

Fair Resource Partitioning Between Cell-center and Cell-edge of FFR-based Multi-tier Wireless Access Networks

Akindele Segun Afolabi[#], Erdenebileg Munkbat[#], Chikara Ohta^{*}, and Hisashi Tamaki^{*}

[#] Department of Computer Science and Systems Engineering,
Kobe University, Japan

^{*}Department of Information Science,
Kobe University, Japan

e-mail: {afolabi, ohta, tamaki}@al.cs.kobe-u.ac.jp, munkhbat45@gmail.com.

Abstract—This paper addresses the problem of fair radio resource partitioning between the cell-edge and cell-center regions in the context of the downlink of a SISO-OFDMA cellular system. Although, a plethora of studies concerning OFDMA-based radio resource allocation exists, it is, however, imperative to accord special attention to the proper design of network resource partitioning variables in order to ensure fairness in cell structures in which cell-wide reuse-1 is impracticable. This work focuses on the mathematical formulation of cell-edge/cell-center radio resource partitioning variables and considers the case of an FFR-based multi-layer cellular structure comprising of femtocell underlay. By way of exhaustive search procedure, we determined the optimal radio resource partitioning parameter values from the perspective of user fairness and throughput maximization.

Keywords- resource partitioning; FFR; macrocell; femtocell

I. INTRODUCTION

Fourth generation (4G) wireless systems (and beyond) are currently being developed to meet the explosive growth in demand for higher data rates by wireless devices. Due to the appealing characteristics of OFDMA, the major 4G standardization bodies such as IEEE and 3GPP/3GPP2 have adopted OFDMA as the main radio access technology for 4G standards such as WiMAX (worldwide interoperability for microwave access) and LTE (long term evolution) [1]–[2]. Besides its inherent capability to combat inter-symbol interference (ISI), OFDMA offers flexibility in resource allocation granularity, since each subcarrier can be allocated, modulated and coded adaptively to exploit multi-frequency and multi-user diversity gains. Further to meeting the high data rate demand in future generation networks, dense reuse, enabled by the introduction of smaller cells such as femtocell, is gaining tremendous attention. A femtocell is a small-size low power indoor base station that operates in the licensed frequency band and backhauls data through a dedicated broadband gateway to the cellular operator's core network. Since femtocell is usually deployed by the user in a plug and play mode, its success depends on to what extent it can automatically configure itself without manual intervention. A number of technical issues therefore needs to be resolved in order to prepare femtocell as a commercial product. The most important is how a femtocell

should allocate spectrum in the presence of femto-macro interference. Fractional frequency reuse (FFR) [3]–[4] is a viable solution to this spectrum allocation problem and in this context, its advantages include; (1) it ameliorates inter-cell interference problem characterized by OFDMA, and (2) it simplifies spectrum allocation problem between macrocell and femtocell layers since femtocell can easily use resources that are not used by the macrocell at the region where the femtocell is deployed.

FFR separates the cell area into two zones, namely, cell-edge (CE) and cell-center (CC) zones, and assigns different reuse factors to these zones. FFR is generally classified into *hard* FFR and *soft* FFR. *Hard* FFR uses separate spectrum for center and edge zones, while in *soft* FFR the center zone can reuse the frequency resources used by its adjacent edge zones. To the best of the authors' knowledge, FFR studies in the literature have not accounted for the mutual relationship existing between cell region partitioning parameter and network resource partitioning parameter. However, judicious selection of values for these parameters is a key step towards achieving user fairness. This paper provides a mathematical formulation for these parameters, and by way of exhaustive search, their optimal values from the perspective of user fairness and throughput maximization are determined. *Hard* FFR is assumed in order to reduce possible interference that center zone femtocell receives from the macrocell.

The remainder of this paper is organized as follows: A model describing cellular network resource partitioning for a single cell macrocell system is presented in Section II. Section III addresses resource partitioning in a multi-layer FFR-enabled network. Optimal selection of resource partitioning parameters is discussed in Section IV, while the paper ends with concluding remarks in Section V.

