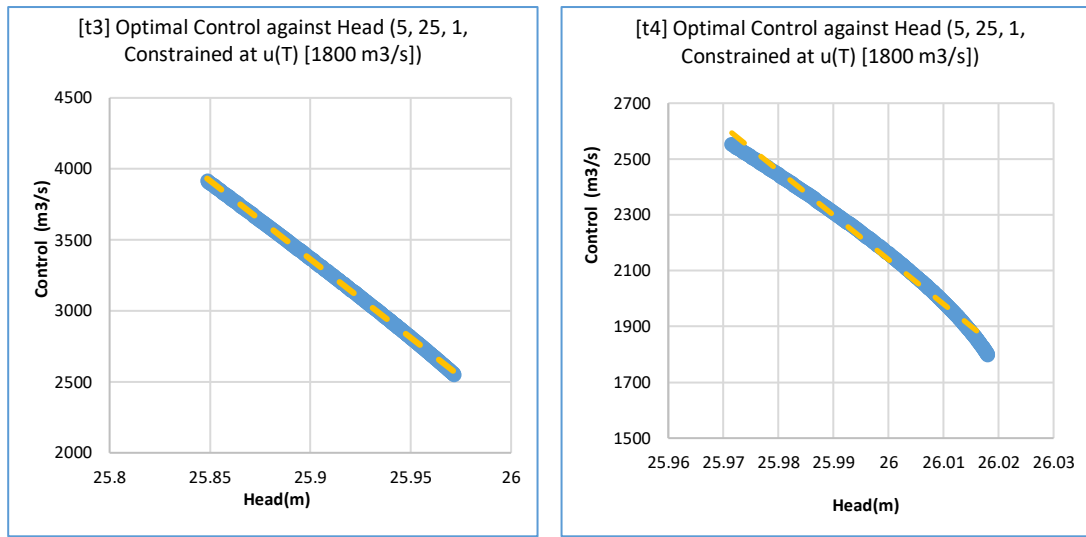
A linearized version can be obtained by partitioning the interval $t_0 \rightarrow t_f$ into four. The relationship between the control and head can then be obtained from Figure 4.55.

$$u_1 = f(h, t_1), u_2 = f(h, t_2), u_3 = f(h, t_3) \text{ and } u_4 = f(h, t_4).$$



$$u_1 = -10022h(t) + 263022$$

$$u_2 = -10204h(t) + 267676$$



$$u_3 = -11059h(t) + 289797$$

$$u_4 = -15942h(t) + 416628$$

Figure 4.55: Approximate Quasi-State Feedback Control for (5, 25, 1, $u(T) = 1800 \text{ m}^3/\text{s}$)

4.6 Chapter Summary

As stated in the Chapter Two and established from the model equations, numerical solutions are employed in solving nonlinear and optimal control problems. Solution to the model equation using the Adams-Moulton technique with a Runge-Kutta starter shows that the technique is stable converges faster and hence appropriate for solving the mild nonlinear model. Where systems or model parameters are not available, the estimation from observation becomes the option. This has been demonstrated in this chapter where the effective reservoir surface area and effective cross-sectional area of the scroll casing were estimated from observations.

The results of optimal control produced by steepest descent, direct conjugate gradient, Progressive expansion of domain and indirect conjugate gradient were presented. Comparisons were made to see the potentials and limit of each techniques. The numerical computation of controls using routines for the Microsoft EXCEL-VBA® environment.

CHAPTER FIVE

Contributions, Conclusion, and Recommendation

5.1 Contributions of the Research

From the results presented earlier, it is possible to see that a causal approach and the effect of cascade operations of hydropower stations, hitherto not considered by earlier research, is the thrust of this work. During the process of the work, the following contributions have been made:

- i. A framework for analysis and control of the cascaded pair, KHEPS and JHEPS, comprising a database of relevant reservoir and power generated in the past ten years and causal models that relate the state of each project to the prevailing power generated depending on the inflows and evaporation.
- ii. The models were posed so that when solved, they provide a basis for predicting reservoir conditions depending on the prevailing history of inflows, evaporation and power generation. Subsequently an optimal control problem that facilitates the management of water at both dams such as to ensure safe operations was formulated and solved.
- iii. A library of macro routines was developed using Microsoft EXCEL-VBA for solving the nonlinear dynamical equations that described the operations of KHEPS and JHEPS. A study of the modelling after calibration of its parameters provided a relative accurate prediction of the state of each of the reservoirs.
- iv. Tools were also developed for the solution of one of the important optimal control problems that best addresses a major difficulty that has confronted and frustrated engineers running the stations from the outset. The tools included steepest descent

and conjugate gradient algorithms for the direct generation of control as well as those that provide accurate solutions of the control canonical equations by an innovative progressive domain extension technique or a conjugate gradient method.

- v. A training module that can be adapted to other hydropower stations was developed and training simulations were conducted for the cascaded pair that is the target of this work.

5.2 Conclusion

In conclusion, the research successfully addressed an extant frustrating difficulty confronting the operators of the cascaded power stations by formulating and solving an optimal control problem using available information and innovative computational methods realized on the ubiquitous EXCEL-VBA® platform. Dynamical models were developed from flow continuity conditions while the optimal control problem was formulated and solved using both direct and indirect methods. The solutions to the indirect optimal control problem which devolved into the well-known canonical equations with its two-point boundary value problem were successfully resolved using PDEM and CGM algorithms. The solutions are highly accurate, less computationally intensive and manifest fast convergence. The developed model, together with the optimal control algorithms will work well to provide a more dependable operating scheme for the operators.

5.3 Recommendation

The work reported here addresses one of a plethora of problems that confront operators of hydropower stations. It is recommended that the ownership of the stations and government be encouraged to adopt the results of this work for planning, operations,

training of engineers and management as well as to support further work to complete the control design process.

5.4 Suggestions for Further Research

This work is the tip of an iceberg that represents the operations of cascaded hydropower systems. Indeed, given the trend to promote renewable power and the length of the River Niger, other stations cannot be ruled out in the future. They will provide a basis for the development of efficient, safe and reliable power if the work reported here is extended to cover other relevant issues. Such issues may include:

- i. Other control problems that will broaden the aspects of operations that are yet to be addressed and the solutions composed into an operational aid for the safe and optimal operation of hydropower stations;
- ii. optimal feedback controls computed using linearized models once the reservoirs are operating within the recommended range of heads; and
- iii. stochastic factors and parametric uncertainties that play important roles in the control process. These can be addressed as stochastic control problems and robust control respectively.

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Appendix A

Sample of Inflow and Elevation Data for KHEPS

MONTH MAX.ENERGY TO DATE :-					0.00	MN.PK :	4635.40	EV.PK:	4101.90			
149.6		TOTAL			K A I N J I P . S.							
DATE	NATIONAL PEAK			NATIONAL L	H.W.E		T.W.E	G.O.H	AV.TD	AV.SP	TOTAL	COMPT D
	DAY	PEAK (MW)	OFF-PEAK (MW)	ENERGY (MWH)								
01/12/2015	TUE	4228.80	2769.80	89641.11	141.07	462.82	103.30	37.77	628.00	400.00	1028	1028.00
02/12/2015	WED	4455.00	3804.30	97916.65	141.07	462.82	103.25	37.82	626.00	400.00	1026	1026.00
03/12/2015	THU	4408.30	3585.10	96885.74	141.08	462.86	103.01	38.07	620.00	400.00	1020	1146.00
04/12/2015	FRI	4092.90	3552.1	90170.01	141.10	462.92	102.95	38.15	628.00	400.00	1028	1324.00
05/12/2015	SAT	4017.50	3533.80	90449.77	141.10	462.92	102.90	38.20	631.00	400.00	1031	1031.00
06/12/2015	SUN	4232.90	3751.80	93524.55	141.12	462.99	102.90	38.22	628.00	400.00	1028	1281.00
07/12/2015	MON	4359.90	3743.00	96192.05	141.12	462.99	102.70	38.42	626.00	400.00	1026	1026.00
08/12/2015	TUE	4135.40	3197.20	90573.87	141.13	463.02	102.55	38.58	567.00	400.00	967	1098.00
09/12/2015	WED	4251.30	3841.80	96097.71	141.14	463.05	102.40	38.74	617.00	400.00	1017	1191.00
10/12/2015	THU	4245.60	3687.90	94275.06	141.16	463.12	103.20	37.96	620.00	400.00	1020	1279.00
11/12/2015	FRI	4151.90	3825.60	94768.31	141.18	463.18	102.40	38.78	622.00	339.00	961	1265.00
12/12/2015	SAT	4193.80	3747.80	94667.10	141.19	463.22	102.40	38.79	749.00	400.00	1149	1280.00
13/12/2015	SUN	4225.30	3386.00	89525.83	141.18	463.18	102.35	38.83	972.00	287.00	1259	1128.00
14/12/2015	MON	4387.50	3871.20	98332.77	141.19	463.22	102.69	38.50	971.00	86.00	1057	1188.00
15/12/2015	TUE	4343.60	3589.40	93953.70	141.24	463.38	102.50	38.74	893.00	16.00	909	1603.00
16/12/2015	WED	3487.30	2931.80	76222.09	141.25	463.41	102.40	38.85	628.00	394.00	1022	1196.00
17/12/2015	THU	3299.10	2758.60	71718.45	141.25	463.41	102.65	38.60	628.00	610.00	1238	1238.00
18/12/2015	FRI	3517.70	3133.70	77258.36	141.25	463.41	102.60	38.65	625.00	505.00	1130	1130.00
19/12/2015	SAT	4260.20	3234.80	86035.90	141.22	463.31	102.20	39.02	686.00	402.00	1088	654.00
20/12/2015	SUN	4243.90	3332.80	92168.68	141.24	463.38	102.45	38.79	944.00	402.00	1346	1606.00
21/12/2015	MON	4344.90	3850.60	96058.29	141.24	463.38	102.74	38.50	966.00	348.00	1314	1314.00
22/12/2015	TUE	4185.80	3717.20	93927.07	141.24	463.38	102.84	38.40	907.00	193.00	1100	1100.00
23/12/2015	WED	4292.10	3899.70	95774.02	141.24	463.38	102.70	38.54	968.00	152.00	1120	1120.00
24/12/2015	THU	4507.50	3827.70	97621.73	141.25	463.41	102.70	38.55	967.00	152.00	1119	1293.00
25/12/2015	FRI	4483.30	3887.70	100484.59	141.26	463.45	102.75	38.51	969.00	152.00	1121	1251.00
26/12/2015	SAT	4635.40	4022.90	102151.59	141.27	463.48	102.75	38.52	959.00	152.00	1111	1242.00
27/12/2015	SUN	4505.40	3991.20	100317.37	141.27	463.48	102.74	38.53	967.00	151.00	1118	1118.00
28/12/2015	MON	4553.20	4101.90	101793.25	141.27	463.48	102.80	38.47	969.00	151.00	1120	1120.00
29/12/2015	TUE	4467.00	3948.10	98779.87	141.28	463.51	102.75	38.53	971.00	141.00	1112	1296.00
30/12/2015	WED	4363.90	3832.70	95949.68	141.29	463.54	102.25	39.04	976.00	151.00	1127	1257.00
31/12/2015	THU	4331.20	3941.90	96671.82	141.30	463.58	102.80	38.50	972.00	177.00	1149	1280.00
TOTAL		131207.60	108748.00	2889906.99	4377.19	14360.68	3183.62	1193.57	24500.00	9361.00	33861.00	37109.00
AVERAGE		4232.50	3624.93	93222.81	141.20	463.25	102.70	38.50	790.32	301.97	1092.29	1197.06
			4101.90									
		4635.40	4101.90	102151.59								

