

APPROXIMATE SOLUTION OF AN ORDINARY DIFFERENTIAL AND INTEGRAL EQUATIONS USING COLLOCATION METHOD BASED ON HERMITE AND LEGENDRE POLYNOMIALS

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ABSTRACT

*In this work, we employed an approximate solution of second and third-order Initial Value Problems (IVPs) in Ordinary Differential Equations (ODEs) by utilizing a standard collocation method based on Hermite and Legendre polynomials. The ODEs were first converted to integral equations and the basis function was substituted to obtain a set of linear algebraic which then form equations that were solved via **maple 2021**. Comparisons were made with the two trial solutions mentioned above in terms of errors obtained. Numerical examples were given to illustrate the performance of the method for various orders. However, the Hermite polynomial basis exhibits better accuracy over the Legendre polynomials in some results as can be seen from the tables of errors presented.*

Keywords: Hermite Polynomial, Initial Value Problems, Integral Equations, Legendre Polynomial, Standard Collocation Method.

INTRODUCTION

Ordinary differential equations assume significant roles in many applied fields such as engineering, electrostatics, electromagnetic, mechanics, mathematical physics, and also in numerical analysis.

Many numerical methodologies have been proposed for solving these problems by some researchers which include Rao (1999), Chung, Sarkar, Jung, & Zhong (2004), and Dianchen, Chen, Rabia, Shamoona & Abdur (2018).

Numerous problems of theoretical physics, engineering, and other disciplines usually lead to linear and nonlinear Integral Equations or Integro-Differential Equations, Ordinary Differential Equations, and Partial Differential Equations. This has drawn the consideration of

researchers to the answers to such problems. Accordingly, various methods and techniques have been developed to discover answers to these problems. These include Skwame et al. (2017) solved order ten implicit one-step hybrid block method for the solution of stiff second-order ordinary differential equations.

Skwame et al. (2019) also developed a numerical application of third derivative hybrid block methods on third-order initial value problem of ordinary differential equation. Kunjan & Twinkle (2015) proposed a solution of the second kind of Volterra Integral and Integro-differential equation using the Homotopy Analysis Method. Oyedepo et al. (2020) proposed a perturbed least squares method with aid of orthogonal polynomial as basis function. Gu & Jiang (1996) constructed a Haar wavelets

operation matrix of integration and Antonia & Nikta (2012) also constructing orthogonal matrix polynomials satisfying differential equations from two different Laguerre weights. Sabo et al. (2020) formulate implicit second-order backward differential Adam's formulae to solve stiff systems of first order initial value problems of ordinary differential equations.

Taiwo & Ishola(2009) presented Collocation Approximation methods for the numerical solution of integral equation of the second kind. Akgonullu et al. (2010) presented a Hermite matrix method to solve high-order linear Fredholm integro-differential equations with variable coefficients, Maleknejad, & Aghazadeh (2005) used the Taylor series expansion approach to solve the Volterra integral equation of the second kind with convolution kernel. In a similar case, Ren, Zhang, & Oiao (1999) came up with a simple Taylor-series expansion method for a class of second kind integral equations. Brunner (1985 and 1990) investigate the application of collocation methods on Volterra integral equations. Xu(2007) applied the Variational iteration method to integral equations. Wazwaz (1996 & 2011) also came up with two methods for solving integral equations.

Moreover, the application of the Chebyshev, Hermite, Laguerre, and Legendre polynomials and their numerical merits in solving Ordinary Differential Equations (ODEs.) and Partial Differential Equations (PDEs) numerically have been discussed in Taiwo et al. (2012), Sohrab (2006), Dianchen *et al.* (2018), Keilson, J., & Nunn (1979) and Baek et al. (2004).

However, the incredible work did by previously mentioned scientists and the researchers motivated us and eventually led us to the

proposal of an Approximate solution of ordinary differential and integral equations using the collocation method based on Hermite and Legendre polynomials.

