

2018-11-05

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Abstract. This paper investigates the flow of viscous incompressible Non-Newtonian Power-law fluid flowing along a vertical plate in an unsteady stream of fluid having viscous dissipation effect in the presence of transverse magnetic field. Governing equations of the problem were non-dimensionalised and solved by employing Crank-Nicolson finite difference scheme using MATLAB package. A discussion is provided for the magnetic field (M) and thermal radiation (N) effects on the flow. It is shown that velocity distribution reduces due to a rise in thermal radiation and magnetic field parameters respectively while the temperature and concentration distributions increases with an increase in magnetic field parameter and thermal radiation parameter.

Keywords: Magnetic Field; thermal radiation; viscous dissipation; Non-Newtonian; power-law fluid.

1. Introduction

Non-Newtonian fluids, unlike Newtonian fluids, failed to obey Newtonian affinity between shear stress and shear rate. Fluids like slurries, pastes, gels, polymer solutions are usually referred to as non-Newtonian fluids. In porous media, non-Newtonian fluids display a nonlinear behaviour different from that of Newtonian fluids. As a result of this, the study of non-Newtonian fluid flow has received great interest for their various engineering applications.

There is no general model that can describe the shear stress against rate of deformation relationship of all kinds of non-Newtonian fluids but mathematical models are available to describe the nonlinear shear stress against deformation rate relationship of different non Newtonian fluids.

However, mathematical model for describing the mechanistic behaviour of a variety of commonly used non-Newtonian fluids is the Power-Law model which is also known as Ostwald-de Waele model (the name is after the scientist who proposed it).

For a Non-Newtonian Power law fluid, the relationship is given by [6] as

$$\tau_{xy} = K \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \quad (1)$$

Where k is the flow consistency index, n is the power-law index, $\frac{\partial u}{\partial y}$ is the velocity gradient perpendicular to the plane of shear, (x, y) are the Cartesian coordinates of any point in the flow domain and τ_{xy} is the shear stress. The result for $n < 1$ is referred to as pseudo-plastic, $n > 1$ is dilatants and $n = 1$ is Newtonian fluids.

Transfer of heat in porous medium is of considerable interest due to its importance to nature. As a result of this, several work has done in this area and more is still in progress.

[13], using regular perturbation method, studied free convection in a non-Darcian porous channel with radiative flux effect. [22] stimulated radiation-convection gas flow in a non-Darcy porous medium with viscous heating effects using Cogley-Vincenti-Giles non-gray model in combination with

implicit difference method. The study of laminar flows which arises in fluids due to the interaction of the force of gravity and density differences caused by the simultaneous action of diffusion of thermal energy and chemical energy was carried out by [8]. Efforts were made to find analytical and numerical solutions, applying certain special conditions and using different mathematical approaches to the same problem but for stationary vertical plate. [11] investigated the problem of unsteady laminar flow of a gas near an infinite flat plate. He obtained solutions which are available only with Prandtl number unity and under transient conditions of step change in the surface temperature. [23] discovered semi similar solutions for the unsteady free-convective boundary layer flow on a vertical plate using an implicit finite difference scheme. [3] provided solution to the steady free-convective boundary layer flow on a non-isothermal vertical flat plate while [1] studied the unsteady case of the previous problem. The problem of thermal radiation effects on unsteady hydromagnetic gas flow along an inclined plane with indirect natural convection was considered by [9]. Thermal radiation on linearly accelerated vertical plate with variable temperature and uniform mass flux was investigated by [10].

[18] Radiation studied the effects of MHD flow through a porous medium with variable temperature. [18] studied radiation effects on MHD free convection flow near a vertical plate with ramped wall temperature. [20] studied a steady flow of MHD free convective non-Newtonian fluid along a semi-infinite moving vertical plate in a porous medium with Soret and Dufour effects. Radiation effects on MHD combined convection and mass transfer flow past a vertical porous plate embedded in a porous medium with heat generation were studied by [20]. All these above mentioned studies have only dealt with Newtonian fluid flows.

In some years back, flow of non-Newtonian fluids has drawn the attention of researchers due to its great importance. Despite the fact that researchers keep reporting on different problems arising from this flow, there are still huge of work to be done. The rheological effects of non-Newtonian fluids through porous media occur in a broad range of engineering applications such as transport processes in chemical industry, storage of

nuclear waste materials, discoveries of the flow of oil in petroleum reservoirs and food processing.

The flow in a power-law fluid over a flat plate moving at constant speed in the direction and opposite to the direction of the main stream was considered by [25]. [6], over an unsteady sheet that is stretching, examined the heat transfer in a power-law fluid.

[2] studied the Flow of a power-law fluid film on an unsteady stretching surface while the Series solutions of unsteady Magnetohydrodynamic flows of non-Newtonian fluids caused by an impulsively stretching plate was investigated by [24]. The Finite difference solution of mass transfer and free convective effects of a Power law fluid past an impulsively started vertical plate was considered by [15] while [7] studied the transient analysis of heat and mass transfer by natural convection in Power-law fluid past a vertical plate immersed in a Porous Medium.

Recently, studies were made on the unsteady free-convective boundary layer flow on moving vertical plate considering the effects of bouyancy forces on the boundary layer. Some of these studies are presented in the works of [16] and [10]. [5] investigated an unsteady natural convective power law fluid flow past a vertical plate embedded in a non-Darcian porous medium in the presence of a homogeneous chemical reaction. The combined effect of magnetic field, dissipation and radiation was not taken into consideration in their work.

In view of this, our aim is to study the effects of these neglected terms together with other physical parameters arising on the flow of a non-Newtonian power law fluid past a vertical plate embedded in the presence transversely applied magnetic field. Numerical results for the dimensionless velocity, temperature and concentration profiles are presented graphically.

2. Problem Formulation

Consider the flow of viscous incompressible Non-Newtonian Power-law fluid past a vertical plate in an unsteady stream of fluid with viscous dissipation effect in the presence of transverse magnetic field. At $t^* > 0$ the plate started moving impulsively with constant

u_o in the vertical direction while the temperature and concentration along the plate was raised and they both maintained constantly. The presence of thermal radiation is represented by y^* transversely to the vertical surface according to [12] as $q_r = -\frac{4\sigma}{3k^1} \frac{\partial T^{*4}}{\partial y^*}$.

The geometry of the problem is given below.