Sample of Inflow and Elevation Data for JHEPS

PREVIOUS MONTH KAINJI LAST TOTAL DIS :- 1042											
MONTH: Dec-15		MONTH MAX.PEAK TO DATE :-									
			J E B B A P. S.								
DATE	ENERGY GENRTD (MWH)	H.W.E METER	T.W.E METER	G.O.H METER	AV.TD CUMECS	AV.SP CUMECS	TOTAL ST.DSCH CUMECS	COMPTD INFLOW CUMECS	LTERAL INFLOW +VE CUMECS	-VE CUMECS	ENERGY GENRTD (MWH)
01/12/2015	5099.00	100.97	74.30	26.67	1460.00	0	1460	904.00	-138.00	138.00	8401.00
02/12/2015	5087.00	100.84	74.30	26.54	1396.00	0	1396	1060.00	1060.00	-1060.00	7921.00
03/12/2015	5055.00	100.71	74.30	26.41	1392.00	0	1392	967.00	-59	59	7889.00
04/12/2015	5123.00	100.56	74.30	26.26	1463.00	0	1463	1012.00	-8	8	8278.00
05/12/2015	5147.00	100.50	73.80	26.7	1217.00	0	1217	1067.00	39	-39	6684.00
06/12/2015	5120.00	100.43	73.80	26.63	1166.00	0	1166	981.00	-50	50	6576.00
07/12/2015	5114.00	100.39	73.80	26.59	1117.00	0	1117	990.00	-38	38	6264.00
08/12/2015	4620.00	100.33	73.80	26.53	1104.00	0	1104	942.00	-84	84	6284.00
09/12/2015	5055.00	100.28	73.80	26.48	1105.00	0	1105	931.00	-36	36	6279.00
10/12/2015	5088.00	100.24	73.80	26.44	1099.00	0	1099	972.00	-45	45	6261.00
11/12/2015	5103.00	100.18	73.80	26.38	1096.00	0	1096	934.00	-86	86	6249.00
12/12/2015	6141.00	100.15	73.80	26.35	1103.00	0	1103	999.00	38	-38	6231.00
13/12/2015	7966.00	100.22	73.80	26.42	1104.00	0	1104	1220.00	71	-71	6256.00
14/12/2015	7947.00	100.22	73.80	26.42	1079.00	0	1079	1079.00	-180	180	6095.00
15/12/2015	7278.00	100.21	73.60	26.61	996.00	0	996	973.00	-84	84	5640.00
16/12/2015	5155.00	100.20	73.60	26.6	999.00	0	999	976.00	67	-67	5667.00
17/12/2015	5148.00	100.24	73.70	26.54	1059.00	0	1059	1152.00	130	-130	6008.00
18/12/2015	5129.00	100.25	73.70	26.55	1068.00	0	1068	1091.00	-147	147	6114.00
19/12/2015	5644.00	100.23	73.70	26.53	1072.00	0	1072	1025.00	-105	105	6141.00
20/12/2015	7761.00	100.30	73.70	26.6	1079.00	0	1079	1299.00	211	-211	6124.00
21/12/2015	7919.00	100.37	73.70	26.67	1094.00	0	1094	1314.00	-32	32	6237.00
22/12/2015	7412.00	100.39	73.70	26.69	1096.00	0	1096	1142.00	-172	172	6139.00
23/12/2015	7911.00	100.36	73.70	26.66	1156.00	0	1156	1087.00	-13	13	6534.00
24/12/2015	7808.00	100.37	73.70	26.67	1091.00	0	1091	1114.00	-6	6	6226.00
25/12/2015	7921.00	100.40	73.70	26.7	1039.00	0	1039	1108.00	-11	11	5928.00
26/12/2015	7852.00	100.42	73.70	26.72	1040.00	0	1040	1109.00	-12	12	5901.00
27/12/2015	7903.00	100.46	73.70	26.76	1010.00	0	1010	1137.00	26	-26	5792.00
28/12/2015	7917.00	100.43	73.80	26.63	1176.00	0	1176	1083.00	-35	35	6674.00
29/12/2015	7937.00	100.44	73.70	26.74	1069.00	0	1069	1104.00	-16	16	6188.00
30/12/2015	7981.00	100.44	73.80	26.64	1165.00	0	1165	1165.00	53	-53	6625.00
31/12/2015	7945.00	100.42	74.00	26.42	1158.00	0	1158	1089.00	-38	38	6577.00
TOTAL	200286.00	3111.95	2288.40	823.55	35268.00	0.00	35268.00	33026.00	300.00	-300.00	200183.00
AVERAGE	6460.84	100.39	73.82	26.57	1137.68	0.00	1137.68	1065.35	9.68	-9.68	6457.52

100.97

Sample of Inflow and Elevation Data for ShHEPS

			S H I R O R O P. S.								DAY
DATE	H.W.E	T.W.E	G.O.H	AV.TD	AV.SP	TOTAL	COMPTD	ENERGY	WATER	DATE	
	METER	METER	METER	CUMECS	CUMECS	CUMECS	CUMECS	(MWH)	CUMECS		
01/12/2015	379.09	270.50	108.59	385.00	0	385.0	154.00	8927.00	0	01/12/2015	TUE
02/12/2015	379.00	270.80	108.2	373.00	0	373.0	123.00	8663.00	0	02/12/2015	WED
03/12/2015	378.86	270.80	108.06	317.00	0	317.0	0.00	7301.00	0	03/12/2015	THU
04/12/2015	378.73	270.50	108.23	296.00	0	296.0	0.00	6866.00	0	04/12/2015	FRI
05/12/2015	378.65	270.80	107.85	360.00	0	360.0	146.00	8305.00	0	05/12/2015	SAT
06/12/2015	378.57	270.50	108.07	345.00	0	345	120.00	7983.00	0	06/12/2015	SUN
07/12/2015	378.47	270.80	107.67	371.00	0	371	108.00	8568.00	0	07/12/2015	MON
08/12/2015	378.36	270.80	107.56	370.00	0	370	67.00	8548.00	0	08/12/2015	TUE
09/12/2015	378.24	270.80	107.44	369.00	0	369	22.00	8511.00	0	09/12/2015	WED
10/12/2015	378.14	270.80	107.34	358.00	0	358	106.00	8223.00	0	10/12/2015	THU
11/12/2015	378.08	270.80	107.28	375.00	0	375	100.00	8602.00	0	11/12/2015	FRI
12/12/2015	377.93	270.80	107.13	370.00	0	370	115.00	8495.00	0	12/12/2015	SAT
13/12/2015	377.81	270.80	107.01	402.00	0	402	76.00	9245.00	0	13/12/2015	SUN
14/12/2015	377.69	270.80	106.89	364.00	0	364	69.00	8334.00	0	14/12/2015	MON
15/12/2015	377.56	270.80	106.76	377.00	0	377	44.00	8661.00	0	15/12/2015	TUE
16/12/2015	377.44	270.80	106.64	387.00	0	387	64.00	8910.00	0	16/12/2015	WED
17/12/2015	377.32	270.80	106.52	386.00	0	386	76.00	8846.00	0	17/12/2015	THU
18/12/2015	377.21	270.50	106.71	337.00	0	337	78.00	7747.00	0	18/12/2015	FRI
19/12/2015	377.15	270.50	106.65	231.00	0	231	81.00	5276.00	0	19/12/2015	SAT
20/12/2015	377.03	270.80	106.23	321.00	0	321	15.00	7232.00	0	20/12/2015	SUN
21/12/2015	376.92	270.80	106.12	367.00	0	367	96.00	8366.00	0	21/12/2015	MON
22/12/2015	376.81	270.80	106.01	368.00	0	368	111.00	8377.00	0	22/12/2015	TUE
23/12/2015	376.71	270.80	105.91	351.00	0	351	88.00	8023.00	0	23/12/2015	WED
24/12/2015	376.58	270.50	106.08	334.00	0	334	6.00	7583.00	0	24/12/2015	THU
25/12/2015	376.46	270.80	105.66	338.00	0	338	60.00	7597.00	0	25/12/2015	FRI
26/12/2015	376.34	270.80	105.54	349.00	0	349	71.00	7935.00	0	26/12/2015	SAT
27/12/2015	376.24	270.80	105.44	327.00	0	327	82.00	7322.00	0	27/12/2015	SUN
28/12/2015	376.13	270.80	105.33	338.00	0	338	38.00	7603.00	0	28/12/2015	MON
29/12/2015	376.03	270.80	105.23	380.00	0	380	121.00	8595.00	0	29/12/2015	TUE
30/12/2015	375.87	270.80	105.07	440.00	0	440	60.00	10002.00	0	30/12/2015	WED
31/12/2015	375.73	270.80	104.93	350.00	0	350	0.00	7930.00		31/12/2015	THU
TOTAL	11701.15	8393.00	3308.15	11036.00	0.00	11036.00	2297.00	252576.00	0.00		
AVERAGE	377.46	270.74	106.71	356.00	0.00	356.00	74.10	8147.61	0.00		
	379.09										