BASIC DEFINITIONS

Definition 1

Collocation Method: This is a method of evaluating a given differential equation at some points to nullify the values of an ordinary differential equation at those points.

Definition 2

Initial Value Problem (IVP): An Initial Value Problem (IVP) can be defined as an ordinary differential equation with a condition specified at an initial point. The general initial value problem is given as

$$y^j(s) + R_1 y^{j-1}(s) + \dots + R_{j-1} y'(s) + R_j y(s) = 0 \quad (1)$$

Subject to initial conditions

$$y(0) = d_0, y'(0) = d_1, y''(0) = d_2, + \dots + y^{(m-1)}(0)(s) = d_{m-1} \quad (2)$$

Definition 3

Hermite Polynomial of order m is defined by $H_j(s)$ and valid in the interval $\gamma \leq s \leq \delta$ is defined as

$$H_m(s) = \sum_{j=0}^{[M]} \frac{(-1)^j m!}{j!(m-2j)!} (2s)^{n-2j}, m \geq 0 \quad (3)$$

where $M = \frac{m}{2}$, if m is even and $M = \frac{(m-2)}{2}$, if m is odd

and also, we can obtain these polynomials using Rodrigue's formula given as

$$H_m(s) = (-1)^m e^{s^2} \frac{d^m}{ds^m} (e^{-s^2}) \quad (4)$$

$$\begin{aligned}
j = 0: H_0(s) &= 1 \\
j = 1: H_1(s) &= 2s \\
j = 2: H_2(s) &= 4s^2 - 2 \\
j = 3: H_3(s) &= 8s^3 - 2s \\
j = 4: H_4(s) &= 16s^4 - 48s^3 + 12 \\
j = 5: H_5(s) &= 32s^5 - 160s^3 + 120s \\
&\vdots
\end{aligned} \tag{5}$$

Hence, the trial solution is now given as

$$y_m(s) = \sum_{j=0}^M d_j H_j(s) \tag{6}$$

where d_j , $j = 0, 1, 2, \dots, M$ are unknown constants and $H_j(s)$ is the approximating polynomials. M is the degree of approximating Polynomials.

Definition 4

Legendre's Polynomial is denoted and defined by Rodrigue's formula

$$L_m(s) = \frac{1}{2^m m!} \frac{d^m}{ds^m} [(s^2 - 1)^m] \tag{7}$$

From the definition given above, it will be observed that m^{th} derivative must be carried out before a polynomial of degree m is obtained. Thus, the first few set of Legendre Polynomials can be obtained as follows:

$L_0(s)$ will not involve any derivative since $m = 0$, hence we obtained $L_0(s) = 1$.

Also, for $m = 1$, we obtain

$$L_1(s) = s$$

$$L_2(s) = \frac{1}{2}(3s^2 - 1) \tag{8}$$

To obtain $L_3(s)$ it will require differentiating three times which will become cumbersome as m increases. With this difficulty that may be

encountered with higher differentiation especially as $m > 2$ in $L_m(s)$ of Rodrigue's formula eqn. (7) above a simple formula for generating the Legendre Polynomials is given by its recurrence relation. This is given as

$$L_{m+1}(s) = \left(\frac{2m+1}{m+1} \right) \cdot s \cdot L_m(s) - \left(\frac{m}{m+1} \right) \cdot L_{m-1}(s); m = 1, 2, \dots \tag{9}$$

Since $L_0(s)$ and $L_1(s)$ are obtained from Rodrigue's formula eqn. (7), we can now switch over to eqn. (9) to generate higher-order polynomials as follows:

$$L_3(s) = \frac{1}{2}(5s^3 - 3s) \tag{10}$$

$$L_4(s) = \frac{1}{8}(35s^4 - 30s^2 + 3) \tag{11}$$

$$L_5(s) = \frac{1}{8}(63s^5 - 70s^3 + 15s) \tag{12}$$

and so on.