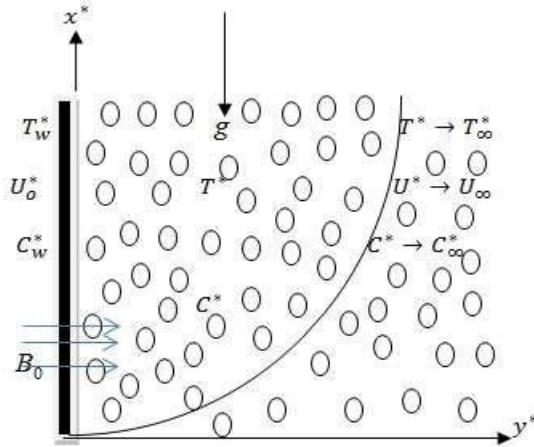


Figure 1: Geometry of the problem

Under constant fluid properties, Boussinesq approximation and assuming a chemical reaction of the first order, the governing equations for the transient incompressible flow of Non-Newtonian electrically conducting and chemically reacting fluid are; continuity, momentum, energy and concentration equations.

Continuity Equation

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Momentum Equation

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} &= \frac{1}{\rho} \frac{\partial}{\partial y^*} (\tau_{xy}) + g\beta_T(T^* - T_\infty^*) + \\ g\beta_C(C^* - C_\infty^*) - \frac{\sigma}{\rho} B_0^2 u^* \end{aligned} \quad (3)$$

Energy Equation

$$\begin{aligned} \frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} &= \alpha \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \\ &\frac{\partial}{\partial y^*} \left[\frac{-4\sigma \partial T^{*4}}{3k^1 \partial y^*} \right] + \frac{k}{\rho C_p} \left\{ \left[\frac{\partial u^*}{\partial y^*} \right] \right\}^{n-1} \frac{\partial u^*}{\partial y^*} \end{aligned} \quad (4)$$

Concentration Equation

$$\frac{\partial C^*}{\partial t^*} + u^* \frac{\partial C^*}{\partial x^*} + v^* \frac{\partial C^*}{\partial y^*} = D \left[\frac{\partial^2 C^*}{\partial y^{*2}} \right] - K_c (C^* - C_\infty^*) \quad (5)$$

By referring to equation (1), the momentum equation becomes

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} &= \frac{k}{\rho} \frac{\partial}{\partial y^*} \left(\left| \left(\frac{\partial u^*}{\partial y^*} \right) \right|^{n-1} \frac{\partial u^*}{\partial y^*} \right) + \\ g\beta_T(T^* - T_\infty^*) + g\beta_C(C^* - C_\infty^*) - \frac{\sigma}{\rho} B_0^2 u^* \end{aligned} \quad (6)$$

where t^* represents dimensionless time, u^* and v^* are the velocity components in x^* and y^* directions, ρ denotes fluid density, g signifies acceleration due to gravity, β_T represents volumetric coefficient of thermal expansion, β_C denotes volumetric coefficient of concentration expansion, T^* represents fluid temperature, C^* signifies fluid concentration, T_∞^* and C_∞^* are the free stream temperature and species concentration, D is the species diffusivity, C_p is the specific heat at constant pressure, σ is the electric conductivity, B_0 is the uniform external magnetic field along the y^* axis and k^1 is the mean absorption coefficient.

The corresponding initial and boundary conditions are given as follows:

$$\begin{aligned} t^* \leq 0: & \quad u^* = 0, v^* = 0, T^* = T_\infty^*, C^* = C_\infty^*, \\ & \quad \text{for every } x^* \text{ and } y^* \\ t^* > 0: & \quad u^* = u_o^*, v^* = 0, T^* = T_w^*, C^* \\ & \quad = C_w^* \quad \text{at } y^* = 0 \\ u^* = 0, & \quad T^* = T_\infty^*, C^* = C_\infty^* \quad \text{at } x^* = 0 \end{aligned}$$

$$u^* \rightarrow 0, T^* \rightarrow T_{\infty}^*, C^* \rightarrow C_{\infty}^* \text{ as } y^* \rightarrow \infty. \quad (7)$$

Where T_w and C_w are the wall temperature and concentration respectively. Following [19], Taylor series for expansion for T^{*4} , without the higher order terms is

$$T^{*4} \approx 4T_{\infty}^{*3}T^* - 3T_{\infty}^{*4}$$

As a result of equation (8), equation (4) becomes

$$\begin{aligned} \frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \alpha \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{16\sigma T_{\infty}^{*3}}{3k^1 \rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} \\ + \frac{k}{\rho C_p} \left[\left| \frac{\partial u^*}{\partial y^*} \right|^{n-1} \frac{\partial u^*}{\partial y^*} \right] \end{aligned} \quad (9)$$

To transform the dimensional governing equations (2), (5), (6) and (9) to dimensionless equations, the following non-dimensional quantities are used.

$$\begin{aligned} x = \frac{x^*}{l}, y = \frac{y^*}{l}, U = \frac{u^*}{u_0^*}, V = \frac{v^*}{u_0^*}, T = \frac{T^* - T_{\infty}^*}{T_w^* - T_{\infty}^*}, \\ C = \frac{C^* - C_{\infty}^*}{C_w^* - C_{\infty}^*}, t = \frac{u_0^* t^*}{l}, u_0^* = \left(\frac{\rho l^n}{k} \right)^{\frac{1}{n-2}} \end{aligned} \quad (10)$$

where l is the length. Introducing equation (10) into the dimensional equations (2), (5), (6) and (9), we arrived at the following dimensionless equations.