Appendix B

Sample of Daily Energy Profile of Nigeria

TRANSMISSION COMPANY OF NIGERIA

To: MD (S/O, M/O), TCN - Abuja														
From: Director (NCC), Osogbo														
<u>DAILY OPERATIONAL REPORT MONDAY 01/08/2016</u>														
FOR: <u>EVENTS OF SUNDAY 31/07/2016</u>														
1.0	EXECUTIVE OPERATIONS SUMMARY:													
1.1	<u>GENERATION PROFILE: NATIONAL STATISTICS</u>													
DETAILS										MW		HRS		DATE
PEAK DEMAND FORECAST (CONNECTED + SUPPRESSED LOAD)										17,720		19:30		31/07/2016
GENERATION CAPABILITY										8037.5		-do-		-do-
UNITS ON BAR CAPABILITY AT PEAK										3127.6		-do-		-do-
PEAK GENERATION										3103.3		-do-		-do-
LOWEST GENERATION										2772.5		10:00		31/07/2016
8-HOURLY DURATION PEAK										3038.5		0000-0800		-do-
										3060.5		0900-1600		-do-
										3103.3		1700-2400		-do-
PEAK GENERATION TO DATE										5074.7		21:30		02/02/2016
MAXIMUM AVAILABLE CAPACITY TO DATE										7492.6		06:00		14/04/2014
MAXIMUM PEAK GENERATION CAPABILITY TO DATE										7099		07:00		13/04/2014
MAXIMUM ENERGY GENERATED (MWH) TO DATE										109,372.01		0000-2400		02/02/2016
1.2	<u>CRITICAL VOLTAGE</u>													
DETAILS					KV			STATION			HRS			
NOMINAL VOLTAGE					330									
STATUTORY LIMITS					280.5 - 346.6									
HIGHEST VOLTAGE FOR THE DAY					346			Omotosho T/S			10:00			
LOWEST VOLTAGE FOR THE DAY					294			Adiabor T/S			20:00			
1.3	<u>FREQUENCY RANGE</u>													

								HZ				HRS		
NOMINAL FREQUENCY								50						
STATUTORY LIMITS								49.75-50.25						
HIGHEST FREQUENCY								51.318				18:07		
LOWEST FREQUENCY								48.896				19:08		
1.4	SYSTEM COLLAPSE UP DATE													
Date of last total collapse: 28 th June , 2016							No. of days since last total collapse: 34							
Date of last partial collapse: 10 th July 2016							No. of days since last partial collapse: 22							
Highest No of days ever attained in-between total collapses: 134							Highest No of days ever attained in-between partial collapses: 240							
		Ja n	Feb.	Mar .	Ap r	Ma y	Jun	Ju l	Au g	Sep.	Oct	No v	De c	Total
2009	Total	1	--	--	--	2	5	5	4	1	--	--	1	19
	Partial	--	5	4	3	3	--	--	1	--	2	1	1	20
2010	Total	---	3	1	--	3	3	3	--	2	2	1	4	22
	Partial	1	--	--	1	1	3	4	4	3	2	--	1	20
2011	Total	--	--	--	--	4	3	1	1	1	2	1	--	13
	Partial	--	1	1	--	--	--	--	1	--	1	2	--	6
2012	Total	--		2	1	5	2	1	--	--	2	2	1	16
	Partial	--		2	3	--	--	--	--	1	--	1	1	8
2013	Total	--	1	2	2	3	4	1	1	1	1	4	2	22
	Partial	--		--	--	--	--	--	--	2				2
2014	Total	2			2		3	1			1			9
	Partial					1						2	1	4
2015	Total	1		1		2		1				1		6
	Partial	--	--	1		2					1			4
2016	Total	--	--	2	3	6	5							16
	Partial	--	--	1		1	3	1						6
2017	Total													0
	Partial													0

1.5 BREAKDOWN OF PEAK & OFF PEAK GENERATION (NATIONAL PEAK & OFF PEAK)						
STATION		TURBINE	PEAK GEN. (MW)		OFF-PEAK GEN(MW)	
1	KAINJI	HYDRO	296		296	
2	JEBBA	HYDRO	438		357	
3	SHIRORO	HYDRO	429		342	
4	EGBIN ST2 - 5	STEAM	250		247	
5	SAPELE	STEAM	65		64	
6	DELTA	GAS	318		318	
7	AFAM IV-V	GAS	0		0	
8	GEREGU GAS	GAS	0		85	
9	OMOTOSHO GAS	GAS	70.6		73.1	
10	OLORUNSOGO GAS	GAS	66.9		60.5	
11	GEREGU NIPP	GAS	130		0	
12	SAPELE NIPP	GAS	110.1		103.1	
13	ALAOJI NIPP	GAS	0		0	
14	OLORUNSOGO NIPP	GAS	0		0	
15	OMOTOSHO NIPP	GAS	110.2		100.2	
16	ODUKPANI NIPP	GAS	0		0	
17	IHOVBOR NIPP	GAS	77		0	
18	OKPAI	GAS	358		350	
19	AFAM VI	GAS	110		110	
20	IBOM POWER	GAS	102.6		103.3	
21	ASCO	GAS	0		0	
22	A.E.S	GAS	0		0	
23	OMOKU	GAS	16.8		12.7	
24	TRANS AMADI	GAS	15.6		12.6	
25	RIVERS IPP	(GAS)	0		0	
26	EGBIN ST6	STEAM	102		100	
27	PARAS ENERGY	GAS	37.5		38	
28	GBARAIN	GAS	0		0	
TOTAL			3103.3		2772.5	
1.6	<u>ENERGY GENERATED/ENERGY SENT OUT:</u>					
STATION		TURBINE	ENERGY GENERATED		ENERGY SENT OUT	
			MWH	MWH/H	MWH	MWH/H
1	KAINJI	HYDRO	7,082.00	295.08	7,068.00	294.50

2	JEBBA	HYDRO	10,047.00	418.63	10,033.80	418.08
3	SHIRORO	HYDRO	10,056.00	419.00	10,025.88	417.75
4	EGBIN ST2 - 5	STEAM	6,066.00	252.75	5,675.10	236.46
5	SAPELE	STEAM	1,435.40	59.81	1,367.60	56.98
6	DELTA	GAS	7,573.90	315.58	7,400.50	308.35
7	AFAM IV-V	GAS	-	0.00	-	0.00
8	GEREGU GAS	GAS	1,118.60	46.61	1,101.91	45.91
9	OMOTOSHO GAS	GAS	1,705.00	71.04	1,687.00	70.29
10	OLORUNSOGO GAS	GAS	1,505.05	62.71	1,489.98	62.08
11	GEREGU NIPP	GAS	1,470.38	61.27	1,459.52	60.81
12	SAPELE NIPP	GAS	2,402.80	100.12	2,386.00	99.42
13	ALAOJI NIPP	GAS	-	0.00	-	0.00
14	OLORUNSOGO NIPP	GAS	-	0.00	-	0.00
15	OMOTOSHO NIPP	GAS	2,322.90	96.79	2,305.10	96.05
16	ODUKPANI NIPP	GAS	-	0.00	-	0.00
17	IHOVBOR NIPP	GAS	1,650.62	68.78	1,628.26	67.84
18	OKPAI	GAS	8,556.00	356.50	8,425.80	351.08
19	AFAM VI	GAS	2,616.69	109.03	2,563.27	106.80
20	IBOM	GAS	2,074.78	86.45	2,066.00	86.08
21	ASCO	GAS		0.00		0.00
22	A.E.S	GAS	-	0.00	-	0.00
23	OMOKU	GAS	351.30	14.64	335.12	13.96
24	TRANS AMADI	GAS	340.90	14.20	332.30	13.85
25	RIVERS IPP	GAS	-	0.00	-	0.00
26	EGBIN ST6	STEAM	2,393.00	99.71	2,242.70	93.45
27	PARAS ENERGY	GAS	882.31	36.76	871.30	36.30
28	GBARAIN	GAS	-	0.00	-	0.00
TOTAL			71,650.63	2,985.44	70,465.14	2,936.05