Hence, the trial solution is now given as

$$y_m(s) = \sum_{j=0}^M d_j L_j(s) \tag{13}$$

where $d_j (j \geq 0)$ are the unknown constants to be determined and $L_j(s)$ Legendre polynomials of the first kind and M is the degree of approximating polynomials.

PROBLEM CONSIDERED AND METHODOLOGY

In this section, we considered an m^{th} order homogeneous linear equation with constant coefficients given as

$$y^m(s) + \gamma_{m-1} y^{(m-1)}(s) + \dots + \gamma_2 y''(s) + \gamma_1 y'(s) + \gamma_0 y(s) = 0 \tag{14}$$

Subject to initial conditions

$$y(0) = \eta_0, y'(0) = \eta_1, y''(0) = \eta_2, + \dots + y^{(m-1)}(0) = \eta_{m-1} \tag{15}$$

where γ^s are constants and m represent the highest-order.

Standard Collocation Method Using Hermite Polynomials

To solve eqn. (14) and (15) using the standard method, we assumed an approximate solution eqn. (6).

Thus, eqn. (6) is differentiated m -times to get

$$y'(s) = \sum_{j=0}^M d_j H_j'(s)$$

$$y''(s) = \sum_{j=0}^M d_j H_j''(s)$$

⋮

$$y^{(m)}(s) = \sum_{j=0}^M d_j H_j^{(m)}(s) \quad (16)$$

Hence, substituting eqn. (16) into eqn. (14), we obtain

$$\begin{aligned} & \sum_{j=0}^M d_j H_j^{(m)}(s) + \gamma_{m-1} \sum_{j=0}^M d_j H_j^{(m-1)}(s) \\ & + \cdots + \gamma_2 \sum_{j=0}^M d_j H_j''(s) + \gamma_1 \sum_{j=0}^M d_j H_j'(s) \\ & + \gamma_0 \sum_{j=0}^M d_j H_j(s) = 0 \end{aligned} \quad (17)$$

Integrating both sides of eqn. (17), with respect to u , we obtain

$$\begin{aligned} & \int_0^v \left(\sum_{j=0}^M d_j H_j^{(m)}(u) \right) du + \\ & \gamma_{m-1} \int_0^v \left(\sum_{j=0}^M d_j H_j^{(m-1)}(u) \right) du + \cdots + \\ & \gamma_2 \int_0^v \left(\sum_{j=0}^M d_j H_j''(u) \right) du \\ & + \gamma_1 \int_0^v \left(\sum_{j=0}^M d_j H_j'(u) \right) du + \\ & + \gamma_0 \int_0^v \left(\sum_{j=0}^M d_j H_j(u) \right) du = 0 \end{aligned} \quad (18)$$

$$\Rightarrow \left[\sum_{j=0}^M d_j H_j^{(m-1)}(v) - \sum_{j=0}^M d_j H_j^{(m-1)}(0) \right] +$$

$$\begin{aligned} & \gamma_{m-1} \left[\sum_{j=0}^M d_j H_j^{(m-2)}(v) - \sum_{j=0}^M d_j H_j^{(m-2)}(0) \right] + \\ & \cdots + \gamma_2 \left[\sum_{j=0}^M d_j H_j'(v) - \sum_{j=0}^M d_j H_j'(0) \right] + \\ & \gamma_1 \left[\sum_{j=0}^M d_j H_j(v) - \sum_{j=0}^M d_j H_j(0) \right] + \\ & \gamma_0 \int_0^v \left(\sum_{j=0}^M d_j H_j(u) \right) du = 0 \end{aligned} \quad (19)$$

Substituting the conditions given in eqn.(15) into eqn. (19), we obtain

$$\begin{aligned} & \left[\sum_{j=0}^M d_j H_j^{(m-1)}(v) - \eta_{m-1} \right] + \\ & \gamma_{m-1} \left[\sum_{j=0}^M d_j H_j^{(m-2)}(v) - \eta_{m-2} \right] + \cdots + \\ & \gamma_2 \left[\sum_{j=0}^M d_j H_j'(v) - \eta_1 \right] + \gamma_1 \left[\sum_{j=0}^M d_j H_j(v) - \eta_0 \right] + \\ & \gamma_0 \int_0^v \left(\sum_{j=0}^M d_j H_j(u) \right) du = 0 \end{aligned} \quad (20)$$

