

EFFICIENT NEW RATIO ESTIMATORS OF POPULATION MEAN USING PRODUCT OF SAMPLE SIZE AND CONVENTIONAL LOCATION PARAMETERS

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ABSTRACT: This paper modified nine ratio-type estimators of population mean of the study variable from auxiliary variable suggested by Abid et al. [1] using product of sample size and conventional location parameters. The results obtained on the Mean Square Errors using the same data set used by Abid et al. shows that the newly modified ratio-type estimators perform better vis-à-vis the Abid et al. estimators, hence, recommended for usage in Sampling.

KEYWORDS: Population, Estimator, Sample size, Parameter, Efficiency

1. INTRODUCTION

One of the major objectives in sampling theory is to estimate the parameter of the study variable with more precision, to achieve this, it has been observed that using auxiliary variable help improving the precision of the estimate of a parameter (Abid et al. [1]). The ratio, regression, and product estimators are used for estimation of population mean when such auxiliary information is available [1]. The ratio-type of estimation is used to obtain an improved estimator of population mean when auxiliary variable is highly positively correlated with variable under study. Various authors like Cochran [3] suggested a classical ratio-type estimator for the estimation of finite population mean using an auxiliary variable under simple random sampling scheme (Subzar et al. [15], Mukhopadhyay [8]). Product type estimator was proposed by Murthy [9], the difference type-ratio estimator proposed by Rao [10] while Upadhyaya and Singh [19]; modified ratio-type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary information. Also, estimators have been proposed by Singh and Tailor [16]; Singh and Chaudhary [12]; Kadilar and Cingi [6][7]; Yan and Tian [22]; Yadav et al. [21]; Gupta and Yadav [4]; Jerajuddin and Kishun [5]; Sisodia and Dwivedi [11]; Singh and Chaudhary [12]; Singh et al. [17]; Subramani [18]; Subramani and Kumarapandiyam [13][14]; and Abid et al. [1][2]. In this paper, we have modified nine

ratio-type estimator suggested by Abid et al. [1] to improve on population mean estimation with higher efficiencies.

Let the population under consideration consists of N distinct and identifiable units and let (x_i, y_i) , $i = 1, 2, \dots, n$ be a two variable sample of size n taken from bivariate variable (X, Y) through simple random sampling without replacement sampling scheme. Let $\bar{X}$ and $\bar{Y}$ be the population mean of the auxiliary and study variable respectively, and let $\bar{x}$ and $\bar{y}$ be respective sample mean. Let ρ be the correlation coefficient between the variable X and Y (Cochran [3]).

2. EXISTING RATIO-TYPE ESTIMATORS IN LITERATURE

Abid et al. [1] suggested some modified ratio estimators for the population mean $\bar{Y}$ in simple random sampling using parameters of auxiliary information such as Tri-mean, Mid-range and Hodges-Lehman which are given as:

$$\hat{\bar{Y}}_1 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + TM)}(\bar{X} + TM)$$

$$\hat{\bar{Y}}_2 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + TM)}(\bar{X}C_x + TM)$$

$$\hat{\bar{Y}}_3 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + TM)}(\bar{X}\rho + TM)$$

$$\hat{\bar{Y}}_4 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + MR)}(\bar{X} + MR)$$

$$\hat{\bar{Y}}_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + MR)}(\bar{X}C_x + MR)$$

$$\hat{\bar{Y}}_6 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + MR)}(\bar{X}\rho + MR)$$

$$\hat{Y}_7 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + HL)}(\bar{X} + HL)$$

$$\hat{Y}_8 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + HL)}(\bar{X}C_x + HL)$$

$$\hat{Y}_9 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + HL)}(\bar{X}\rho + HL)$$

Abid et al. [1] estimators biases, constants and the mean square error (MSE) are given below respectively as:

$$B(\hat{Y}_1) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_1^2, R_1 = \frac{\bar{Y}}{(\bar{X} + TM)},$$

$$MSE(\hat{Y}_1) = \frac{1-f}{n} [R_1^2 S_x^2 + S_y^2 (1 - \rho^2)]$$

$$B(\hat{Y}_2) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_2^2, R_2 = \frac{\bar{Y}C_x}{(\bar{X}C_x + TM)},$$

$$MSE(\hat{Y}_2) = \frac{1-f}{n} [R_2^2 S_x^2 + S_y^2 (1 - \rho^2)]$$

$$B(\hat{Y}_3) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_3^2, R_3 = \frac{\bar{Y}\rho}{(\bar{X}\rho + TM)},$$