II. MACROCELL NETWORK

In cellular networks, if network resources are not adequately shared between the CE and CC regions, there will be non-uniformity in the per user data rate of user terminals (UTs) located at the CE and CC regions. This may result from the fact that UTs at the CC region of the cell experience low

path-loss and are also more robust against inter-cell interference compared to those at the CE region who are suffering from high interference as well as high path-loss. For this reason, it is necessary to determine the optimal resource partitioning ratio that minimizes the margin between the per user capacities of UTs located at the CC and CE. In order to address this, we consider the downlink of a SISO-OFDMA macrocell system illustrated in Fig. 1(a), and assume that network resources are partitioned based on FFR-scheme as illustrated in Fig 1(b). The coverage region of the macrocell base station (MBS) has a radius denoted as R , and the base station transmits data to N_m macrocell users that are assumed to be uniformly distributed in the coverage area of the cell.

A. Signal to Noise Ratio

The factor representing the fraction of R that constitutes the radius of the CC region is denoted by α , such that the radius of the CC region is αR , for $0 \leq \alpha \leq 1$. n is the pathloss exponent, d is the distance from a UT to the MBS, and σ is the additive white Gaussian noise. Assuming the power transmitted at the MBS on a sub-carrier is P_m , then, the power received at the UT, P_r , is a decayed version of P_m and it is expressed as [5],

$$P_r = \frac{P_m}{d^n}, \quad (1)$$

$$0 \leq d \leq \alpha R, \text{ for CC UTs,}$$

$$\alpha R \leq d \leq R, \text{ for CE UTs,}$$

and the SNR (signal to noise ratio), γ , at the UT is

$$\gamma = \frac{P_r}{\sigma}. \quad (2)$$

B. Network Capacity

The number of UT per unit area is μ , such that,

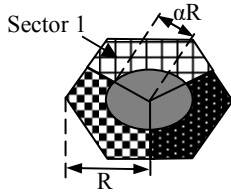


Fig. 1(a) Cell structure.

Sector 1	Macro user network resources	Femto user network resources
Cell center		
Cell edge		

Fig. 1 (b) Network resource partitioning architecture.

$$\mu = \frac{N_m}{\pi R^2}, \quad (3)$$

and the system bandwidth, W , is approximated as:

$$W = \int_0^{2\pi} \int_0^R W_u \mu r dr d\theta, \quad (4)$$

where W_u is the per user bandwidth. Using the Shannon capacity equation, W_u is expressed as:

$$W_u = \frac{C_u}{\log_2(1 + \gamma_u)}, \quad (5)$$

where C_u is the capacity achieved per UT, while γ_u is the corresponding SNR of the UT. Substituting for μ and W_u in (4), W is re-expressed as:

$$\begin{aligned} W &= \int_0^{2\pi} \int_0^R \frac{C_u}{\log_2(1 + \gamma_u)} \frac{N_m}{\pi R^2} r dr d\theta \\ &= \frac{2N_m C_u}{R^2} \int_0^R \frac{1}{\log_2(1 + \gamma_u)} r dr. \end{aligned} \quad (6)$$

Re-arrange (6), we have

$$C_u = \frac{WR^2}{2N \int_0^R \frac{1}{\log_2(1 + \gamma_u)} r dr}. \quad (7)$$

In FFR, the bandwidth W is divided between UTs located at the CC and UTs located at the CE such that the factor representing the fraction of W assigned for CC data transmission is denoted by β as illustrated in Fig 1(b). $C_{u,c}$, which is the per user capacity at the CC is expressed as:

$$C_{u,c} = \frac{\beta WR^2}{2N_m \int_0^{\alpha R} \frac{1}{\log_2(1 + \gamma_{u,c})} r dr}, \quad (8)$$

while $C_{u,e}$, which is the per user capacity at the CE is

$$C_{u,e} = \frac{(1 - \beta)WR^2}{2N_m \int_{\alpha R}^R \frac{1}{\log_2(1 + \gamma_{u,e})} r dr}, \quad (9)$$

where $\gamma_{u,c}$ and $\gamma_{u,e}$ are the per user SNR of CC and CE UTs, respectively. The difference between the per user capacity of CC and CE UTs is denoted by δ , and it is desired to minimize δ in order to achieve fairness between these classes of users (CC and CE UTs). In this vain, δ should be brought close to zero as much as possible. Since δ is a function of β and α , they (β and α) should be appropriately tuned in order to keep δ low.