Expanding eqn. (20) further, we obtain

$$\begin{aligned} & \sum_{j=0}^M d_j H_j^{(m-1)}(v) - \eta_{m-1} + \\ & \gamma_{m-1} \sum_{j=0}^M d_j H_j^{(m-2)}(v) - \gamma_{m-1} \eta_{m-2} + \cdots + \\ & \gamma_2 \sum_{j=0}^M d_j H_j'(v) - \gamma_2 \eta_1 + \gamma_1 \sum_{j=0}^M d_j H_j(v) - \\ & \gamma_1 \eta_0 + \gamma_0 \int_0^v \left(\sum_{j=0}^M d_j H_j(u) \right) du = 0 \end{aligned} \quad (21)$$

Integrating both sides of eqn. (21) with respect to v , we obtain

$$\begin{aligned} & \int_0^x \left(\sum_{j=0}^M d_j H_j^{(m-1)}(v) \right) dv - \gamma_m \eta_{m-1} \int_0^x dv \\ & + \gamma_{m-1} \int_0^x \left(\sum_{j=0}^M d_j H_j^{(m-2)}(v) \right) dv - \\ & \gamma_{m-1} \eta_{m-2} \int_0^x dv \\ & + \cdots + \gamma_2 \int_0^x \left(\sum_{j=0}^M d_j H_j'(v) \right) dv - \gamma_2 \eta_1 \int_0^x dv \\ & - \gamma_1 \int_0^x \left(\sum_{j=0}^M d_j H_j(v) \right) dv \\ & - \gamma_1 \eta_0 \int_0^x dv + \gamma_0 \int_0^x \int_0^v \left(\sum_{j=0}^M d_j H_j(u) \right) dudv = 0 \end{aligned} \quad (22)$$

$$\begin{aligned} & \Rightarrow \int_0^x \left(\sum_{j=0}^M d_j H_j^{(m-1)}(v) \right) dv - \gamma_m \eta_{m-1} x \\ & + \gamma_{m-1} \int_0^x \left(\sum_{j=0}^M d_j H_j^{(m-2)}(v) \right) dv - \gamma_{m-1} \eta_{m-2} x \\ & + \cdots + \gamma_2 \left[\sum_{j=0}^M d_j H_j(v) - \sum_{j=0}^M d_j H_j(0) \right] - \gamma_2 \eta_1 x \\ & + \gamma_1 \int_0^x \left(\sum_{j=0}^M d_j H_j(v) \right) dv - \gamma_1 \eta_0 x \end{aligned}$$

$$+\gamma_0 \int_0^x \int_0^v (\sum_{j=0}^M d_j H_j(u)) du dv = 0 \quad (23)$$

Expanding eqn. (24) further, we obtain

$$\begin{aligned} & \int_0^x (\sum_{j=0}^M d_j H_j^{(m-1)}(v)) dv - \\ & \gamma_m \eta_{m-1} x + \gamma_{m-1} \int_0^x (\sum_{j=0}^M d_j H_j^{(m-2)}(v)) dv - \\ & \gamma_{m-1} \eta_{m-2} x + \dots + \gamma_2 \sum_{j=0}^M d_j H_j(x) - \gamma_2 \eta_0 - \\ & \gamma_2 \eta_1 x + \gamma_1 \int_0^x (\sum_{j=0}^M d_j H_j(v)) dv - \gamma_1 \eta_0 x \\ & + \gamma_0 \int_0^x \int_0^v (\sum_{j=0}^M d_j H_j(u)) du dv = 0 \quad (24) \\ & \int_0^x (\sum_{j=0}^M d_j H_j^{(m-1)}(v)) dv - \\ & \gamma_m \eta_{m-1} x + \gamma_{m-1} \int_0^x (\sum_{j=0}^M d_j H_j^{(m-2)}(v)) dv - \\ & \gamma_{m-1} \eta_{m-2} x + \dots + \gamma_2 \sum_{j=0}^M d_j H_j(x) - \gamma_2 \eta_0 - \\ & \gamma_2 \eta_1 x + \gamma_1 \int_0^x (\sum_{j=0}^M d_j H_j(v)) dv - \\ & \gamma_1 \eta_0 x + \gamma_0 \int_0^x ((x-v) \sum_{j=0}^M d_j H_j(v)) dv = 0 \quad (25) \end{aligned}$$