$$MSE(\hat{Y}_3) = \frac{1-f}{n} [R_3^2 S_x^2 + S_y^2 (1 - \rho^2)]$$

$$B(\hat{Y}_4) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_4^2, R_4 = \frac{\bar{Y}}{(\bar{X} + MR)},$$

$$MSE(\hat{Y}_4) = \frac{1-f}{n} [R_4^2 S_x^2 + S_y^2 (1 - \rho^2)]$$

$$B(\hat{Y}_5) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_5^2, R_5 = \frac{\bar{Y}C_x}{(\bar{X}C_x + MR)},$$

$$MSE(\hat{Y}_5) = \frac{1-f}{n} [R_5^2 S_x^2 + S_y^2 (1 - \rho^2)]$$

$$B(\hat{Y}_6) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_6^2, R_6 = \frac{\bar{Y}\rho}{(\bar{X}\rho + MR)},$$

$$MSE(\hat{Y}_6) = \frac{1-f}{n} [R_6^2 S_x^2 + S_y^2 (1 - \rho^2)]$$

$$B(\hat{Y}_7) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_7^2, R_7 = \frac{\bar{Y}}{(\bar{X} + HL)},$$

$$MSE(\hat{Y}_7) = \frac{1-f}{n} [R_7^2 S_x^2 + S_y^2 (1 - \rho^2)]$$

$$B(\hat{Y}_8) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_8^2, R_8 = \frac{\bar{Y}C_x}{(\bar{X}C_x + HL)},$$

$$MSE(\hat{Y}_8) = \frac{1-f}{n} [R_8^2 S_x^2 + S_y^2 (1 - \rho^2)]$$

$$B(\hat{Y}_9) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_9^2, R_9 = \frac{\bar{Y}\rho}{(\bar{X}\rho + HL)},$$

$$MSE(\hat{Y}_9) = \frac{1-f}{n} [R_9^2 S_x^2 + S_y^2 (1 - \rho^2)]$$

3. NEWLY MODIFIED RATIO TYPE ESTIMATORS

Motivated by Abid et al. [1], we have used a product of sample size and location parameter of auxiliary information given by Abid et al. as:

$$\kappa_{p1} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + TM * n)}(\bar{X} + TM * n) \quad (1)$$

$$\kappa_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + TM * n)}(\bar{X}C_x + TM * n) \quad (2)$$

$$\kappa_{p3} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + TM * n)}(\bar{X}\rho + TM * n) \quad (3)$$

$$\kappa_{p4} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + MR * n)}(\bar{X} + MR * n) \quad (4)$$

$$\kappa_{p5} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + MR * n)}(\bar{X}C_x + MR * n) \quad (5)$$

$$\kappa_{p6} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + MR * n)}(\bar{X}\rho + MR * n) \quad (6)$$

$$\kappa_{p7} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + HL * n)}(\bar{X} + HL * n) \quad (7)$$

$$\kappa_{p8} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + HL * n)}(\bar{X}C_x + HL * n) \quad (8)$$

$$\kappa_{p9} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + HL * n)}(\bar{X}\rho + HL * n) \quad (9)$$

We have used the following approximations in order to study the large sample approximations so also used by Abid et al. [1] as:

$$\bar{y} = \bar{Y}(1 + e_0), \bar{x} = \bar{X}(1 + e_1), E(e_i) = 0, (i = 0, 1),$$

$$E(e_0^2) = \frac{1-f}{n} C_y^2$$

$$E(e_1^2) = \frac{1-f}{n} C_x^2, \quad E(e_0 e_1) = \frac{1-f}{n} \rho C_x C_y, \quad f = \frac{n}{N},$$

$$C_y^2 = \frac{S_y^2}{\bar{Y}^2}, \quad C_x^2 = \frac{S_x^2}{\bar{X}^2}$$

Using above and expand (Wolter [Wol85]) up to the first order of approximations, the followings are biases and the mean squared errors of proposed estimators, given as

$$B(\kappa_{pi}) = \frac{1-f}{n} \frac{S_x^2}{\bar{Y}} R_{pi}^2, (i=1,2,\dots,9.) \quad (10)$$

$$MSE(\kappa_{pi}) = \frac{1-f}{n} [R_{pi}^2 S_x^2 + S_y^2 (1-\rho^2)], (i=1,2,\dots,9.) \quad (11)$$