For numerical evaluation, we assume an OFDMA network in which the MBS transmit power is maintained at 43 dBm while the number of subcarriers available for distribution is set to 600 with each subcarrier spanning a bandwidth of 8 kHz.

The number of MBS users is set to 1,000, and the coverage radius of the MBS is set to 1,000 m. Fig. 2 shows the per UT capacity at the CC as a function of α and β . A decrease in α and an increase in β results in a corresponding increase in $C_{u,c}$. This means that the per user capacity of CC UTs increases when the number of UTs at the CC is reduced while the CC bandwidth is increased. Fig. 3 shows the per UT capacity at the CE as a function of α and β . Contrary to Fig. 2, it is observable in Fig. 3 that an increase in α and a decrease in β results in a corresponding increase in $C_{u,e}$. This means that the per user capacity of the UTs at the CE increases when the number of CE UTs is reduced while the CE bandwidth is increased. If these two graphs (Figs. 2 and 3) are merged together, the regions around their points of intersection is the desired operating region where the margin between $C_{u,e}$ and $C_{u,c}$ is minimum. We use *Simpson's* numerical method to compute $C_{u,e}$ and $C_{u,c}$ while the *brute-force* search algorithm shown in Fig. 4 is used to obtain the values of α that minimizes δ for each value of β considered.

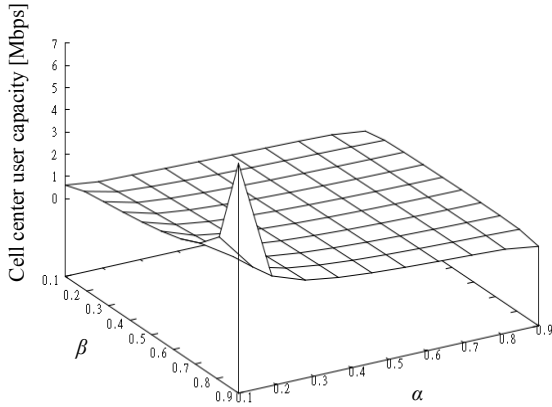


Fig. 2 Per user capacity of macrocell center user, $C_{u,c}$, as a function of α and β .

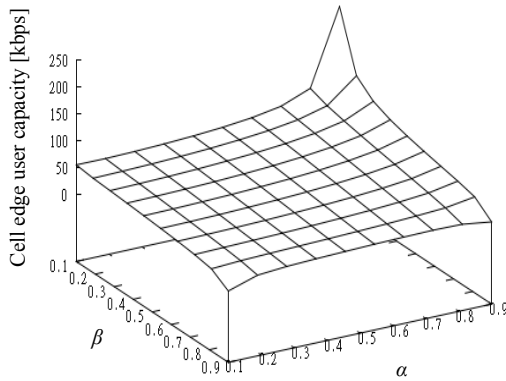


Fig. 3 Per user capacity of macrocell edge user, $C_{u,e}$, as a function of α and β .

```

Initialize:  $\beta = 0.0, \Delta\beta = \Delta\alpha = 0.01, \beta_{\max} = \alpha_{\max} = 1.0$ 
do
     $\alpha_{\text{candidate}} = 0.0, \alpha = 0.0,$ 
    compute  $C_{u,c}$ ,
    compute  $C_{u,e}$ ,
     $\delta_{\text{old}} = |C_{u,c} - C_{u,e}|,$ 
    do
        compute  $C_{u,e}$ 
        compute  $C_{u,c}$ 
         $\delta_{\text{new}} = |C_{u,c} - C_{u,e}|$ 
        if  $\delta_{\text{new}} < \delta_{\text{old}}$  then
             $\delta_{\text{old}} = \delta_{\text{new}},$ 
             $\alpha = \alpha_{\text{candidate}},$ 
        end if
         $\alpha_{\text{candidate}} \leftarrow \alpha_{\text{candidate}} + \Delta\alpha$ 
    while ( $\alpha_{\text{candidate}} < \alpha_{\max}$ )
     $\beta \leftarrow \beta + \Delta\beta$ 
while ( $\beta < \beta_{\max}$ )