This implies $\varphi(d_0, x) = \sum_{j=0}^M d_j H_j(x)$,

$$\varphi(d_1, x) = \int_0^x (\sum_{j=0}^M d_j H_j^{(m-1)}(v)) dv,$$

$$\varphi(d_2, x) = \int_0^x ((x-v) \sum_{j=0}^M d_j H_j(v)) dv,$$

$$\varphi(d_{m-1}, x) = \int_0^x (\sum_{j=0}^M d_j H_j^{(m-1)}(v)) dv,$$

and

$$\varphi(d_{m-2}, x) = \int_0^x (\sum_{j=0}^M d_j H_j^{(m-2)}(v)) dv \quad (26)$$

Thus, substituting eqn. (26) into eqn.(27), we obtain

$$\begin{aligned} & \varphi(d_{m-1}, x) - \gamma_m \eta_{m-1} x + \gamma_{m-1} \varphi(d_{m-2}, x) - \\ & \gamma_{m-1} \eta_{m-2} x + \dots + \gamma_2 \varphi(d_0, x) - \gamma_2 \eta_0 - \gamma_2 \eta_1 x + \\ & \gamma_1 \varphi(d_1, x) - \gamma_1 \eta_1 x + \gamma_0 \varphi(d_2, x) = 0 \quad (27) \end{aligned}$$

Thus, eqn.(27) is collocated at point $x = x_i$, to obtain

$$\begin{aligned} & \varphi(d_{m-1}, x_i) - \gamma_m \eta_{m-1} x_i + \gamma_{m-1} \\ & \varphi(d_{m-2}, x_i) - \gamma_{m-1} \eta_{m-2} x_i + \dots \\ & \varphi(d_1, x_i) - \gamma_1 \eta_1 x_i + \gamma_0 \varphi(d_2, x_i) = 0 \quad (28) \end{aligned}$$

where

$$x_i = \vartheta + \frac{(\sigma - \vartheta)i}{M+1}, \quad i = 1, 2, \dots, M \quad (29)$$

Standard Collocation Method Using Legendre Polynomials

To discuss this method, we assumed an approximate solution of eqn. (14). This method is similar to the one we discussed above under section (3.2). We have (M+1) algebraic linear system of equation (M+1) unknown constants $d_j (j \geq 0)$. The (M+1) algebraic system of linear equations is then solved by software **Maple 2021** to obtain the unknown constants $d_j (j \geq 0)$ which are then substituted back in eqn.(14).

NUMERICAL EXAMPLES

In this section, we have demonstrated the Standard Collocation Approximation Method on higher-order ordinary differential equations via Hermite and Legendre Polynomials as the basis functions and compared it with each other. The examples are solved to illustrate the accuracy, efficiency, and time of execution of the method using the software Maple 2021.