1. 7 A	GENERATION POSITION AT 06:00HRS OF MONDAY 01/08/2016							
	STATIONS	AVAILA BLE UNITS	INSTA LLED GENE RATIO N CAPA CITY (MW)	GENERATI ON CAPABILIT Y(MW)	UNITS ON BAR (MW)	CAPABI LITY OF UNITS ON BAR (MW)	GENERA TION AT 06:00HRS (MW)	REMARKS
PRIVATISED COMPANIES	KAINJI HYDRO	1G5, 6, 11 & 12 4	440	440	1G5, 6, 11 & 12 4	295	295	1G7 - Out on stator winding failure. 1G8 - Out due to loss of supply to the auxiliary. 1G9 - Out due to burnt 7.5MVA 33/0.415kV & 183.6MVA 16/330kV station service transformer and generator transformer. 1G10 - Shut down due to thrust bearing temperature problem. Generation reduced due to low operating head.
	JEBBA HYDRO	2G1 - 5 5	482	450	2G1 - 5 5	431	431	2G6 – Out due to burnt generator winding and AVR.
	SHIRORO HYDRO	411G1 - 3 3	450	450	411G1 - 3 3	425	425	411G4 - Out on maintenance. Generation reduced due to low operating head.
	EGBIN STEAM	ST2 - 5 4	880	880	ST2 & 3 2	271	271	ST1 - Tripped on generator CB trouble. ST2 & 3 - Not on spinning reserve due to Egbin G/S management decision. ST4 & 5 - Out due to gas constraints. Generation limited to 271MW due to gas constraints.
	SAPELE (STEAM)	ST1 1	120	65	ST1 1	65	65	ST1 - Generation limited to 65MW due to technical reason. ST2 - Tripped on vacuum switch problem. ST3 - Undergoing rehabilitation. ST4 & 5 - Awaiting major overhaul. ST6 – Tripped on gas control valve not following reference point.
	DELTA (GAS)	GT4, 6 - 13, 16,18 & 19 12	525	480	GT6 - 8, 10 -13, 16 & 18 9	318	318	GT3 & 5 – Out due to generator winding fault. GT4, 9 & 19 - Out due to gas constraints. GT14 - Out on major overhaul. GT15 – Commissioning test is on going. GT17 - Tripped on excitation trouble. GT16 & 18 - Not on spinning reserve due to Delta P/S management decision. GT20 - Out on major inspection. Generation reduced due to gas constraints.
	AFAM IV & V (GAS)	0	0	0	0	0	0	GT1 -12 - De-commissioned and

		0			0			scrapped. GT13-16 – Out on blade failure. GT17 & 18 - Out due to burnt generator transformer. GT19 & 20 - Awaiting major overhaul.
	GEREGU GAS	GT11 & 12 2	290	290	0 0	0	0	GT11 & 12 - Out due to gas constraints. GT13 - Out on maintenance.
	OMOTOSHO GAS	GT1 - 8 8	336	304	GT2 & 6 2	76	74.2	GT1,3 - 5, 7 & 8 - Out due to gas constraints. GT2 & 6 - On frequency response.
	OLORUNSOG O GAS	GT1 - 5, 7 & 8 7	294	266	GT5 & 8 2	76	70.9	GT5 - On frequency response. GT1 - 4 & 7 - Out due to gas constraints. GT6 – Out on vibration trouble.
NIPP	GEREGU NIPP	GT21 - 23 3	435	435	GT22 1	130	130	GT21 & 23 - Out due to gas constraints.
	SAPELE NIPP	GT2 1	112.5	112.5	GT2 1	103.4	103.4	GT1 - Tripped on transformer neutral O/C. GT3 - Out due to 125v D.C ground E/F alarm. GT4 - Out on maintenance.
	ALAOJI NIPP GAS	GT3 & 4 2	250	240	0 0	0	0	GT1 - Out on maintenance. GT2 - Tripped on excessive gas flow. GT3 & 4 - Out due to gas pipe line vandalism.
	OLORUNSOG O NIPP GAS	GT1 - 3 & ST2 4	500	480	0 0	0	0	GT1 - 3 & ST2 - Out due to gas constraints. GT4 - Out due to water leakage on the generator cooler. ST1 - Out on maintenance.
	OMOTOSHO NIPP GAS	GT1 - 4 4	500	480	GT4 1	120	110	GT1 - 3 - Out due to gas constraints. GT4 - On spinning reserve.
	ODUKPANI NIPP	GT3 & 5 2	250	240	0 0	0	0	GT1 - Out due to some missing parts. GT2 & 4 - Out due to units transformer problem. GT3 & 5 - Out due to gas constraints.
	IHOVBOR NIPP	GT1 - 4 4	450	450	GT3 1	79.6	79.6	GT1, 2 & 4 - Out due to gas constraints. Generation reduced due to gas constraints.
	OKPAI GAS/STEAM	GT11, 12 & ST18 3	450	450	GT11, 12 & ST18 3	357	357	Available units on bar.
	AFAM VI GAS/STEAM	GT11 -13 & ST10 4	650	580	GT13 1	110	110	GT11, 12 & ST10 - Out due to gas constraints. Generation reduced due to gas constraints.
	IBOM POWER GAS	GT2 & 3 2	168	150	GT3 1	51.2	51.2	GT1 - Out on guide vane problem. GT2 - Out due to gas constraints. Generation reduced due to gas constraints.
IPP	AES	0	0	0	0	0	0	

		0			0			No communication and out of production.
	ASCO	0	0	0	0	0	0	GT1 - Shut down due to oil leakage.
		0			0			
	OMOKU GAS	GT1, 3, 5 & 6	100	80	GT1 & 3	15.1	15.1	GT1 & 3 - On island operation. GT2 - Out on maintenance. GT4 - Out for overhaul. GT5 & 6 - Out due to gas constraints. Generation reduced due to gas constraints.
		4			2			
	TRANS-AMADI	GT1, 2 & 4	75	60	GT2	13.7	13.7	GT1 & 4 - Out due to gas constraints. Generation reduced due to gas constraints.
		3			1			
	RIVERS	GT1	180	160	0	0	0	GT1 - Out due to condensate problem in the gas pipe line.
		1			0			
	GBARAIN GAS	GT2	112.5	112.5	0	0	0	GT2 - Shut down due to gas constraints.
		1			0			
	SUB TOTAL	83	7937.5	7542.5	40	2937	2920.1	Frequency = 49.51Hz.
1.7B	BILATERAL AGREEMENT MONDAY 01/08/2016	UNITS						
	EGBIN	ST6	220	220	ST6	102	102	ST6 - Not on spinning reserve due to Egbin P/S management decision. Generation reduced due to gas constraints.
		1			1			
	PARAS ENERGY	GT5 - 9	58	50	GT5 - 8	37.5	37.5	GT4 - Shut down due to gas constraints. Generation reduced due to gas constraints.
		5			4			
	SUB TOTAL	7	278	270	5	139.5	139.5	
	GRAND TOTAL	90	8215.5	7812.5	45	3,076.50	3,059.60	Frequency = 49.51Hz.

1.7 C	BILATERAL AGREEMENT UNITS DISPATCH - SUNDAY 31/07/2016				
	FEEDERS NAME	MAX. LOAD (MW)	TIME (HRS)	HOURS IN SERVICE	REMARK
	100MW EGBIN ST6 GENERATION TO EKO ELECTRICITY DISTRIBUTION COMPANY				
	CHEVRON	15	7:00	19	FORCED OUTAGE
	MAIN ONE	1	1:00	24	NIL
	MAROKO	19	22:00	24	NIL
	ADEMOLA	OUT	OUT	OUT	OUT
	ANIFOWESE	16.5	18:00	23	PLANNED OUTAGE
	BANANA/BECKLEY	3.7	21:00	23	PLANNED OUTAGE
	FOWLER #1	OUT	OUT	OUT	OUT
	FOWLER #2	19.2	18:00	23	PLANNED OUTAGE
	TOTAL	74.4			

100MW EGBIN ST6 GENERATION TO IKEJA ELECTRICITY DISTRIBUTION COMPANY										
	U-STEEL	ON	1:00	24	NIL					
	PTCEXPRESS	6	21:00	20	FORCED OUTAGE					
	SANKYO	10	21:00	24	NIL					
	FEEDER 8	7	21:00	24	NIL					
	PULK IT	6	23:00	23	URGENT OUTAGE					
	INDUSTRIAL	15	7:00	21	URGENT OUTAGE					
	SPINTEX	3	11:00	24	NIL					
	DANGOTE	OUT	OUT	OUT	OUT					
	LAND CRAFT	52	1:00	24	NIL					
	TR4 15MVA	3.1	1:00	24	NIL					
	TR5 -15MVA	2.8	22:00	24	NIL					
	ALAUSSA	9.7	19:00	24	NIL					
	TOTAL LOAD	114.6								
	GRAND TOTAL	189								
40MW PARAS ENERGY GENERATION TO EKO ELECTRICITY DISTRIBUTION COMPANY										
	FEEDERS NAME	MAX. LOAD (MW)	TIME (HRS)	HOURS SERVICE	IN	REMARK				
	ILASAN	13.5	22:00	20		FORCED OUTAGE				
	IBEJU	15	7:00	24		NIL				
	IKATE EXPRESS	6	1:00	12		FORCED OUTAGE				
	LEEKI	17	22:00	24		NIL				
	ELEGUSHI	1	1:00	24		NIL				
	ALAGBON LOCAL	10.7	18:00	23		PLANNED OUTAGE				
	TOTAL	63.2								

