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Taguchi's Loss Function Approach to Economic Design of Rectifying Single Sampling Plan by Variable With Asymmetric Quality Cost

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ABSTRACT: The quadratic Taguchi (quality) loss function has been given wide attention in the application of statistical (Economic) process control, which in most cases is always aimed at obtaining optimal setting for a process. This article is an extension of Chung-Ho Chen (2005) and focuses on the use of Taguchi loss function with truncated normal probability density function in designing optimal inspection policy and economic specification limits for rectifying single sampling plan by variable with unequal quality costs ($C_s \neq C_r \neq C_i$).

Keywords: Rectifying Single Sampling plan (RSSP), Taguchi's Loss function, Target value, Specification limits, Cost of scrap (C_s), Cost of rework (C_r), Cost of Inspection (C_i).

1. Introduction

Acceptance sampling is the process of randomly inspecting a sample of goods and deciding whether to accept the entire lot based on the results of an appraisal of the sample. It determines whether a batch of goods should be accepted or rejected. In the current manufacturing world, the path from raw materials to final product often takes place over multiple branches of the manufacturers and across multiple states of the country.

A manufacturing company may likely sub-contract different parts to different sub-contractors. For instance, the assembling of a product might take place in yet

another plant and the packing and shipping, from yet another location. In order to assure a certain quality level, companies use inspection at the different supply chain stages.

Sampling is used everywhere in quality assurance. It should be noted that inspection at different supply chain stages involves sampling some of the items produced. Even if all items produced or services performed are inspected for conformance to specifications, we are still sampling from the universe of all like items or service that have been or will be produced.

Rectifying an inspection plan arises when a lot is rejected and then requires 100% inspection and the accepted lot only takes sampling inspection. The statistical theory of sampling inspection by variables has been treated extensively in many standard statistical quality control textbooks and by many authors such as Lieberman and Resnikoff [1955], and D. C. Montgomery (1991).

Economic design on the type of inspection is to select the type of inspection that will minimize the sum of production costs, acceptance costs and unsatisfactory product costs influenced by the decision regarding inspection. Certain conditions are evident that are favorably to no (0%) inspection, 100% inspection and to sampling inspection.

The traditional concept of conformance to specifications is that items meet the specification limits. Prior to Taguchi (1986), producers believe that any item produced within specifications is as good as any other. Taguchi argues that best quality is achieved by minimizing deviation from the target value. The cost of quality then can be measured as a function of process deviation from target value. The basic philosophy is that whenever an item is produced, a cost is imposed on the society. Part of that cost is borne by the producer and part is borne by the consumer as a result of using the product or services. If these costs are plotted as a function of "quality", producer costs tend to increase with increased "quality"; customer costs tend to decrease because of greater efficiency, less breakdowns, etc.

According to Chandra (2001), a product is said to be scrapped if its quality characteristics under study is less than the lower specification limit. Hence, Cost of scrap is the loss incurred when an item is produced below the specification limits. It is proportional to the quality level of the item produced and can be obtained as follows:

$$C_s = C_o - C_r$$

Where;

C_s is the cost of scrap

C_o Is the cost of producing the item.

C_r Is any amount received by disposing the item as a "secondary" product.

Leavenworth and Grant (1999) stated that the higher the percentage of unsatisfactory product (scrap) submitted and the higher the unsatisfactory product cost per unit of such product, the more favourable the condition for 100% inspection as compared with no (0%) inspection. This can be shown by the figure below:

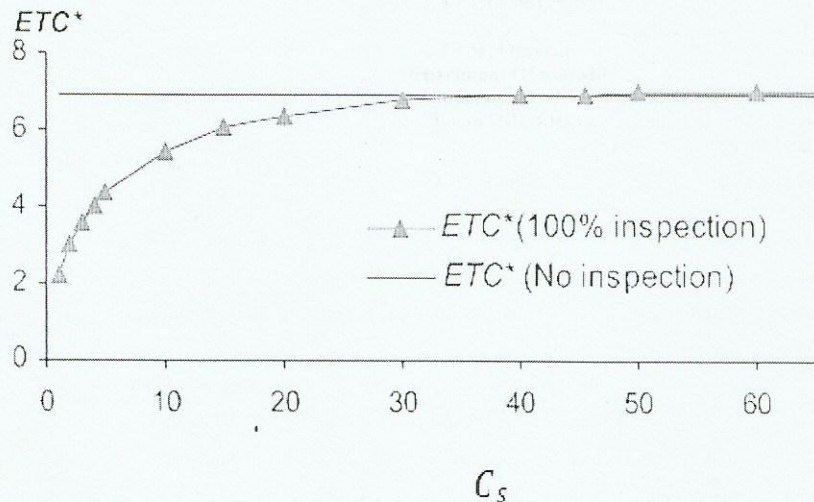


Figure 1.0 Relationship between ETC^* and C_s .