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$\begin{aligned} \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} \left(\left| \frac{\partial U}{\partial y} \right|^{n-1} \frac{\partial U}{\partial y} \right) \\ + GrT + GmC - MU \end{aligned} \quad (12)$$

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \frac{1}{Pr} \left[1 + \frac{4}{3N} \right] \frac{1}{Re} \frac{\partial^2 T}{\partial y^2} + Ec \left[\left| \frac{\partial U}{\partial y} \right|^n \frac{\partial U}{\partial y} \right]$$

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = \frac{1}{ScRe} \frac{\partial^2 C}{\partial y^2} - \gamma C$$

where $Sc = \frac{\nu}{D}$ stands for Schmidt number, x and y are dimensionless coordinates, U and V are dimensionless velocities, t denotes dimensionless time, n is the fluid indices, T denotes dimensionless temperature function, C signifies dimensionless concentration function, $N = \frac{k^1 k}{4\sigma T_{\infty}^{*3}}$ is the radiation heat transfer parameter, $Pr = \frac{\rho \nu C_p}{k}$ stands for Prandtl number, $Re = \frac{u_0^* l}{\nu}$ is the Reynold number, $Ec = \frac{u_0^{*2}}{C_p(T_w^* - T_{\infty}^*)}$ is the Eckert number, $Gr = \frac{g \beta_T (T_w^* - T_{\infty}^*)}{u_0^{*2}}$ stands for thermal Grashof number, $Gm = \frac{g \beta_C (T_w^* - T_{\infty}^*)}{u_0^{*2}}$ is the species Grashof number, $\gamma = \frac{k_c l}{u_0^*}$ stands for chemical reaction parameter and $M = \frac{\sigma B_0^2 \nu}{\rho u_0^{*2}}$ is the Magnetic field parameter.

Under these initial and boundary conditions:

$$t \leq 0, U = 0, V = 0, T = 0, C = 0, \forall X, Y$$

$$t > 0, U = 1, V = 0, T = 1, C = 1, \text{ at } Y = 0$$

$$U = 0, T = 0, C = 0, \text{ at } X = 0$$

$$U = 0, T = 0, C = 0 \text{ as } Y \rightarrow \infty.$$

Skin friction, Nusselt and Sherwood numbers are of great importance in analyzing the importance of this flow. The Skin friction is the friction force acting on a solid body that is moving past a fluid. The Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer while the Sherwood number often known as the rate at which mass is transferred at wall is the dimensionless group in support of convective mass transfer in fluid flow and the ratio between mass transfer by convection and mass transfer by diffusion.

The skin-friction is defined in terms of a non-dimensional skin-friction coefficient C_f as follows;

(13)

$$C_t Re^{1/2} = \left| \frac{\partial U}{\partial Y} \right|_{Y=0}^n \quad (16)$$

The dimensionless Nusselt and Sherwood numbers are given as

$$Nu Re^{-1/2} = - \left(\frac{\partial T}{\partial Y} \right)_{Y=0} \quad (17)$$

$$Sh Re^{-1/2} = - \left(\frac{\partial C}{\partial Y} \right)_{Y=0} \quad (18)$$

3. Solution to The Problem

To solve these transient partial differential equations (11), (12), (13) and (14) with the boundary conditions (15), we employ a finite difference scheme of the Crank-Nicolson type. The partial differential equations were converted to difference equation. The Crank-Nicolson method converges faster and it is unconditionally stable. It has been used in several heat transfer, radiation and convection flow problems. [17] used Crank-Nicolson scheme to analyze the transient convective heat and mass transfer with thermal radiation effects along a vertical impulsively started plane. The co-ordinates (x, y, t) are defined by $x = i\Delta x$, $y = j\Delta y$ and $t = k\Delta t$ while the values of u at these mesh points is given by $U(i\Delta x, j\Delta y, k\Delta t) = U_{i,j}^k$ with positive integers i, j, k . The finite difference equations corresponding to the transient coupled non-linear partial differential equations are as follows:

$$\begin{aligned} & \left[\frac{U_{i,j}^{k+1} - U_{i-1,j}^{k+1} + U_{i,j}^k - U_{i-1,j}^k + U_{i,j+1}^{k+1} - U_{i-1,j+1}^{k+1}}{4\Delta x} + \right. \\ & \left. \frac{U_{i,j+1}^k - U_{i-1,j+1}^k}{4\Delta y} \right] + \\ & \left[\frac{V_{i,j}^{k+1} - V_{i,j-1}^{k+1} + V_{i,j}^k - V_{i,j-1}^k}{2\Delta y} \right] = 0 \end{aligned} \quad (19)$$

$$= \left(\frac{1}{\Delta y} \right) \left[\left| \frac{U_{i,j+1}^k - U_{i,j}^k}{\Delta y} \right|^{n-1} \left(\frac{U_{i,j+1}^k - U_{i,j}^k}{\Delta y} \right) \right]$$

$$\left(\frac{1}{\Delta y} \right) \left[\left| \frac{U_{i,j}^k - U_{i,j-1}^k}{\Delta y} \right|^{n-1} \left(\frac{U_{i,j}^k - U_{i,j-1}^k}{\Delta y} \right) \right]$$

$$Gr \frac{[T_{i,j}^{k+1} + T_{i,j}^k]}{2} + Gm \frac{[C_{i,j}^{k+1} + C_{i,j}^k]}{2} - \quad (20)$$

$$M \frac{[U_{i,j}^{k+1} + U_{i,j}^k]}{2}$$

$$\begin{aligned} & \frac{[T_{i,j}^{k+1} - T_{i,j}^k]}{\Delta t} + U_{i,j}^k \frac{[T_{i,j}^{k+1} - T_{i-1,j}^{k+1} + T_{i,j-1}^{k+1} - T_{i-1,j-1}^{k+1}]}{2\Delta x} \\ & V_{i,j}^k \frac{[T_{i,j+1}^{k+1} - T_{i,j-1}^{k+1} + T_{i,j+1}^k - T_{i,j-1}^k]}{4\Delta y} = \end{aligned}$$

$$\frac{1}{PrRe} \left[1 + \frac{4}{3N} \right] \frac{[T_{i,j-1}^{k+1} - 2T_{i,j}^{k+1} + T_{i,j+1}^{k+1} + T_{i,j-1}^k - 2T_{i,j}^k + T_{i,j+1}^k]}{2(\Delta y)^2} +$$

$$Ec \left[\left| \frac{U_{i,j+1}^k - U_{i,j-1}^k}{2\Delta y} \right|^n \left(\frac{U_{i,j+1}^k - U_{i,j-1}^k}{2\Delta y} \right) - \right]$$

$$\begin{aligned} & \frac{[C_{i,j}^{k+1} - C_{i,j}^k]}{\Delta t} + U_{i,j}^k \frac{[C_{i,j}^{k+1} - C_{i-1,j}^{k+1} + C_{i,j}^k - C_{i-1,j}^k]}{2\Delta x} + \\ & V_{i,j}^k \frac{[C_{i,j+1}^{k+1} - C_{i,j-1}^{k+1} + C_{i,j+1}^k - C_{i,j-1}^k]}{4\Delta y} = \frac{1}{ScRe} \left[\left| \frac{C_{i,j+1}^{k+1} - C_{i,j-1}^{k+1}}{2\Delta y} \right|^n \left(\frac{C_{i,j+1}^{k+1} - C_{i,j-1}^{k+1}}{2\Delta y} \right) - \right. \end{aligned}$$