Numerical Example 1

Consider the second order differential equation

$$L(y(s)) = y'' + 2y' + y = 0 \quad (31)$$

with initial condition

$$y(0) = 0, \quad y'(0) = 1 \quad (32)$$

and exact solution

$$y(s) = x e^{-s} \quad (33)$$

Table 1 below, shows the numerical solution obtained in terms of exact solution and the errors for the ordinary differential equations

solved through Hermite and Legendre Polynomials basis function. It was observed that the Hermite Polynomial performed better than

Legendre Polynomials method in some a the results obtained

Table 1a: Results and Absolute errors obtained for example 1

s	Exact	Hermite Computed Results For Case M = 3	Legendre Computed Results For Case M = 3	Error in Hermite.	Error in Legendre.
0.0	0.000000000000	-0.006624378800	-0.006624378800	6.62438E-03	6.62438E-03
0.2	0.16374615062	0.1632918637792	0.1632918637792	4.54287E-04	4.54287E-04
0.4	0.26812801841	0.2672534430016	0.2672534430016	8.74575E-04	8.74576E-04
0.6	0.32928698166	0.3289908160544	0.3289908160544	2.96166E-04	2.96166E-04
0.8	0.35946317129	0.3589856825248	0.3589856825248	4.77489E-04	4.77489E-04
1.0	0.36787944117	0.3677197420000	0.3677197420000	1.59699E-04	1.59691E-04

Table 1b: Results and Absolute errors obtained for example 1

s	Exact	Hermite Computed Results For Case M = 6	Legendre Computed Results For Case M = 6	Error in Hermite.	Error in Legendre.
0.0	0.000000000000	-0.000004191554	-0.000004191688	4.19155E-06	4.19169E-06
0.2	0.16374615062	0.1637457936756	0.1637457938254	3.56900E-07	3.56800E-07
0.4	0.26812801841	0.2681278265263	0.2681278266841	1.91900E-07	1.91700E-07
0.6	0.32928698166	0.3292868447436	0.3292868449008	1.37000E-07	1.36800E-07
0.8	0.35946317129	0.3594630501394	0.3594630502863	1.21200E-07	1.21000E-07
1.0	0.36787944117	0.3678793598510	0.3678793599759	8.13000E-08	8.12000E-08

Numerical Example 2

Consider the second order differential equation

$$L(y(z)) = y'' + 5y' + 6y = 0 \quad (34)$$

with initial condition

$$y(0) = 0, \quad y'(0) = 1 \quad (35)$$

and exact solution

$$y(s) = -e^{-3s} + e^{-2s} \quad (36)$$

Table 2 below, shows the numerical solution obtained in terms of exact solution and the errors for the ordinary differential equations solved through Hermite and Legendre Polynomials basis function. It was observed that the results obtained by the Hermite Polynomial performed better than Legendre Polynomials method in a few iterations and a lower error.

Table 2a: Results and Absolute errors obtained for example 2

s	Exact	Hermite Computed Results For Case M = 3	Legendre Computed Results For Case M = 3	Error in Hermite.	Error in Legendre.
0.0	0.00000000000000	-0.0313428910000	-0.0313428910000	3.13429E-02	3.13429E-02
0.2	0.1215084099416	0.1156228766512	0.1156228766320	5.88553E-03	5.88553E-03
0.4	0.1481347522050	0.1497239508160	0.1439723950480	4.16236E-03	4.16236E-03
0.6	0.1358953236906	0.1357012700464	0.1357012700080	1.94054E-04	1.94054E-04
0.8	0.1111785647052	0.1101193253008	0.1101193252720	1.05924E-03	1.05924E-03
1.0	0.0855482148687	0.0865363846000	0.0865363846000	9.88161E-04	9.88161E-04

Table 2b: Results and Absolute errors obtained for example 2

s	Exact	Hermite Computed Results For Case M = 6	Legendre Computed Results For Case M = 6	Error in Hermite.	Error in Legendre.
0.0	0.00000000000000	0.00040258320000	0.00040258420000	4.02583E-04	4.02584E-04
0.2	0.1215084099416	0.12145574045807	0.12145574133967	5.26694E-05	5.26686E-05
0.4	0.1481347522050	0.14811723274056	0.14811723344616	1.75195E-05	1.75188E-05
0.6	0.1358953236906	0.13589011524248	0.13589011577208	5.20850E-06	5.20790E-06
0.8	0.1111785647052	0.11117599983263	0.11117600028227	2.56490E-06	2.56440E-06
1.0	0.0855482148687	0.08554672450000	0.08554672510000	1.49037E-06	1.48977E-06