So also from Chandra (2001), product is said to be reworked if its quality characteristics under study is greater than the upper specification limit. Hence, the cost of rework is the extra payments made to make the unacceptable product acceptable. A product is inspected if 100% or sampling inspection is required. Hence, cost of inspection is the time that the inspection personnel spend in evaluating the technical performance of the product (A.V. Feigenbaum (1983)).

Leavenworth and Grant (1999) stated that "the higher the unit cost of inspection, then the lower the effectiveness of 100% inspection and this gives a more favorable condition for no inspection." This can be represented by the figure below:

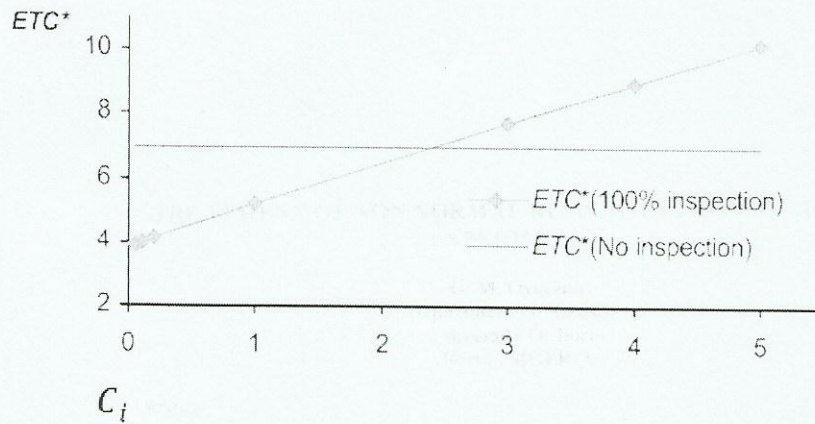


Figure 2.0 relationship between ETC^* and C_i .

In this paper, an extension was attempted to Taguchi's loss function approach to economic design of rectifying single sampling plan by variables with asymmetric quality cost. ($C_{10} \neq C_{15} \neq C$). Just like Chung-Ho Chen (2005), the economic specification limits and optimal inspection policy was obtained.

2. MATHEMATICAL MODEL AND ASSUMPTIONS

Recall, for a variable sampling plan with two specification limits and known variance; the lot is rejected if $\mu - m\sigma < LSL$ and if $\mu + m\sigma > USL$, therefore, $LSL = \mu - m\sigma$ and $USL = \mu + m\sigma$. Where, $m\sigma = \Delta$ is the allowance on either side of the nominal size, hence $USL = \mu + \Delta$ and $LSL = \mu - \Delta$, where $\mu = T$.

According to Leavenworth and Grant (1999), no economic comparison of alternative plans for acceptance inspection is possible without making assumptions regarding the quality of the product submitted for acceptance and the conclusion of any study will depend on these assumptions. Hence, for formulating the mathematical model, the following assumptions are made:

1. The quality characteristics y follows a normal distribution (Truncated normal distribution) with parameters mean μ and variance σ^2 .
2. A loss is incurred when the quality characteristics y , deviates from the target value T , and this loss is governed by the quadratic loss function.

3. The quality characteristics y is nominal-the-best type.
4. The process mean, μ , is centered at the target value T .
5. Cost of scrap (C_s) is not equal to Cost of rework (C_w).
6. It is assumed that when $Y < LSL$, the loss $L(Y) = C_s$, and when $Y > USL$, the associated loss $L(Y) = C_w$.
7. Variances of the two groups are equal, hence the need for pooled variance.