The finite difference equations can now be written in the simplified form:

$$-V_{i,j-1}^{k+1} + V_{i,j}^{k+1} = F_1 \quad (23)$$

$$\begin{aligned} & \frac{U_{i,j}^{k+1} - U_{i-1,j}^{k+1} + U_{i,j}^k - U_{i-1,j}^k + U_{i,j+1}^{k+1} - U_{i-1,j+1}^{k+1}}{\Delta t} + U_{i,j}^k \frac{[U_{i,j}^{k+1} - U_{i-1,j}^{k+1} + U_{i,j}^k - U_{i-1,j}^k + U_{i,j+1}^{k+1} - U_{i-1,j+1}^{k+1}]}{2\Delta x} + \\ & V_{i,j}^k \frac{[U_{i,j+1}^{k+1} - U_{i,j-1}^{k+1} + U_{i,j+1}^k - U_{i,j-1}^k]}{4\Delta y} = F_2 \end{aligned} \quad (24)$$

$$\frac{U_{i,j}^{k+1} - U_{i,j}^k}{\Delta t} + U_{i,j}^k \frac{[U_{i,j}^{k+1} - U_{i,j}^k]}{\Delta x} + V_{i,j}^k \frac{[U_{i,j+1}^{k+1} - U_{i,j-1}^{k+1} + U_{i,j+1}^k - U_{i,j-1}^k]}{4\Delta y} = F_3 \quad (25)$$

$$V_{i,j}^k \frac{[U_{i,j+1}^{k+1} - U_{i,j-1}^{k+1} + U_{i,j+1}^k - U_{i,j-1}^k]}{4\Delta y} - A_4 C_{i,j-1}^{k+1} + B_4 C_{i,j}^{k+1} + D_4 C_{i,j+1}^{k+1} = F_4 \quad (26)$$

where

$$E_1 = -\frac{\Delta y}{2\Delta x}; E_2 = \frac{\Delta t}{2\Delta x} U_{i,j}^k; E_3 = \frac{\Delta t}{4\Delta y} V_{i,j}^k;$$

$$E_4 = \frac{\Delta t Gr}{2}; E_5 = \frac{\Delta t Gm}{2};$$

$$E_8 = \frac{\Delta t M}{2}; E_9 = \frac{1}{PrRe} \left[1 + \frac{4}{3N} \right] \frac{\Delta t}{2(\Delta y)^2};$$

$$E_{11} = \frac{\Delta t}{2(\Delta y)^2 ScRe}; E_{12} = \Delta t Ec; E_{13} = \frac{\Delta t \gamma}{2};$$

$$W = \frac{1}{\Delta y} \left[\left| \frac{U_{i,j+1}^k - U_{i,j}^k}{\Delta y} \right|^{n-1} \left(\frac{U_{i,j+1}^k - U_{i,j}^k}{\Delta y} \right) \right] -$$

$$\frac{1}{\Delta y} \left[\left| \frac{U_{i,j}^k - U_{i,j-1}^k}{\Delta y} \right|^{n-1} \left(\frac{U_{i,j}^k - U_{i,j-1}^k}{\Delta y} \right) \right]$$

$$F_1 = E_1 \left[\frac{U_{i,j}^{k+1} - U_{i-1,j}^{k+1} + U_{i,j}^k - U_{i-1,j}^k + U_{i,j+1}^{k+1} - U_{i,j+1}^k}{U_{i-1,j-1}^{k+1} + U_{i,j-1}^k - U_{i-1,j-1}^k} \right] + V_{i,j-1}^k - V_{i,j}^k$$

$$F_2 = (E_3) U_{i,j-1}^k + (1 - E_2 + E_8) U_{i,j}^k + (-E_3) U_{i,j+1}^k +$$

$$E_4 [T_{i,j}^{k+1} + T_{i,j}^k] + E_5 [C_{i,j}^{k+1} + C_{i,j}^k] + E_2 U_{i-1,j}^{k+1} +$$

$$E_2 U_{i-1,j}^k + W$$

$$F_3 = (E_3 + E_9) T_{i,j-1}^k + (1 - E_2 - 2E_9) T_{i,j}^k - (E_3 - E_9) T_{i,j+1}^k + E_2 T_{i-1,j}^{k+1}$$

$$F_4 = (E_3 + E_{11}) C_{i,j-1}^k + (1 - E_2 - 2E_{11} - E_{13}) C_{i,j}^k$$

$$- (E_3 - E_{11}) C_{i,j+1}^k + E_2 C_{i-1,j}^{k+1} + E_2 C_{i-1,j}^k$$

where

$$A_2 = E_3; B_2 = 1 + E_2 - E_8; D_2 = E_3;$$

$$A_3 = E_3 + E_9; B_3 = 1 + E_2 + 2E_9; D_3 = E_3 - E_9;$$

$$A_4 = E_3 + E_{11}; B_4 = 1 + E_2 + 2E_{11} + E_{13}; D_4 = E_3 - E_{11};$$

The grid point along x - direction is denoted by i - while j - and k - stands for the grid

point along y - and t - directions respectively. The grids along x and y are divided into P and Q grid spacing respectively with $\Delta X = 0.05$, $\Delta Y = 0.25$ and $\Delta t = 0.01$ as the mesh sizes.

In the computations, the coefficients $U_{i,j}^k$ and $V_{i,j}^k$ at any one time step are taken to be constants. The values of C , T , U and V are known at all grid points at $t=0$ from the initial conditions. The values of C , T , U and V at time level $(k+1)$ are calculated using the known values at previous time level (k)

(27) as follows:

Equation (26) on a given i - level at every internal nodal point forms a tri-diagonal system of equation. Thomas algorithm which was discussed in [4] was used to provide solution to the problem. With this, the value of C on a given i - level at time $(k+1)$ level was known at every nodal point. Also, the value of T at time $(k+1)$ level was calculated in the same manner we calculated C using equation (25).

The results gotten from C and T at time $(k+1)$ level are used in equation (24) to calculate U at time $(k+1)$ level.