1.8 HYDROLOGICAL DATA: MONDAY 01/08/2016											
						KAINJI		JEBBA		SHIRORO	
HEAD WATER ELEVATION (METERS)						132.13		101.45		365.88	
TAIL WATER ELEVATION (METERS)						103.68		74.90		271.00	
GROSS OPERATING HEAD (METERS)						28.45		26.55		94.88	
AVERAGE TURBINE DISCHARGE (M3/SEC)						1057		1758		491	
AVERAGE SPILLAGE (M3/SEC)						0		0		0	
AVERAGE TOTAL STATION DISCHARGE (M3/SEC)						1057		1758		491	
COMPUTED INFLOW (M3/SEC)						1140		1584		773	
STORAGE DIFFERENTIAL						83		-174		282	
MAXIMUM LEVEL FOR SPILLAGE COMMENCEMENT						141.73		103		382.5	
MINIMUM LEVEL BELOW WHICH NO GENERATION						130		99		355	

3.0 SYSTEM SPINNING RESERVE AS AT 0600HRS OF MONDAY 01/08/2016 = 10MW. FREQUENCY: 49.51Hz.					
S/N	POWER STATION	UNIT ON SPINNING RESERVE	RESERVE CAPABILITY	ACTUAL RESERVE (MW)	TOTAL (MW)
1	Egbin	ST2	0	0	0
		ST4		0	
		ST5		0	
		ST6		0	
2	Kainji	1G11	0	0	
3	Delta	GT16	40	0	0
		GT17	40	0	
4	Olorunsogo NIPP	GT1	0	0	0
		GT2	40	0	
		GT3	40	0	
		GT4	0	0	
5	Geregu NIPP	GT22	35	0	0
6	Omotosho NIPP	GT1	0	0	10
		GT2	40	0	
		GT3	40	0	
		GT4	40	10	
TOTAL (MW)			315	10	10
4.0	UNUTILIZED GENERATION CAPABILITY				
The unutilized generation capability for Monday 01/07/2016 as at 0600Hrs was = 4662MW.					
S/NO	STATION	ACTUAL GENERATION CAPABILITY (MW)	GENERATION AT 0600HRS (MW)	UNUTILIZED GENERATION CAPABILITY (MW)	SUB TOTAL
4.1 DUE TO GAS CONSTRAINTS:					
1	Olorunsogo NIPP GT1 - 3 & ST2	480	0	480	4662
2	Omotosho Gas GT1, 3 - 5, 7 & 8	228	0	228	
3	Geregu NIPP GT21 - 23	435	130	305	
4	Omotosho NIPP GT1 - 3	360	0	360	
5	Delta GT4, 6 - 13, 16,18 & 19	480	318	162	
6	Olorunsogo Gas GT1 - 4, 7 & 8	228	0	228	
7	Sapele NIPP GT2	112.5	103.4	9.1	
8	Afam VI GT11 - 13 & ST10	650	110	540	
9	Egbin ST2 - 6	1100	373	727	
10	Trans-Amadi GT1, 2 & 4	60	13.7	46.3	
11	Ihovbor NIPP GT1 - 4	450	79.6	370.4	
12	Alaoji NIPP GT3 & 4	240	0	240	
13	Rivers IPP GT1	160	0	160	
14	Geregu Gas GT11 & 12	290	0	290	
15	Gbarain GT2	112.5	0	112.5	
16	Ibom GT2 & 3	150	51.2	98.8	
17	Omoku GT1, 3, 5 & 6	80	15.1	64.9	
18	Odukpani GT3 & 5	240	0	240	

IT IS ENVISAGED WITH IMPROVED PLANT AVAILABILITY AND RELIEF FROM EXISTING POWER SYSTEM NETWORK RELATED CONSTRAINTS, ADDITIONAL 1818.83MSCF OF GAS (EQUIVALENT TO 6820.6MW WILL BE REQUIRED WHEN ALL THERMAL UNITS ARE ON BAR.					
4.2 DUE TO LINE CONSTRAINTS/LOAD REJECTION BY DISCOS					
					0
				0	
				0	
				0	
				0	
4.3 DUE TO WATER MANAGEMENT/MAINTENANCE					
				0	0
				0	
4.4 DUE TO HIGH FREQUENCY OCCASIONED BY RAINFALL					
				0	0
				0	
Total					4662

5.0	<u>OUTAGES:</u>													
5 (A)	<u>FORCED OUTAGES (GENERATION):-</u>													
1	08:12 through 24:00Hrs, Ihovbor P/S GT1 tripped on high voltage protection trip. LHR = 80.6MW.													
5 (B)	<u>FORCED OUTAGES (TRANSMISSION):-</u>													
1	12:40 – 12:51Hrs, Umuahia T/S 40MVA 132/33kV transformer ET2 primary CB tripped on Phase instantaneous O/C. LHR = 4MW.													
2	15:14 through 24:00Hrs,Irrua T/S 30MVA 132/33kV transformer T2 primary and secondary CBs tripped on differential and O/C Blue phase respectively. Simultaneously Agbor 33kV feeder CB tripped on E/F. LHR = 2.4MW. The transformer was left out for maintenance attention.													
3	22:27Hrs, of 29/07/2016 - 10:55Hrs of 30/07/2016, Ijebu-Ode T/S 30MVA 132/33kV transformer T2 primary CB tripped on bias differential Red & Blue phases while the sec CB tripped without relay indication. A python that bridged the sec side of the transformer was removed, also insulation resistance test conducted on the transformer was certified okay before restoration. LHR = 5.5MW.													
5 (C)	<u>PLANNED OUTAGES:</u>													
5 (D)	<u>URGENT OUTAGES</u>													
1	08:48 – 12:12Hrs, Jebba 2G2 was shut down to facilitate the job at Ikeja West T/S for maintenance crew to top up low SF6 gas pressure to normal. LHR = 80MW. Frequency was 51.35Hz.													
2	09:01 – 11:38Hrs, Ikeja West/Akangba 330kV lines 1 & 2 (ccts W3L and W4L) were taken out at Ikeja West T/S to enable maintenance crew top up low SF6 gas pressure to normal level on cct W3L line CB. LHR = 20MW each. Cct W4L out as it was feeding from tie breaker of cct W3L.													

5 (E)	<u>EMERGENCY OUTAGES: The under mentioned lines were opened on frequency control:-</u>
1	04:27 - 05:56Hrs, 16:14 - 16:23Hrs & 19:52 - 18:16Hrs, Akangba/Ojo 132kV line 2. LHR = 15MW, 16MW & 20MW . Frequency was 49.30Hz, 49.20Hz & 49.25Hz respectively.
2	04:27 - 05:57Hrs, 16:14 - 16:23Hrs & 17:52 - 18:16Hrs, Akangba/Itire 132kV line 1 CB. LHR = 25MW, frequency was 49.40Hz, 12MW & 15MW, frequency was 49.00Hz, 49.12Hz & 49.20Hz respectively.
3	18:30 - 18:37Hrs, 18:47 - 19:01Hrs, 19:44 - 20:15Hrs, Osogbo/Akure/Ado 132kV line CB. LHR = 24MW, 22MW & 23MW, frequency was 48.94Hz, 49.82Hz respectively.
4	20:11 - 20:16Hrs, Osogbo/Ilesha/Ife 132kV line CB. LHR = 10MW, frequency was 48.78Hz respectively.
5 (F)	<u>AUTOMATIC UNDER FREQUENCY OPERATION:</u>
1	11:30 - 11:45Hrs & 12:10 - 12:25Hrs, Apapa Road T/S 45MVA 132/33kV transformer TR2. LHR = 4MW & 4MW respectively.
2	05:41 - 05:59Hrs, 15:18 - 15:47Hrs, 18:27 - 19:44Hrs, Akangba T/S 60MVA 132/33kV transformer 10T1A sec breaker. LHR = 10MW, 9MW.
5 (G)	<u>330KV LINES THAT ARE OUT ON VOLTAGE CONTROL:-</u> ccts U2A, J2L, A1S, J1H, B11J & N8K.
5 (H)	<u>330KV LINES THAT ARE OUT ON FAULT:-</u> cct R4B
5 (I)	<u>330KV LINES THAT ARE OUT ON BREAKER PROBLEM:-</u> ccts K1J & N4J.
5 (J)	<u>330KV LINES THAT ARE OUT ON PLANNED OUTAGE:-</u> NIL
9	<u>WEATHER REPORT:-</u> Rainfall was reported at
10	<u>INTERNATIONAL LINES:-</u>
A	<u>IKEJA WEST/SAKATE 330KV LINE</u> Maximum load - 127MW at 21:00Hrs. Minimum load - 88MW at 24:00Hrs.
B	<u>BIRNIN KEBBI/NIAMEY 132KV LINE</u> Maximum load - 75.0MW at 15:00Hrs Minimum load - 63.0MW at 08:00Hrs
C	<u>KATSINA/GAZAOUA/MARADI 132KV LINE</u> Maximum load - 36MW at 22:00Hrs. Minimum load - 17MW at 08:00Hrs.