```

Fig. 4. Brute-force search algorithm for determining the value of α that gives the minimum δ for each value of β considered.

For each value of β , the entire α -space is searched for an optimal value that minimizes δ using the brute-force search algorithm illustrated in Fig. 4. Optimal values of α resulting from the exhaustive search algorithm are plotted against corresponding values of β as illustrated in Fig. 5. Also illustrated in Fig. 5, is the graph of the quadratic expression $\alpha^2 - \beta = 0$. The two graphs in Fig. 5 are observed to deviate only slightly from each other which indicates that β is approximately equal to the square of α at the point where $|C_{u,e} - C_{u,c}|$ is minimum.

III. FFR-ENABLED MACROCELL AND FEMTOCELL NETWORK

In this section, we further our investigation by considering

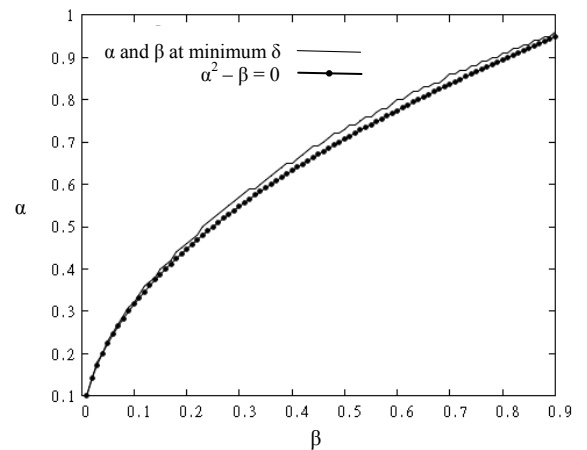


Fig. 5. Comparison between the graph of α plotted against β at minimum δ and the graph of the quadratic expression $\alpha^2 - \beta = 0$.

a heterogeneous architecture of macrocell and femtocell layers with FFR functionality enabled. The structure of resource partitioning is illustrated in Fig 1(b). In Fig. 1(b), it can be observed that the femtocell users are prevented from using a portion of the network resources depending on their positions in the coverage area of the MBS. Considering femtocell UTs located in the center region of the macrocell, the size of the bandwidth that the femtocell is allowed to use is $2/3(1-\beta)W$.

In the case of CE femtocell UT, the size of usable bandwidth is,

$$\frac{2}{3}(1-\beta)W + \beta W = \frac{(2+\beta)W}{3}. \quad (10)$$

In order to estimate the capacity of the femtocell, we assume that femtocells are deployed on the nodes of a grid network. The power transmitted by a serving femtocell BS, s , to femto UT, i , is $P_{s,i}$ while the total antenna gain (receiver and transmitter combined) is $G_{s,i}$. Assuming the distance between the femtocell UT and the femtocell BS is denoted as $l_{s,i}$, the SINR, γ_i , of UT i can be estimated as,

$$\gamma_i = \frac{P_{s,i} G_{s,i} / L(l_{s,i})}{\sum_{j=1}^J \frac{P_{j,i} G_{j,i}}{L(l_{j,i})} + \sigma}, \quad (11)$$

where j represents interfering femtocell BS. $L(l_{s,i})$ is the loss which is a function of the separation distance between the UT and the serving femtocell BS while $L(l_{j,i})$ is the loss due to the UT's distance from neighbor femtocell BS. J denotes the total number of interfering femtocell BSs in the neighborhood of the serving femtocell BS. Due to the way the bandwidth resources are partitioned and due to the equal distance between adjacent nodes of the grid network, the SINR seen at each femtocell will be similar. Assuming the total number of femtocells deployed in the network is denoted as N_f , then, the total number of femtocells at the CC region is,