Hence, the values of C , T and U are known on a particular i - level. This process was carried out several times at various i - levels. In this way, the values of C , T and U are known at all grid point.

The values of V are also calculated at every nodal point explicitly using equation (23) on a particular i - level at $(k+1)$ th time level.

(28)

4. Discussion of Results

Computations were conducted to examine the influence of magnetic field (M), thermal radiation parameter (N), Schmidt number (Sc), viscous dissipation function (Ec), thermal Grashof number (Gr), modified Grashof number (Gm) and Prandtl number (Pr) that are arising from the flow on the velocity, temperature and concentration profiles. To

(29) check the correctness of this work, it was compared with the Newtonian work of Anwar et al. (2008) that is a specific case ($n=1$) of this study and was found to be in good agreement as shown in figure 2.

Figures 3, 4 and 5 represent the impact of magnetic field parameter on the profiles of Velocity, temperature and concentration respectively. Retarding effect on velocity profile is experienced on figure 3 as a result of increasing magnetic field parameters giving boundary layer thickness significant changes. This retarding effect is as a result of the presence of Lorentz force, with his power to slow down the motion of the fluid. It was observed that temperature and concentration profiles increased with an increase in magnetic field on figures 4 and 5. On Figure 6, with a rise in the N values, velocity profile experienced a decrease which is therefore accompanied by reduction in the momentum boundary layers.

Figures 7 and 8 display the temperature and concentration profiles for various thermal radiation values N . It was observed from these figures a fall in the fluid temperature as N increases and a rise in fluid concentration as thermal radiation parameter increases. Low radiation flux was experienced due to a high value of N and the rate at which energy was transported to the fluid was reduced by thermal radiation.

Figure 9, 10 and 11 displays the impacts of Schmidt number Sc on fluid velocity, temperature and concentration fields. Fluid velocity and concentration profiles decreases as a result of increasing Schmidt number. This causes a reduction in velocity, diffusivity and the concentration layer thickness because schmidt number measures the ratio of thickness of the viscous and concentration boundary layer. Low diffusion property was seen with a high value of Sc . This makes concentration boundary layer become thinner than velocity boundary layer. Increasing Schmidt number causes a slight increases the fluid temperature.

Figure 12 reports on velocity distribution having influence of thermal Grashof Gr number. A rise in velocity distribution was caused by an increase in Gr and this increase boosts the buoyancy force which increases the fluid flow.

Figure 13 represent the impact of thermal Grashof number on temperature distribution. This shows in the boundary layer the influence of thermal buoyancy force to viscous hydrodynamic force. An increase in Gr produced a decrease in the fluid

temperature with a smooth decrease from 1 at the wall down the free stream.

Figure 14 represents the temperature distribution for modified thermal Grashof numbers. Increasing Gm leads to an increase in the velocity of the fluid while the fluid temperature decreases down the stream.

Figures 15 and 16 show the Velocity and temperature distributions having the impact of Prandtl number Pr . Increasing the value of Pr from caused the velocity and thermal boundary layer thickness to decrease because Pr with higher values reaches zero faster while smaller of Pr values causes a rise in the thermal conductivity across the boundary layer.

The effect of increasing Eckert number on velocity and temperature is presented on figures 17 and 18. With the values used in plotting the graphs, it was discovered that both velocity and temperature profile decreases with significant changes in both momentum and thermal boundary layer thicknesses.

Different values of the power law index n is plotted for both velocity and temperature profiles on figures 19 and 20. As n increases, significant changes on velocity profile and the thermal boundary layer is observed. Temperature profile decreases with no significant changes in the thermal boundary layer thickness.

Also, Skin friction coefficient, Nusselt and Sherwood numbers were expressed as non-dimensional velocity, temperature and concentration gradients. Different values of fluid parameters are shown on Table 1 and contributions of these parameters were tabulated. The results from table 1 show that increasing either thermal Grashof number Gr and power law index n causes the skin friction to reduce whereas an increase in any of thermal radiation parameter N , Magnetic field parameter M , Schmidt number Sc , viscous dissipation parameter Ec and Prandtl number Pr increases the skin friction coefficient. Increasing thermal Grashof number implies increasing buoyancy and this in turn accelerates the flow producing decrease in the boundary layer skin friction coefficient. A rise in thermal radiation decelerates the wall velocity gradient thereby increasing the skin friction coefficient.

As thermal radiation N , Prandtl number Pr , viscous dissipation parameter Ec and Power-law index n increases, the rate of heat transfer

increases while a reduction is seen with an increase in Schmidt number Sc , magnetic field parameter M and thermal Grashof number Gr . For Sherwood number, as Schmidt number Sc , viscous dissipation parameter Ec , thermal Grashof number Gr and Power-law index n increases, the Sherwood number also increases while it reduces with an increase in Prandtl number Pr , Magnetic field parameter M and thermal radiation N .

Grashof number Gr increases, the Sherwood number increases while it reduces with an increase in Prandtl number Pr , Magnetic field parameter M and thermal radiation N .

5. Conclusion

In this study, a problem of transient free convection flow of Power-law fluid flowing past a vertical plate was considered. The governing equations were non-dimensionalised and solution was provided using Crank-Nicolson finite difference scheme with the help of MATLAB programming package. The investigation shows that: velocity increases with an increase in thermal Grashof number, Power-law index and modified Grashof number whereas it decreases with an increase in magnetic field parameter, viscous dissipation parameter, thermal radiation, Schmidt number, Prandtl and permeability parameter.

Temperature increases with a rise in magnetic field parameter and thermal radiation parameter while it decreases as Schmidt number, thermal Grashof number, viscous dissipation parameter, modified Grashof number and Prandtl number increases.

Concentration increases as magnetic field and thermal radiation parameters increases while it decreases with an increase in Schmidt, Prandtl, thermal Grashof and modified Grashof numbers respectively. Also, increasing thermal Grashof number Gr and power law index n reduces skin friction while a rise in Magnetic fields parameter M , thermal radiation parameter N , Schmidt number Sc , Prandtl number Pr and viscous dissipation parameter Ec causes skin friction to increase.