Numerical Example 3

Consider the third order differential equation

$$L(y(s)) = y''' - y'' - y' + y = 0 \quad (37)$$

with initial condition

$$y(0) = 4, \quad y'(0) = 2, \quad y''(0) = 3 \quad (38)$$

and exact solution

$$y(s) = e^s + se^s \quad (39)$$

Table 3 below, shows the numerical solution obtained in terms of exact solution and the errors for the ordinary differential equations solved through Hermite and Legendre Polynomials basis function. It was observed that the results obtained by the Hermite Polynomial performed better than Legendre Polynomials method in a few iterations and a lower error

Table 3a: Results and Absolute errors obtained for example 3

s	Exact	Hermite Computed Result For Case M = 3	Legendre Computed Result For Case M = 3	Error in Hermite.	Error in Legendre.
0.0	1.00000000000	1.000000000000	0.9396663550000	4.4E-02	6.03336E-02
0.2	1.46568330980	1.221392913839	1.2213929138135	7.50495E-03	7.50495E-03
0.4	2.08855457661	1.491802720235	1.4918027200548	2.02931E-02	2.02931E-02
0.6	2.91539008062	1.822084149092	1.8220841481468	8.70426E-02	8.70426E-02
0.8	4.00597367129	2.225492448817	2.2254924462281	2.43713E-01	2.43713E-01
1.0	5.43656365692	2.718170481120	2.7181704760200	5.24802E-01	5.24802E-01

Table 3b: Results and Absolute errors obtained for example 3

s	Exact	Hermite Computed Result For Case M = 6	Legendre Computed Result For Case M = 6	Error in Hermite.	Error in Legendre.
0.0	1.00000000000	0.9995557455800	0.9995574562000	4.42544E-04	4.42544E-04
0.2	1.4656833098	1.4686246879341	1.4686246884931	2.94138E-03	2.94138E-03
0.4	2.08855457661	2.1148280102664	2.1148280190397	2.62734E-02	2.62734E-02
0.6	2.91539008062	3.0138569816983	3.0138569823181	9.84669E-02	9.84669E-02
0.8	4.00597367129	4.2628231540792	4.2628231545474	2.56849E-01	2.56849E-01
1.0	5.43656365692	5.9796925703000	5.9796925704000	5.43129E-01	5.43129E-01

CONCLUSION

In this work, we have demonstrated the collocation approximation method for solving second and third-order Initial Value Problems (IVPs) of various orders using the Hermite and Legendre Polynomials as basis functions. We have also compared our results with each other. The results obtained by the Hermite Polynomial basis performed better over the Legendre Polynomial in some examples.

However, we also observed that Hermite Polynomial with the initial approximation obtained from initial conditions yielded a good approximation to the exact solution as the degree of M approximation increases.