6	<u>GENERATOR MERIT ORDER BASED ON SYSTEM RESPONSE AND GRID RELIABILITY</u>												
	GROUP 1 - HYDRO IN HIGH FLOOD PERIOD SEASON, KAINJI, JEBBA AND SHIRORO.												
	GROUP 2 - ALL UNITS OFFERING REGULATING RESERVE.												
	GROUP 3 - ALL UNITS OFFERING SPINNING RESERVE.												
	GROUP 4 - ALL UNITS OFFERING FREE GOVERNOR FREQUENCY RESPONSE.												
	GROUP 5 - ALL UNITS OFFERING VOLTAGE SUPPORT OUTSIDE NORMAL LEVEL.												
	GROUP 6 - ALL UNITS USED TO AVOID OVERLOADING TRANSMISSION LIMITS.												
	GROUP 7 - ALL UNITS IN LOCATION TO DIMINISH LOSSES												
	GROUP 8 - ALL UNITS NOT OFFERING ANY SERVICES.												
8 (A)	<u>COMMUNICATION MEDIA</u>												
	COMMUNICATION MEDIA				NO. INSTALLED			NO. WORKING					
	PLC/PAX				36			33					
	OPTIC FIBRE				13			11					
	PLS				14			6					
8 (B)	<u>SCADA RTU STATUS</u>												
	STATIONS		TOTAL NO OF STATIONS		NO OF RTU INSTALLED			NO OF RTU WORKING					
	POWER STATIONS		23		8			3					
	330kV		36		17			14					
	132kV		111		103			19					

12.0	<u>UNAVAILABLE EQUIPMENTS</u>												
12(A)	<u>UNAVAILABLE TRANSFORMERS/REACTORS:</u>												
	<u>SHIRORO REGION:</u>												
1	Apo T/S 60MVA 132/33kV transformer TR3 - Burnt.												
	<u>KADUNA REGION:</u>												
1	Kumbotso 150MVA 330/132/33kV transformer T1A - Burnt control cables.												
	<u>ENUGU REGION:</u>												
1	Onitsha T/S 90MVA 330/132/13.8kV transformer 9T2 - Out due to differential fault.												
12(B)	<u>UNAVAILABLE BREAKERS/ISOLATORS</u>												
	<u>LAGOS REGION:</u>												
1	Akangba T.S. 132kV Bus Coupler CB - Serious Air leakage since 1994.												
2	Egbin/Aja 330kV line 2 (cct N4J) - faulty breaker at Aja T/S.												
3	Otta/Ogba 132kV line isolator - defective control shaft.												
4	Osogbo/Ikeja West and Olorunsogo/Ikeja West tie CB at Ikeja West T/S - defective pole												
5	Oke-Aro/Ikeja West 330kV lines 1 & 2 tie CB at Ikeja West T/S - Defective pole.												
	<u>OSOGBO REGION:</u>												

1	Omotosho 330kV T/S, 2nd series 330kV CB #3322 out due to control circuit problem.																		
2	Akure T/S 60MVA 132/33kV transformer T3A primary CB - defective spring charging mechanism.																		
	ENUGU REGION:																		
1	New Haven/Yendev 132kV line CB - Faulty Red phase CB pole. Protection is transferred to the bus coupler CB.																		
2	Abakaliki T/S 30MVA 132/33kV transformer T1A and 60MVA 132/33kV transformer T2 primary CBs - Breakers were not provided since commissioning but the transformers are protected by Nkalagu 132kV line CB.																		
3	Oji River T/S 30MVA 132/33kV transformer T1A primary CB - The breaker is completely out of service and bypassed. The transformer is either protected by Onitsha/Oji 132kV line CB or New Haven/Oji 132kV line CB.																		
4	Oturkpo/Yendev 132kV By pass isolator - Canibalized to repair the line isolator.																		
	SHIRORO REGION																		
1	Kainji/Jebba 330kV line 1 - CB problem at Kainji T/S.																		
12 (C)	UNAVAILABLE/DEFECTIVE GROUND SWITCHES																		
	Benin Region																		
1	Irrua T/S: Benin/Irrua 132kV line Earth Switch - faulty.																		
	ENUGU REGION:																		
1	New Haven/Nkalagu 132kV line 2 ground switch - Canibalized Red phase female contact and canibalized Yellow phase male contact.																		

12 (D)	UNAVAILABLE TRANSMISSION LINES:
1	Egbin/Aja 330kV line 2 (cct N4J) - faulty breaker at Aja T/S.
2	132kV Akangba/Itire line 1- Faulty line isolator at Akangba T/S.
3	Aja/Alagbon 132kV line 2 - detached conductor on tower 56 and 57. The line was isolated on tower 47 to feed Lekki T/S.
4	Agbara/Ojo line 1 on soak.
12(E)	UNAVAILABLE CAPACITOR BANK:
	(i) Ogba T.S, (ii) Maiduguri T/S, (iii) Isolo T/S, (iv) Ado-Ekiti T/S, (v) Ilorin T/S, (vi) Akure T/S.
12 (F)	LINES WITH MISSING SKY WIRE
	LAGOS REGION
1	Omotosho/Ikeja West line (M5W) and Egbin/Ikeja West line 3 (N6W) - 21 spans.
2	Ikeja West/Olorunsogo line (cct R1W) - 80 spans.
3	330kV Ikeja West/Osogbo line. - 95 spans.
4	Ikeja West/Alimosho/Ogba 132kV lines 1 & 2 - 15 spans.
5	Ikeja West/Akangba 330kV line 2 - 1 span.
6	Ikeja West/ Ilupeju 132kV lines 1 & 2 - 8 spans.
7	Akangba/Itire 132kV lines 1 & 2 - 1 span.
8	Ikeja West/Agbara 132kV lines 1 & 2 - 61 spans.
9	Ikeja West/Otta 132kV lines 1 & 2- 17 spans.

	<u>SHIRORO REGION</u>
1	Jebba T/S 330kV Osogbo lines 1 & 2 and Ganmo line - 9 spans
2	Minna T/S 132kV Bida line - 32 spans
3	Apo T/S 132kV Keffi line - 36 spans
4	Shiroro T/S 330kV Jebba line 2 - 28 spans
12 (G)	<u>UNAVAILABLE SYNCHROSCOPES :</u>
1	Benin , Ikeja West, Onitsha, Egbin, Osogbo, Jebba, Kaduna & Alaoji T/S.
	<u>FOR MORE INFORMATION: NCC Telephone Lines -</u>
a	PAX 09/300, 309, 303 & 305.
b	NCC GSM - 08074888524, 08069379367, 08023572651 & 08051401331. CUG Nos. 08129458488, 08129458489, 08129458489; 08129458490.
c	PLS - 622
d	Director NCC -08129929806
e	GM (NCC) -08033711297, 08054911239, CUG Airtel No. 08129458479.
f	AGM (S/O) - 08062273616, 08054684495, CUG: 08129458486
g	Coordinator (S/O) - 08037868916, 08054353904,08028332125, CUG Airtel 08129458481.
h	SYSTEM OPERATIONS WEB SITE: www.nigeriasystemoperator.org

GENERATION POSITION AT PEAK (19:30HOURS) SATURDAY 30/07/2016											
S/N	STATIONS	AVAILABLE UNITS	INSTALLED CAPACITY (MW)	GENERATION CAPABILITY(MW)	UNITS ON BAR	CAPABILITY OF UNITS ON BAR (MW)	GENERATION AT PEAK	REMARKS			
1	KAINJI HYDRO	1G5, 6, 11& 12 4	440	440	1G5, 6, 11 & 12 4	296	296	1G7 - Out on stator winding failure. 1G8 - Out due to loss of supply to the auxiliary. 1G9 - Out due to burnt 7.5MVA 33/0.415kV station service transformer and 183.6MVA 16/330kV generator transformer. 1G10 - Out due to thrust bearing temperature. Generation reduced due to low operating head.			
2	JEBBA HYDRO	2G1 - 5 5	482.5	450	2G1 - 5 5	438	438	2G6 - Burnt generator winding and AVR.			
3	SHIRORO HYDRO	411G1 - 3 3	450	450	411G1 - 3 3	429	429	411G4 - Out on maintenance. Generation reduced due to low operating head.			
4	EGBIN STEAM	ST2 - 5 4	880	880	ST2 & 3 2	250	250	ST1 - Tripped on generator CB trouble. ST2 & 3 - Not on spinning reserve due to Egbin P/S management decision. ST4 & 5 - Out due to gas constraints. Generation limited to 250MW due to gas constraints.			
5	SAPELE (STEAM)	ST 1 1	120	65	ST1 1	65	65	ST1 - Generation limited to 65MW due to technical reason. ST2 - Tripped on vacuum switch problem. ST3 - Undergoing rehabilitation. ST4 & 5 - Awaiting major overhaul. ST6 - Tripped and shut down on gas control valve not following reference point.			
6	DELTA (GAS)	GT4, 6 - 13, 16,18 & 19 12	525	480	GT6, 7, 8, 10 - 13, 16 & 18 9	318	318	GT3 & 5 - Out due to generator winding fault. GT4, 9 & 19 - Out due to gas constraints. GT14 Out on major overhaul. GT15 - Commissioning test is on going. GT17 - Tripped on excitation trouble. GT18 & 19 - Not on spinning reserve due to Delta P/S management decision. GT20 - Out on major inspection. Generation reduced due to gas constraints.			
7	AFAM (GAS)	0 0	0	0	0 0	0	0	GT1-12 - De-commissioned and scrapped. GT13-16 - Out on blade failure. GT17 & 18 - Out due to burnt generator transformer. GT19 & 20 - Awaiting major overhaul.			