Increasing thermal radiation N , Prandtl number Pr , viscous dissipation parameter Ec and Power-law index n increases Nusselt number while it reduces with an increase in Schmidt number Sc , magnetic field parameter M and thermal Grashof number Gr . For Sherwood number, as Schmidt number Sc , viscous dissipation parameter Ec and thermal

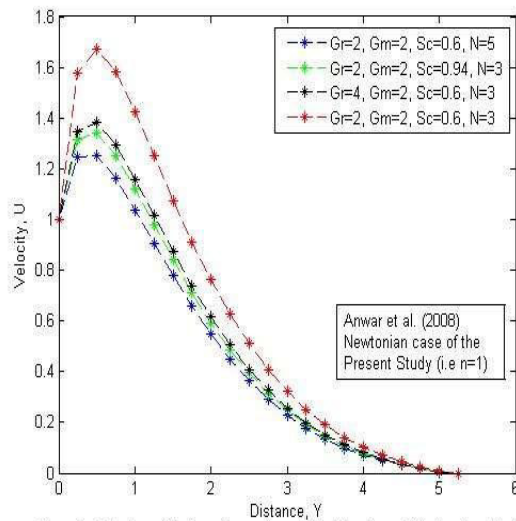


Figure 2: Velocity profile for various values of Gr , Gm , Sc and N at $n=1$ and $t=0.75$