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REFERENCES

- Akgonullu, N, Sahin, N. & Sezer, M. (2010). A Hermite collocation method for the approximate solutions of high-order linear Fredholm integro-differential equations, *Wiley Periodicals, Inc*, 27, 1707-1721
- Antonio, J. D., & Nikta, S. (2012). Constructing orthogonal matrix Polynomials differential equations from two different Laguerre weights *Taylor & Francis*, 24(4), 263-279.
- Baek, H. J., Tapan, K. S. Young, S. C., Magdalena, S. P., & Zhong, J. (2004). Time-domain combined field integral equations using Laguerre polynomials as temporal basis functions, *Int. J. Numer. Model*, 17, 251-268.
- Brunner, H. (1985). The numerical solution of weakly singular Volterra integral equations by collocation on graded meshes, *Appl. Math. Compu.*, 45, 417-437.
- Brunner, H. (1990). On the numerical solution of Nonlinear Volterra – Fredholm Integral Equations by collocation methods, *SIAM Jour. Numer. Anal.*, 27, 987-1000.
- Chung, Sarkar, Jung, & Zhong (2004). Solution of time domain electric field integral equation using an unconditionally stable methodology, *IEEE Trans. Antennas Propagat.*, 52(9), 2319-2329
- Dianchen, L. Chen, Y., Rabia, M., Shamoona, J., & Abdur, R. (2018). Approximate solution of Multi-Pantograph equations with variable coefficients via collocation method based on Hermite polynomials *Communications in mathematics and Applications.*, 9(4), 601-614
- 52(9), 2319-2329
- Erdelyi, A. (1963). An Integral Equation Involving Legendre's Polynomials *Mathematical Association of America*, 70(6), 651-652
- Guo, N., & Yunpeng, M. (2018). Numerical Algorithm to solve fractional IDES Based on Legendre Wavelets method *IAENG International Journal of Applied Mathematics.*, 48(2),
- Gu, J. S., & Jiang (1996). The Haar wavelets operational matrix of Integration, *Int. J. Syst. Sci.* 27, 623-628.
- Keilson, J., & Nunn (1979). Laguerre transformation as a tool for the numerical solution of integral equations of convolution type, *Applied Mathematics and Computation*, 5, 313-359
- Kunjah, S., & Twinkle, S. (2015). Solution of second kind Volterra integral and integro-differential equation by Homotopy analysis method. *International Journal of Mathematical Archive*. 6(4), 49-59
- Maleknejad, K., & Agazadeh, N. (2005). Numerical solution of Volterra integral equations of second kind with convolution

- kernel by using Taylor series expansion method, *Appl. Math. Compu.*, 161, 647-655.
- Oyedepo, T., Adebisi, F. A., Tayo, M. R., Ayinde, A. M. & Peter, J. O. (2020). Perturbed least squares techniques for solving Volterra fractional integro-differential equations based on constructed orthogonal polynomials *J. Math. and Comput. Sci.*, 10(40), 10-20.
- Rao, S. M. (1999). *Time dom electromagnetics*, Academic Press, New York.
- Ren, Y., Zhang, B., & Qiao, J. O. (1999). A simple Taylo-series expansion method for a class of second kind integral equation, *J. Comput. Appl. Math.*, 110(40), 15-24.
- Sabo, J., Kyagya, T. Y., & Ayinde, A. M. (2020). The formulation of implicit Second-order backward difference Adam's formulae for solving stiff systems of first-order initial value problems of ordinary differential equations, *Asian Journal of Advanced Research and Reports*. 10(4), 21-29.
- Skwame, Y., Dalatu, P. I., Sabo, J., & Mathew, M. (2019). Numerical application of third-order derivative hybrid block methods on third order initial value problem of ordinary differential equations. *International Journal of Statistics and Applied Mathemtics*, 4(6), 90-100.
- Skwame, Y., Sabo, J., Tumba, P. & Kyagya, T. Y. (2017). Order ten implicit one-step hybrid block method for solution of stiff second-order ordinary differential equations, *International Journal of Engineering and Applied Sciences*, 4, 43-47.
- Sohrab, A. Y. (2006). Numerical solution of Abel's integral equations by Legendre Wavelets. *Applied Mathematics and Computation*, 175, 574-580.
- Taiwo, O. A., & Ishola, C. Y. (2009). Collocation approximation methods for the numerical solution of integral equations, *Int. Journal Pure and Applied Science*, 2, 29-37.
- Taiwo, O. A., Ayinde, M. A., & Olagunju, S. (2012). Collocation approximation method for special linear integro-differentials using Chebyshev basis function, *Int. Journal of Engineering Research and Application (IJERA)*, 2(6), 12-14.
- Wazwaz, A. M. (1996). Two Method for solving integral equations, *Appl. Math. Compu.*, 77, 79-89.
- Wazwaz, A. M. (2011). *Linear and nonlinear integral equations*, (c). Higher Education Press, Beijing and Springer-Verlag Berlin.
- Xu, L. (2007). Variational iteration method for solving integral equations, *I. Jour. Appl.*, 54, 1071-1078.