where,

$$R_{p1} = \frac{\bar{Y}}{(\bar{X} + TM * n)}, \quad R_{p2} = \frac{\bar{Y} C_x}{(\bar{X} C_x + TM * n)},$$

$$R_{p3} = \frac{\bar{Y} \rho}{(\bar{X} \rho + TM * n)}, \quad R_{p4} = \frac{\bar{Y}}{(\bar{X} + MR * n)},$$

$$R_{p5} = \frac{\bar{Y} C_x}{(\bar{X} C_x + MR * n)}, \quad R_{p6} = \frac{\bar{Y} \rho}{(\bar{X} \rho + MR * n)},$$

$$R_{p7} = \frac{\bar{Y}}{(\bar{X} + HL * n)}, \quad R_{p8} = \frac{\bar{Y} C_x}{(\bar{X} C_x + HL * n)},$$

$$R_{p9} = \frac{\bar{Y} \rho}{(\bar{X} \rho + HL * n)}.$$

4. EFFICIENCY COMPARISON

From equation (11), the MSE of the proposed estimators, κ_{pi} ($i=1,2,\dots,9$) and Abid et al. [1] estimators; the estimators κ_{pi} ($i=1,2,\dots,9$) is better than sample mean $\hat{\bar{Y}}_i$ ($i=1,2,\dots,9$) (Abid et al. [1]) whenever:

$$MSE(\kappa_{pi}) \leq MSE(\hat{\bar{Y}}_i),$$

$$\frac{1-f}{n} [R_{pi}^2 S_x^2 + S_y^2 (1-\rho^2)] \leq \frac{1-f}{n} [R_i^2 S_x^2 + S_y^2 (1-\rho^2)]$$

$$[R_{pi}^2 S_x^2 - R_i^2 S_x^2] \leq 0$$

$$R_{pi} \leq \pm R_i, (i=1,2,\dots,9)$$

5. EMPIRICAL STUDY

Here, we also used Murthy [9] data set given below as used by Abid et al. [1] for better judgment on newly proposed modified ratio-type estimators:

$$N = 80, n = 20, \bar{Y} = 5182.637, \bar{X} = 1126.463, \rho = 0.941, S_y = 1835.659, C_y = 0.354$$

$$S_x = 845.610, C_x = 0.751, \beta_1 = 1.050, \beta_2 = -0.063,$$

$$M_d = 757.500, QD = 588.125$$

$$HL = 1040.5, TM = 931.562, MR = 1795.5$$

6. RESULTS

Table 1: Mean Squared Errors obtained on the newly modified Ratio-Type Estimators and other existing ones considered.

Estimator	Constant	Bias	Mean Squared Error
$\hat{\bar{Y}}_1$	2.518	32.81	184446.20
$\hat{\bar{Y}}_2$	2.189	24.79	142903.20
$\hat{\bar{Y}}_3$	2.449	31.03	175238.70
$\hat{\bar{Y}}_4$	1.774	16.23	98755.61
$\hat{\bar{Y}}_5$	1.473	11.23	72582.52
$\hat{\bar{Y}}_6$	1.708	15.10	92644.60
$\hat{\bar{Y}}_7$	2.392	29.59	167778.6
$\hat{\bar{Y}}_8$	2.063	22.01	128487.60
$\hat{\bar{Y}}_9$	2.322	27.90	158990.70
κ_{p1}	0.26231	0.35560	16315.83
κ_{p2}	0.19983	0.20661	15541.59
κ_{p3}	0.24767	0.31736	16115.58
κ_{p4}	0.13993	0.10131	14995.88
κ_{p5}	0.10589	0.05802	14771.48
κ_{p6}	0.13191	0.09003	14937.42
κ_{p7}	0.23626	0.28879	15967.53
κ_{p8}	0.17973	0.16713	15336.97
κ_{p9}	0.22299	0.25728	15804.19

From Table 1, the results reveal that the newly modified ratio-type estimators are having lesser biases and mean squared errors as compared to Abid et al. [1] estimators. Hence, these newly modified ratio-type estimators are more efficient than their respective Abid et al. [1] estimators.

CONCLUSION

From the empirical study, it has been found and established that these newly modified ratio-type estimators are more efficient than Abid et al. [1]

existing ratio-type estimators. They are respectively more efficient but the newly modified ratio-type estimator, K_{p_5} , is the best since it has the smallest mean squared error, thus, it is recommended for use in the future for practical applications.

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