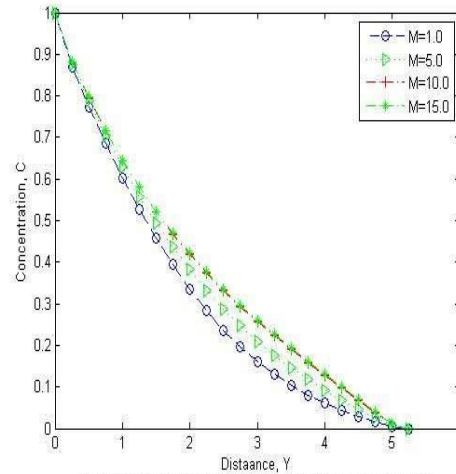


Figure 5: Concentration profile for various values of Magnetic Field

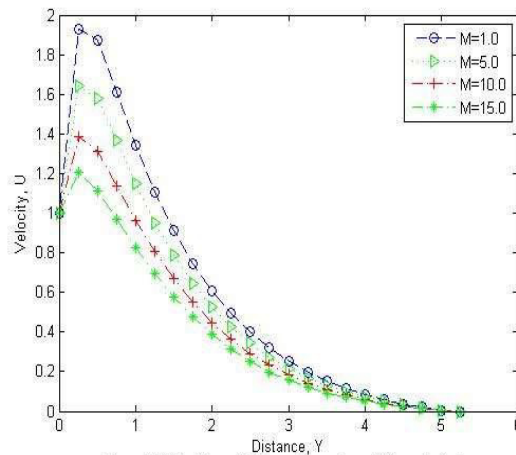


Figure 3: Velocity profile for various values of Magnetic field

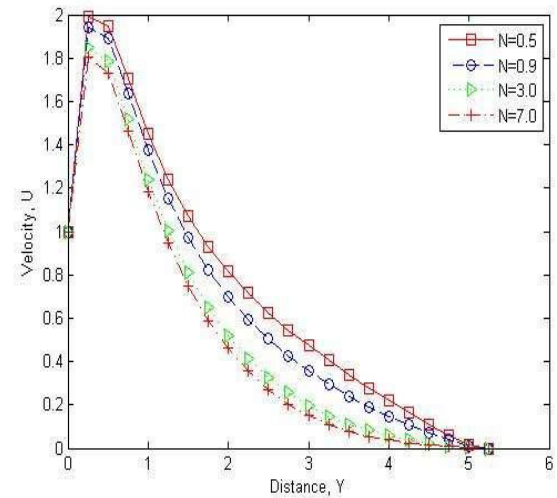


Figure 6: Velocity profile for various values of thermal radiation

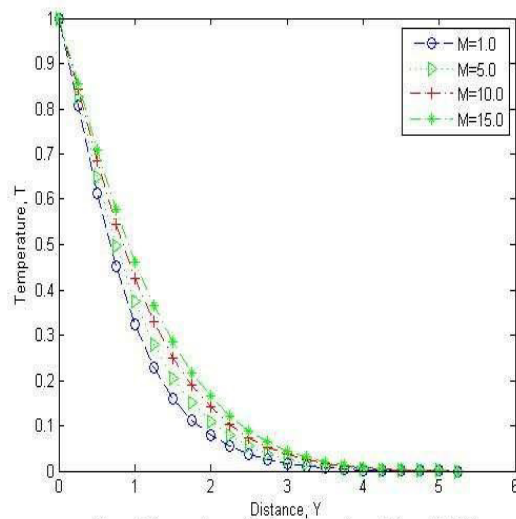


Figure 4: Temperature profile for various values of Magnetic field

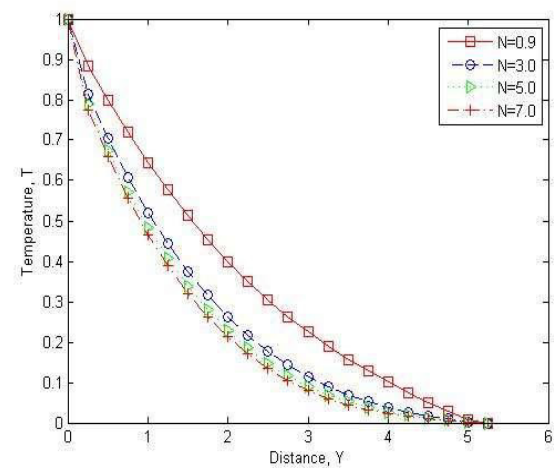


Figure 7: Temperature profile for various values of thermal radiation

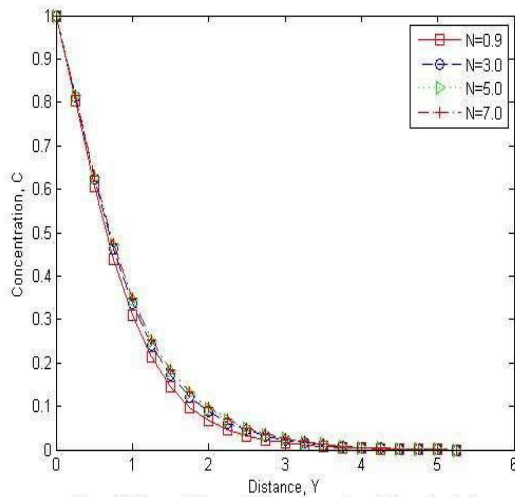


Figure 8: Concentration profile for various values of thermal radiation

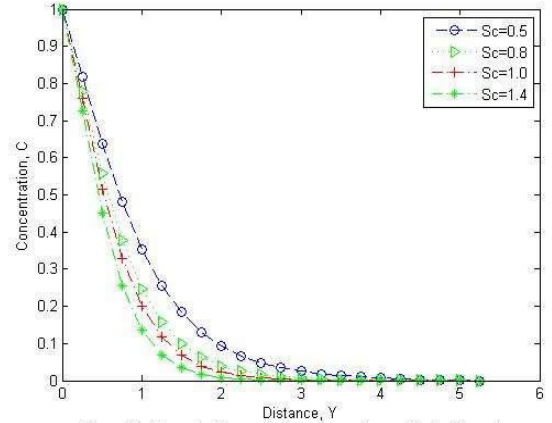


Figure 11: Concentration profile for various values of Schmidt number

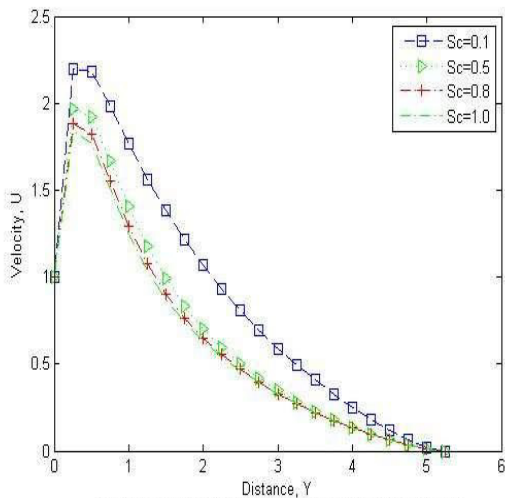


Figure 9: Velocity profile for various values of Schmidt number

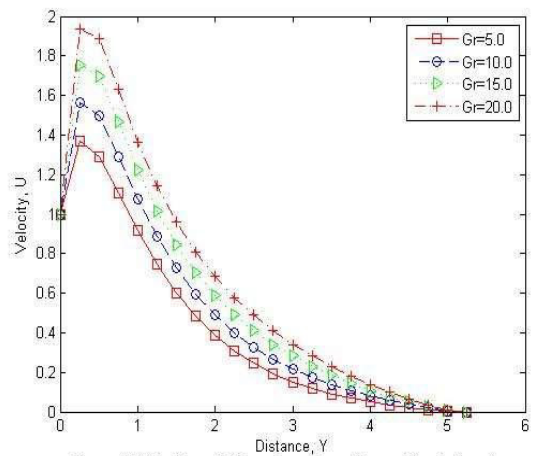


Figure 12: Velocity profile for various values of thermal Grashof number

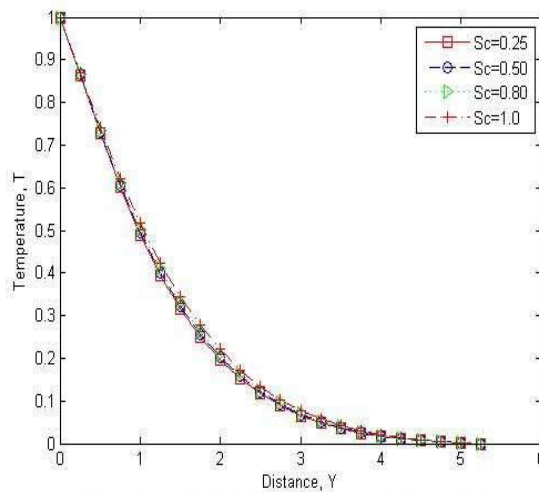


Figure 10: Temperature profile for various values of Schmidt number

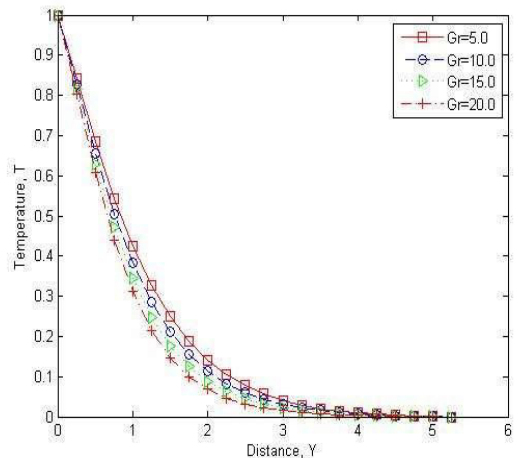


Figure 13: Temperature profile for various values of thermal Grashof number

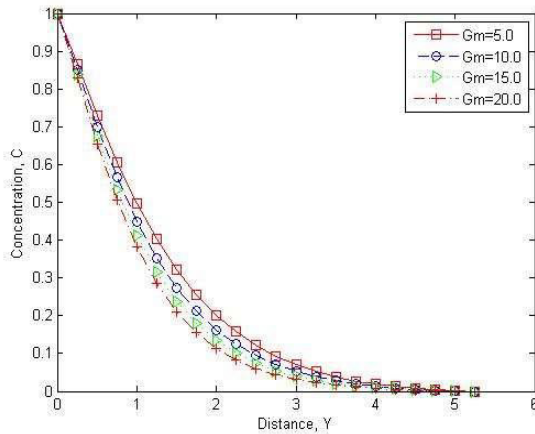


Figure 14: Concentration profile for various values of modified Grashof number

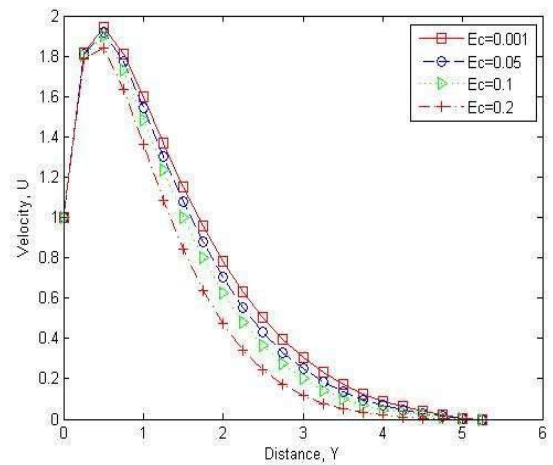


Figure 17: Velocity profile for various values of Eckert number