8	GEREGU GAS	GT11 & 12 2	290	290	0 0	0	0	GT11 & 12 - Out due to gas constraints. GT13 - Out on maintenance.
9	OMOTOSHO GAS	GT1 - 8 8	336	304	GT2 & 6 2	76	70.6	GT2 & 6 - On frequency response. GT1, 3 - 5, 7 & 8 - Out due to gas constraints.
10	OLORUNSOGO GAS	GT1 - 5, 7 & 8 7	294	266	GT5 & 8 2	76	66.9	GT1 - 4 & 7 - Out due to gas constraints. GT5 & 8 - On frequency response. GT6 - Out on vibration trouble.
11	GEREGU NIPP	GT21 - 23 3	435	435	GT22 1	130	130	GT21 & 23 -Out due to gas constraints.
12	SAPELE NIPP	GT2 & 3 2	225	225	GT2 1	110.1	110.1	GT1 - Tripped on transformer neutral O/C. GT3 - Out due to gas constraints. GT4 - Out on maintenance.
13	ALAOJI NIPP GAS	GT3 & 4 2	250	240	0 0	0	0	GT1 - Out on maintenance GT2 - Tripped on excessive gas flow trip. GT3 & 4 - Out due to gas pipeline vandalization.
14	OLORUNSOGO NIPP GAS	GT1 - 3 & ST2 4	500	480	0 0	0	0	GT1 - 3 & ST2 - Out due to gas constraints. GT4 - Out due to water leakage on the generator cooler. ST1 - Out on maintenance.
15	OMOTOSHO NIPP GAS	GT1 - 4 4	500	480	GT4 1	120	110.2	GT1 - 3 - Out due to gas constraints. GT4 - On spinning reserve.
16	ODUKPANI NIPP	GT3 & 5 2	250	240	0 0	0	0	GT1 - Out due to some missing parts. GT2 - Out due to faulty unit transformer. GT3 & 5 - Out due to gas constraints. GT4 - Tripped without relay indication.
17	IHOVBOR NIPP	GT1 - 4 4	450	450	GT3 1	77	77	GT1, 2 & 4 - Out due to gas constraints. Generation reduced due to gas constraints.
18	OKPAI GAS/STEAM	GT11, 12 & ST18 3	450	450	GT11, 12 & ST18 3	358	358	Available units on bar.
19	AFAM VI GAS/STEAM	GT11- 13 & ST10 4	650	580	GT 13 1	110	110	GT11,12 & ST10 - Out due to gas constraints. Generation reduced due to gas constraints.
20	IBOM POWER GAS	GT2 & 3 2	168	150	GT 3 1	102.6	102.6	GT1 - Out on guide vane problem. GT2 - Shut down due to line constraints. Generation reduced due to line constraints.
21	ASCO	0 0	0	0	0 0	0	0	GT1 - Shut down due to oil leakage.
22	A. E.S GAS	0 0	0	0	0 0	0	0	No communication and out of production.
23	OMOKU GAS	GT1, 3, 5 & 6 4	100	80	GT1 & 3 2	16.8	16.8	GT1 & 3 - On Island operation. GT2 - Undergoing maintenance. GT4 - Out for overhaul. GT5 & 6 - Out due gas constraints. Generation reduced due to gas constraints.
24	TRANS-AMADI	GT1, 2 & 4 3	75	60	GT2 1	15.6	15.6	GT1 & 4 - Out due to gas constraints. GT3 - Out on maintenance. Generation reduced due to gas constraints.
25	RIVERS	GT1 1	180	160	0 0	0	0	GT1 - Out due to condensate problem in the gas pipe line.
26	GBARAIN GAS	GT2 1	112.5	112.5	0 0	0	0	GT2 - Shut down due to gas constraints.
TOTAL		84	8050.5	7767.5	40	2988.1	2963.8	Frequency = 49.74Hz.
BILATERAL AGREEMENT UNITS (GENERATION AT PEAK) (19:30HRS) SATURDAY 29/07/2016								
27	Egbin	ST6 1	220	220	ST6 1	102	102	ST6 - Not on spinning reserve due to Egbin G/S management decision. Generation reduced due to gas constraints
28	Paras Energy	GT4 - 9 6	58	50	GT5 - 9 5	37.5	37.5	GT4 - Shut down due to gas constraints.
TOTAL		7	278	270	6	139.5	139.5	
GRAND TOTAL		90	8050.5	8037.5	45	3127.6	3,103.30	Frequency = 49.74Hz.

Appendix C

Summary Monthly Average Lake Evaporation in Cubic Meters Per Second at KHEPS (1974 - 2006)

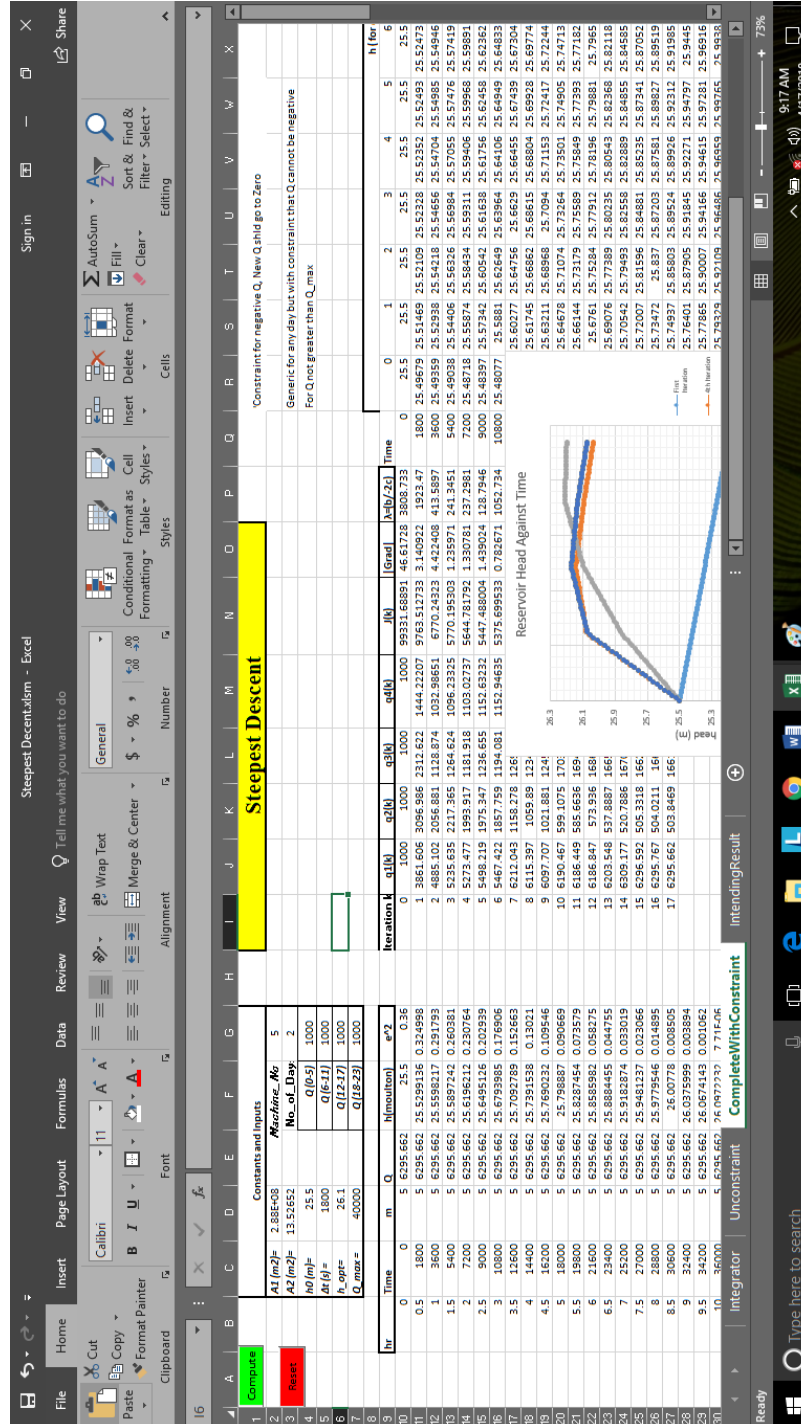
		NATIONAL ELECTRIC POWER AUTHORITY										
		KAINJI POWER STATION										
TABLE 6		MONTHLY AVERAGE LAKE EVAPORATION IN CUBIC METERS PER SECOND (1974 - 2006)										
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1974	66	81	117	85	64	62	29	28	19	48	76	79
1975	88	113	127	75	28	26	28	16	25	27	70	82
1976	85	106	129	95	26	39	27	21	34	19	59	55
1977	64	93	97	99	27	39	28	18	23	40	44	41
1978	62	88	78	56	36	32	28	19	24	39	58	58
1979	65	97	98	85	51	26	24	22	26	43	49	58
1980	58	92	109	86	58	39	29	25	26	32	41	53
1981	67	96	97	86	48	37	24	23	23	45	56	25
1982	85	94	101	78	51	31	26	24	24	43	49	51
1983	91	96	104	87	59	39	28	21	25	47	58	62
1984	78	99	94	71	41	38	27	24	25	37	54	56
1985	72	93	87	77	48	31	19	22	23	24	56	59
1986	68	89	89	86	59	41	19	26	19	43	49	52
1987	58	103	84	62	36	25	24	24	18	39	46	44
1988	51	76	80	68	56	26	27	17	25	55	60	65
1989	91	103	91	90	46	34	24	15	29	32	57	53
1990	65	86	100	68	46	25	16	17	21	40	54	56
1991	57	79	90	73	31	23	18	18	32	42	54	69
1992	86	93	93	79	44	28	20	23	18	45	58	60
1993	93	89	87	82	55	28	14	20	22	39	49	49
1994	49	78	92	63	34	26	17	16	28	39	62	64
1995	68	86	92	82	59	28	32	17	24	39	45	45
1996	58	84	95	85	49	38	26	17	25	38	55	59
1997	78	108	92	84	38	36	27	17	29	34	51	58
1998	79	105	103	84	50	21	20	20	30	35	58	60
1999	70	97	99	73	51	29	16	10	24	40	60	67
2000	69	104	111	94	62	27	30	20	26	41	54	60
2001	77	97	104	78	52	32	26	19	28	39	60	70
2002	85	90	98	71	68	35	22	16	26	40	67	72
2003	79	87	94	63	61	25	29	17	32	46	65	76
2004	78	105	103	99	42	33	23	21	24	43	57	66
2005	87	99	89	78	45	26	20	20	16	38	54	59
2006	61	89	86	83	44	32	24	23	24	42	70	69
2007	83	94	98	65	39	26	23	14	27	45	58	58
2008	75	89	92	64	41	23	19	18	29	44	56	51
2009	58	82	88	67	56	38	19	15	24	46	61	62
Mean	72.47	92.73	94.73	77.53	48.97	30.67	22.93	19.30	24.87	40.40	55.80	58.33
S.D	12.45	8.46	7.42	9.87	9.11	5.71	4.69	3.75	3.94	5.68	6.33	10.01
Skew	-0.15	-0.11	0.32	0.09	0.01	0.22	-0.02	-0.20	-0.30	-0.39	-0.12	-1.13
CV	0.17	0.09	0.08	0.13	0.19	0.19	0.20	0.19	0.16	0.14	0.11	0.17
Max	93.00	108.00	111.00	99.00	68.00	41.00	32.00	26.00	32.00	55.00	70.00	76.00
Min	49.00	76.00	80.00	62.00	31.00	21.00	14.00	10.00	16.00	24.00	41.00	25.00