$$N_f \cdot \frac{\pi(\alpha R)^2}{\pi R^2} = N_f \alpha^2, \quad (12)$$

while the total number of femtocells at the CE region is,

$$N_f - N_f \alpha^2 = (1-\alpha^2)N_f. \quad (13)$$

Combining (10), (11), (12), and (13), the total femtocell capacity, C_f , is therefore,

$$\begin{aligned} C_f &= \frac{2(1-\beta)W\alpha^2 N_f}{3} \log_2(1+\gamma_i) \\ &\quad + \frac{(2+\beta)(1-\alpha^2)W N_f}{3} \log_2(1+\gamma_i) \\ &= \frac{2+\beta-3\alpha^2}{3} N_f W \log_2(1+\gamma_i). \end{aligned} \quad (14)$$

IV. SELECTION OF VALUES OF RESOURCE PARTITIONING PARAMETERS

In this section, proper selection of values of α and β that maximizes femtocell throughput and at the same time maintains user fairness at the MBS is the main focus. As explained in Section II, MBS user fairness is achieved if $\beta = \alpha^2$. Substituting this in (14),

$$C_f = \frac{2+\alpha^2-3\alpha^4}{3} N_f W \log_2(1+\gamma_i), \quad (15)$$

and

$$C_{f,\max} = C_f \Big|_{\frac{\partial C_f}{\partial \alpha}=0, \text{ and } \frac{\partial^2 C_f}{\partial \alpha^2} < 0}, \quad (16)$$

where $C_{f,\max}$ is the maximum total throughput of the femtocell. Solving (16), the value of α that maximizes the femtocell capacity is $(1/6)^{1/2}$. It follows that $\beta = \alpha^2 = 1/6$. These values of α and β ensure user fairness at the MBS layer and also maximizes the total femtocell throughput.

V. CONCLUSIONS

Fourth generation cellular networks will witness a dense deployment of small cell base stations (femtocell) as underlay which makes careful consideration of radio resource (e.g. spectrum) partitioning among the layers of the network imperative. Furthermore, ensuring fairness among user terminals (UTs) is also vital in order to reduce user churn rate. In this paper FFR (fractional frequency reuse) scheme is used for resource partitioning and an exhaustive search algorithm is used to establish a mathematical relationship between network resource partitioning parameters that minimizes the throughput margin between cell-edge and cell-center UTs. Further, exact values of partitioning parameters that maximizes the femtocell throughput are also determined. These parameter values are relevant in the practical implementation of multi-layer FFR-based 4G networks.

REFERENCES

- [1] I. N. Stiakogiannakis, and D. I. Kaklamani "Fractional frequency reuse techniques for multi-cellular WiMAX networks," Proc. the IEEE International Symposium on Personal Indoor and Mobile Communications (PIMRC), pp. 2432–2437, Sept. 2010.
- [2] R. An, X. Zhang, G. Cao, R. Zheng, and L. Sang, "Interference avoidance and adaptive fractional frequency reuse in a hierarchical cell structure," Proc. the IEEE Wireless Communications and Networking Conference (WCNC), April 2010.
- [3] M. Rahman, and H. Komeroglu, "Enhancing cell-edge performance: A downlink dynamic interference avoidance scheme with inter-cell coordination," IEEE Transaction on Wireless Communications, Vol. 9, No. 4, pp. 1414–1425, April 2010.
- [4] X. Wu, A. Das, J. Li, and R. Laroia, "Fractional power reuse in cellular networks" Proc. the 44th Allerton Conference on Communication, Control and Computing, 2006.
- [5] K.-L. Du, and M. N. S. Swamy, "Wireless communication systems: from RF subsystems to 4G enabling technologies," Cambridge University Press, 2010.