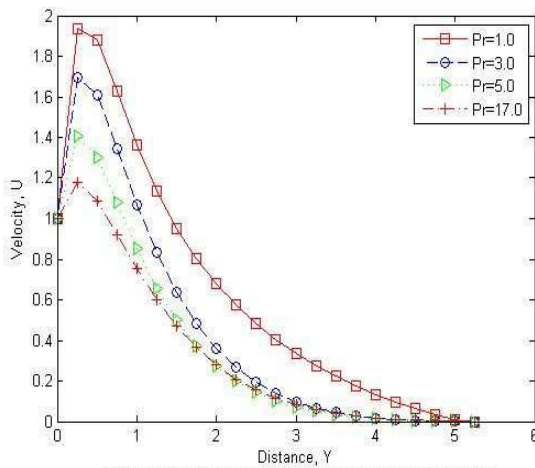


Figure 15: Velocity profile for various values of Prandtl number

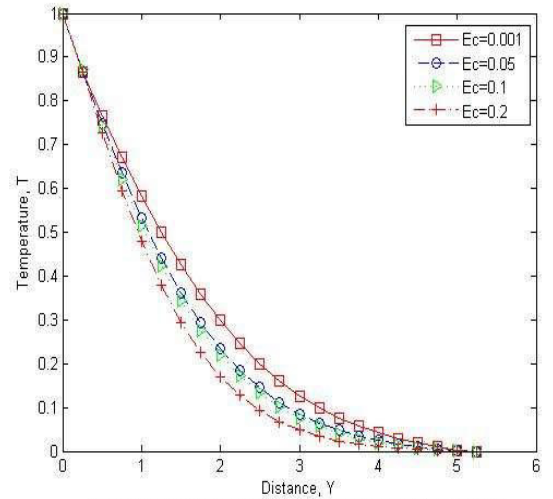


Figure 18: Temperature profile for various values of Eckert number

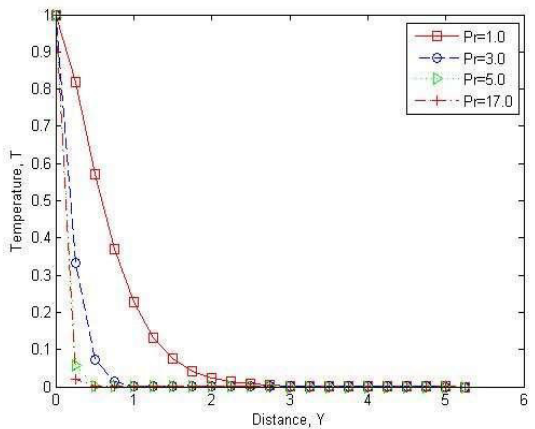


Figure 16: Temperature profile for various values of Prandtl number

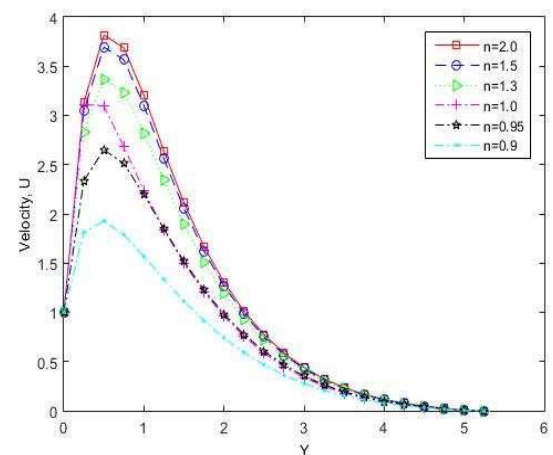


Figure 19: Velocity profile for various values of power law index

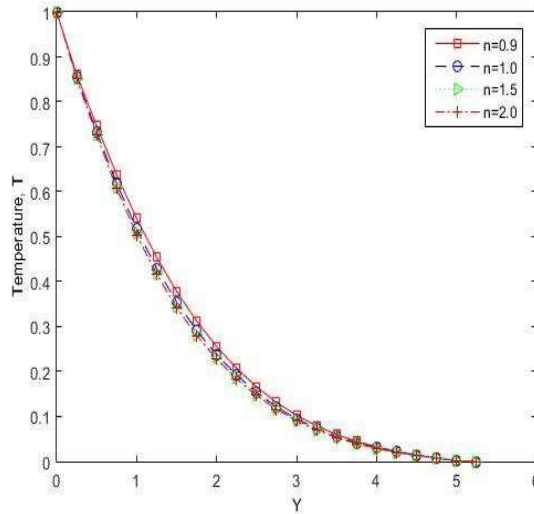


Figure 20: Temperature profile for various values of power law index

Table 1: The Values of Skin Friction, Nusselt and Sherwood numbers for at $Pr = 1.0$, $Sc = 0.62$, $N = 3.0$, $M = 1.0$, $Gr = 15.0$, $Gm = 15.0$, $Re = 1.0$, $Ec = 0.001$, $n = 1.5$

Parameters	Cf	Nu	Sh
M = 1.00	3.1232	0.2894	0.5471
M = 5.00	4.5742	0.2849	0.5026
M = 10.00	6.0477	0.2815	0.4693
N = 0.50	5.7094	0.3441	0.5405
N = 0.90	5.9131	0.4674	0.5294
N = 3.00	6.0046	0.7734	0.5121
Ec = 0.01	6.0551	0.2844	0.5473
Ec = 0.05	6.0612	0.2623	0.5480
Ec = 0.10	6.1201	0.2344	0.5489

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Gr = 3.00	3.0750	0.2767	0.4610
Gr = 5.00	2.6065	0.2779	0.4687
Gr = 7.00	2.1340	0.2791	0.4773
Sc = 0.10	5.9478	0.3045	0.2512
Sc = 0.50	6.1075	0.2903	0.5079
Sc = 0.80	6.4755	0.2882	0.6102
n = 0.90	6.6508	0.2771	0.4687
n = 1.00	5.3415	0.2802	0.4919
n = 1.50	4.0121	0.2837	0.5189
Pr = 17.0	2.5732	0.3055	0.6501
Pr = 21.0	2.8764	0.3566	0.6650
Pr = 30.0	2.8816	0.3883	0.6842

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