From Chandra (2001), the general form of the loss function for unequal quality cost is given as;

$$L(y) = K_1(y - T), \quad LSL \leq y \leq T \\
= K_2(y - T), \quad T \leq y \leq USL \\
= 0, \quad \text{otherwise.}$$

Now the expected value of the total loss function is defined as:

$$E[L(y)] = K_1 \int_{\mu - m\sigma}^T (y_1 - T)^2 f(y_1) dy_1 + K_2 \int_T^{\mu + m\sigma} (y_2 - T)^2 f(y_2) dy_2$$

$$E[L(y)] = (K_1 + K_2) \sigma_p^2 \left[1 - \frac{m\phi(m)}{\Phi(m) - 1/2} - \left(\frac{\phi(0) - \phi(m)}{\Phi(m) - 1/2} \right)^2 \right]$$

Where

$$\phi(0) = 0.3989 \text{ (Constant)}$$

T is the target value, K_1 is the coefficient of the quality loss for observations $LSL \leq y \leq T$, K_2 is the coefficient of the quality loss for observations $T \leq y \leq USL$, m is the coefficient of the specification limits,

$$f(y_1) = \frac{e^{-\frac{1}{2}\left(\frac{y_1 - \mu}{\sigma}\right)^2}}{[\Phi(m) - 1/2]\sigma\sqrt{2\pi}}, \quad \text{and} \quad f(y_2) = \frac{e^{-\frac{1}{2}\left(\frac{y_2 - \mu}{\sigma}\right)^2}}{[\Phi(m) - 1/2]\sigma\sqrt{2\pi}}$$

are p.d.f of a truncated normal random variable for observations $LSL \leq y \leq T$ and $T \leq y \leq USL$ respectively.

$\mu - m\sigma$ and $\mu + m\sigma$ are the respective lower and upper specification limits for screening the product.

m is the coefficient of the specification limits.

$\Phi(m)$ is the cumulative distribution function for the standard normal random variable with density function $\phi(m)$.

$$\phi(m) = \frac{1}{\sqrt{2\pi}} e^{-\frac{m^2}{2}}, \quad -\infty < m < \infty$$

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3. EXPECTED MATHEMATICAL DECISION FUNCTION

When 0% inspection is performed, the expected total cost per unit is due to variation in the process. Hence the expected total cost per unit for 0% inspection is:

$$ETC_0 = (K_1 + K_2)\sigma_p^2$$

But for 100% inspection, we have to take into consideration the expected total loss per unit for 100% inspection, expected cost per unit due to scrap, expected cost per unit due to rework and expected cost per unit due to inspection. Hence, the expected total cost per unit for 100% inspection is obtained as:

$$\begin{aligned} ETC_{100} &= E[L(y)] + C_s \int_{-\infty}^{LSL} f(y) dy + C_w \int_{USL}^{\infty} f(y) dy + C_i \\ &= (K_1 + K_2)\sigma_p^2 \left[1 - \frac{2m\phi(m)}{2\Phi(m) - 1} - \left(\frac{\phi(0) - \phi(m)}{\Phi(m) - 1/2} \right)^2 \right] + [1 - \Phi(m)](C_s + C_w) + C_i \end{aligned}$$

Where ETC_0 is the expected total cost per unit for no (0%) inspection, ETC_{100} is the expected total cost per unit for 100% inspection, C_s is the scrap cost per unit, C_w is rework cost per unit, C_i is the cost of inspection.

$$\hat{\sigma}_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

is the estimate of the pooled variance.

Chung-Ho Chen (2005) shows that a unique minimum value exists for ETC_{100} on varying $m \in [0, \infty)$ which implies an optimal value of m at the minimum cost for ETC_{100} . This can be obtained by the use of MATLAB software in minimizing ETC_{100} to obtain the optimum m .

To obtain the optimum inspection policy, the rejected lot needs to take the 100% inspection and the accepted lot only takes sampling inspection. The ATI denotes the average total inspection per lot. Leavenworth and Grant (1999) show that in analyzing and evaluating various sampling plans that minimize the ATI, it is convenient to state the problem in terms of Average Fraction Inspected (AFI) which is obtained as:

$$ATI = n + (N - n)(1 - P_a)$$

$$AFI = \frac{ATI}{N}$$

Where;

n is the sample size, N is the lot size and P_a is the probability of acceptance and

$$0 \leq AFI \leq 1$$

From Chung-Ho Chen (2005), for $m > 0, \forall n$, either $ETC_{100} \geq ETC_0$ or $ETC_{100} < ETC_0$. If the minimum AFI is obtained and corresponds to a pair of values $0 < m^* < \infty$, and $0 \leq n^* \leq N$, then we have the following conclusion as regard the optimal inspection policy:

1. If the min AFI = 0, and corresponds to min $m > 0$ and $n=0$,

Hence, $ATI = N(1 - P_a)$

Therefore, $AFI = (1 - P_a)$

But $AFI = 0$, thus we have $1 - P_a = 0$

Implies $P_a = 1$, and this corresponds to no (0%) inspection.

2. If min AFI = 1, and corresponds to min $m \rightarrow \infty$ and $n=N$, then, $ATI=N$

$$AFI = \frac{ATI}{N}$$

$$= 1$$

And this also corresponds to $n=N$, i.e. 100% inspection.