Summary Monthly Average Lake Evaporation in Cubic Meters Per Second at JHEPS (1987 -2010)

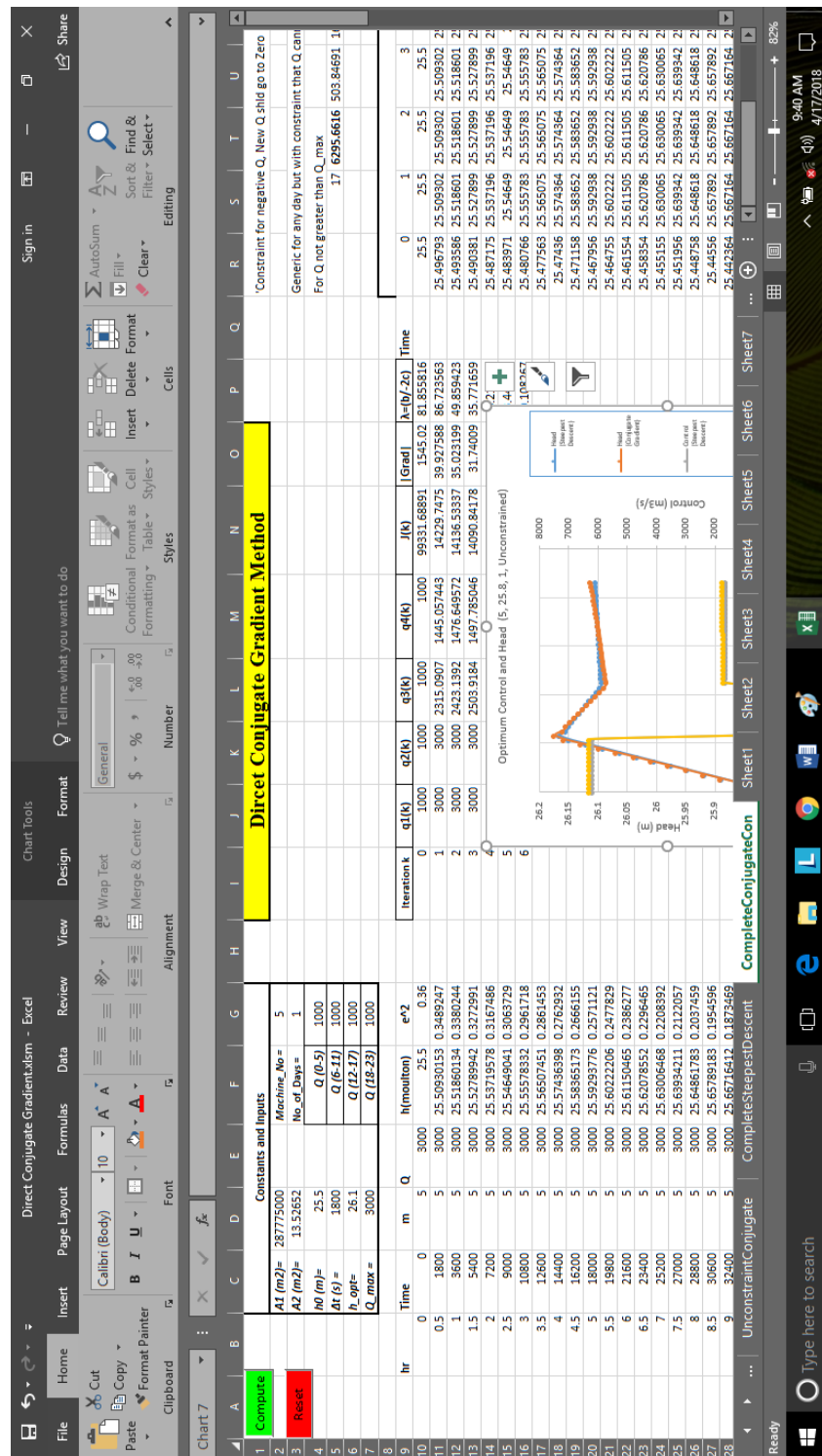
JEBBA HYDROPOWER GENERATING STATION (1987 -2010)																		
Aver Jebba Monthly Reservoir Evaporation Loss (m³/s)																		
	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	Total loss (m³/s)	Mean	S.D	Skew	Max	Min
1987	-	25	26	27	28	17	14	13	14	16	16	16	212.00	19.27	5.88	0.58	28.00	13.00
1988	16	23	25	24	19	15	12	11	14	18	19	17	213.00	17.75	4.54	0.23	25.00	11.00
1989	21	25	25	26	19	16	13	13	14	15	17	16	220.00	18.33	4.81	0.60	26.00	13.00
1990	18	25	28	24	18	16	12	14	14	15	16	18	218.00	18.17	4.95	0.93	28.00	12.00
1991	20	25	26	21	17	14	12	11	10	16	18	17	207.00	17.25	5.14	0.32	26.00	10.00
1992	22	26	29	24	19	15	16	13	13	18	19	18	232.00	19.33	5.05	0.58	29.00	13.00
1993	21	26	25	27	20	14	13	14	14	18	19	18	229.00	19.08	4.93	0.38	27.00	13.00
1994	19	25	30	25	19	16	15	13	15	18	20	19	234.00	19.50	4.95	0.89	30.00	13.00
1995	20	27	30	27	20	17	14	14	14	17	17	16	233.00	19.42	5.60	0.93	30.00	14.00
1996	18	24	28	25	22	15	12	13	15	16	18	18	224.00	18.67	5.03	0.56	28.00	12.00
1997	19	25	27	24	17	14	14	15	15	17	19	18	224.00	18.67	4.42	0.85	27.00	14.00
1998	21	29	30	28	17	18	14	12	11	15	18	16	229.00	19.08	6.57	0.72	30.00	11.00
1999	18	25	26	23	19	18	14	13	13	14	19	17	219.00	18.25	4.49	0.52	26.00	13.00
2000	19	24	28	27	23	16	13	13	14	16	16	17	226.00	18.83	5.34	0.67	28.00	13.00
2001	19	25	29	24	19	15	12	12	13	17	20	19	224.00	18.67	5.35	0.49	29.00	12.00
2002	20	27	28	26	23	18	14	12	14	15	17	19	233.00	19.42	5.47	0.38	28.00	12.00
2003	19	24	26	24	23	13	13	14	15	17	18	18	224.00	18.67	4.60	0.29	26.00	13.00
2004	17	24	26	26	16	16	12	12	14	16	17	17	213.00	17.75	4.92	0.84	26.00	12.00
2005	19	24	27	25	18	15	13	13	13	15	18	16	216.00	18.00	4.90	0.79	27.00	13.00
2006	16	24	26	27	20	16	14	10	14	16	18	17	218.00	18.17	5.17	0.52	27.00	10.00
2007	19	25	27	27	19	15	14	13	15	16	17	19	226.00	18.83	4.95	0.78	27.00	13.00
2008	22	26	28	24	17	15	12	11	13	17	17	19	221.00	18.42	5.53	0.42	28.00	11.00
2009	21	25	29	20	17	14	12	10	13	17	17	18	213.00	17.75	5.45	0.70	29.00	10.00
2010	18	22	18	26	16	14	10	11	14	12	17	18	196.00	16.33	4.60	0.66	26.00	10.00
Mean	19.22	25.00	26.96	25.04	19.38	15.50	13.08	12.50	13.67	16.13	17.79	17.54						
S.D	1.68	1.41	2.49	1.97	2.78	1.38	1.28	1.32	1.20	1.42	1.18	1.06						
Skew	-0.19	0.70	-1.93	-0.85	1.45	0.38	-0.03	-0.31	-1.57	-0.93	0.27	-0.12						
Max	22.00	29.00	30.00	28.00	28.00	18.00	16.00	15.00	15.00	18.00	20.00	19.00						
Min	16.00	22.00	18.00	20.00	16.00	13.00	10.00	10.00	10.00	12.00	16.00	16.00						

Appendix D

Overview of the Toolbox for Steepest Descent Method



Overview of the Toolbox for Direct Conjugate Gradient Method



Overview of the Toolbox for Progressive Domain Expansion Method