Hence, the optimum inspection is either 0% inspection or 100% inspection. The economic specification limits is then defined through the optimal m value that minimizes ETC_{100} , which will be used to screen the product that will be shipped to the customer when 100% inspection is adopted.

4. NUMERICAL EXAMPLE

Assuming the following parameters

$$T = \mu = 14,$$

$$\hat{\sigma}_p^2 = 0.006, K_1 = 62.81, K_2 = 0.83, C_s = 3.04, C_w = 0.04, C_i = 0.15$$

where, T is the target value, μ is the process mean, K_1 is the coefficient of the quality loss for observations $LSL \leq y \leq T$, K_2 is the coefficient of the quality loss for observations $T \leq y \leq USL$, m is the coefficient of the specification limits, C_s is the scrap cost per unit, C_w is rework cost per unit, C_i is the cost of inspection. By minimizing ETC_{100} , we obtain $m = 2.876$. The optimum decision is to perform 100% inspection because $\min ETC_{100} (0.287) < ETC_0 (0.382)$ and the economic specification limits to screen the product is set as $LSL = \mu - m\hat{\sigma}_p = 13.78$, and $USL = \mu + m\hat{\sigma}_p = 14.22$.

By increasing the cost of inspection in the example above as mentioned by Grant and Leavenworth, we observe a favourable condition for 0% inspection. i.e. assuming all the parameters above are the same except $C_i = 0.25$, we obtain $\min ETC_{100} (0.387) > ETC_0 (0.382)$, hence 0% inspection is performed.

5. CONCLUSION .

This study is a review of an economic design of Chung-Ho Chen which was a generalization of Kapur and Wang's study.

An extension of the study was attempted to determine when the cost of not meeting the target value is not constant and when the scrap and rework cost per item are not equal i.e.

$$C_s \neq C_w \neq C_i .$$

The optimum inspection policy is either no (0%) inspection i.e. acceptance without control or 100% inspection. The 100% inspection arises when the minimum of the expected total cost per unit for 100% inspection (ETC_{100}) is less than the expected total cost per unit for 0% inspection (ETC_0) and no inspection if otherwise.

ETC_0 implies the cost (loss) per unit due to variation in the process.

ETC_{100} implies the loss per unit due to not meeting the target value, unsatisfactory product (scrap or rework) cost and cost of inspection

The economic (optimal) specification limits are defined through the optimal value of m that minimizes ETC_{100} , which is then used to obtain Lower and upper specification limits.

Since this is an economic design, the type of rectifying the single sampling plan becomes irrelevant.

References

1. Bisgaard S, Hunter G, and Pallesen G: Economic Selection of Quality of Manufactured Product. *TECHNOMETRICS* vol. 26 No 1: 9 – 18. 1984.
2. Chandra MJ: Statistical Quality Control. CRC press LLC: 52 – 79. 2001.
3. Chung-Ho Chen: Economic Design of Dodge-Romig AOQL Single Sampling Plans by Variables with the Quadratic Loss Function. *Tamkang Journal of Science and Engineering*, Vol. 8, No 4: 313 – 318. 2005.
4. Feigenbaum AV: Total Quality Control (3rd edition). McGraw-Hill: 109 – 119. 1983.
5. Feng Q and Kapur KC: Economic development of specifications for 100% inspection based on asymmetric quality loss functions. *IIE Transactions*. 38: 659 – 669. 2006.
6. Grant EL and Leavenworth RS: Statistical Quality Control (International edition). McGraw-Hill: 631 – 643. 1999.
7. Lieberman GJ and Resnikoff J: Sampling Plans for Inspection by variables. *Journal of the American Statistical Association*, Vol. 50, issue 270: 457 – 516. 1955.
8. Montgomery DC: Introduction To Statistical Quality Control, John Wiley, New York, NY, U.S.A. 1991.
9. Osanaiye PA: An Economic choice of sampling inspection plans under varying process quality. *Appl. Statist.* Vol. 38, No 2: 301 – 308. 1989.
10. Ross SM: Introduction to Probability and Statistics for Engineers and Scientists (3rd edition). Elsevier Academic press, 2004.
11. Tagaras G: Economic Acceptance Sampling by Variables with Quadratic Quality Costs, *IIE Transactions*, 1992.
12. Wen-Hsien T: Quality cost measurement under activity-based costing. *International Journal of Quality & Reliability Management*, Vol. 15 No. 7: 719 – 752. 1998.