

**MODIFICATION TO SELECTION PROCEDURES IN PROBABILITY  
PROPORTIONAL TO SIZE SAMPLING WITHOUT REPLACEMENT**

**BY**

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## CERTIFICATION

This is to certify that this doctoral thesis was carried out by DAWODU Omotola Omotayo (04/55EG056) in the Department of Statistics, in partial fulfillment of the requirements for the award of Doctor of Philosophy in Statistics of the University of Ilorin, Ilorin, Kwara State, Nigeria and has not been presented elsewhere for the award of a degree, or any other purpose.

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## **DEDICATION**

To all who have added to my knowledge in Statistics.

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## ABSTRACT

Probability proportional to size sampling (or unequal probability sampling) is a probability sampling in which every unit in the population has different probabilities of being selected in the sample. It can be with or without replacement. Most sample selection procedures in probability proportional to size sampling without replacement (PPSSWOR) focus on selection of sample size two which is the challenge of this study. A procedure involves selection of the first unit with probability proportional to  $\frac{P_i}{2 - p_i}$  without replacement, then the second unit is selected with probability  $\frac{P_i}{1 - p_j}$ , where  $P_i$  and  $P_j$  are the non – zero known probability values assigned to each unit of the population at any specific draw for  $i, j = 1, 2, \dots, N$ , and  $N$  is the total number of units. Another procedure involves selection of the first unit with probability proportional to  $\frac{p_i^\alpha (1 - p_i^\beta)}{(1 - 2p_i^\beta)}$  without replacement, and then a random sample of size  $n-1$  from the remaining  $N-1$  units is selected, where  $\alpha$  and  $\beta$  are integer constants. This study aimed at modifying the sample selection procedures in PPSSWOR described above to produce more efficient ones that can be used for any sample size. The specific objectives of this study were to: (i) obtain two modified sample selection procedures (I and II) in PPSSWOR for any sample size; (ii) derive the probability of inclusion of single unit  $\pi_i$  and the joint probability of inclusion of units  $\pi_{ij}$  in the sample for procedures I and II; and (iii) determine the most efficient sample selection procedure with minimum variance using both real life and simulated data. Modified Procedure I involves selection of the first unit with probability proportional to  $\frac{P_i}{2 - P_i}$  without replacement, and then, a random sample of size  $n - 1$  from the remaining  $N - 1$  units will be selected while the modified procedure II involves selection of the first unit with probability proportional to  $\frac{P_i^\alpha}{2 - P_i^\beta}$  without replacement, and then a random sample of size  $n - 1$  from the remaining  $N - 1$

units is selected. The optimum values of  $\alpha$  and  $\beta$  would be determined through empirical studies.

The findings were:

- (i). two modified sample selection procedures (I and II) in PPSSWOR were obtained. Both procedures can be used to select sample of any size for specified values of two constants  $(\alpha, \beta)$  ;
- (ii). the probabilities of inclusion of single unit,  $\pi_i$ , and the joint probability of inclusion of units  $\pi_{ij}$  in the sample in PPSSWOR for these modified procedures were derived; and
- (iii). using both real life and simulated data, modified procedure II was found to have minimum variance, hence, most efficient sample selection procedure for any sample size.

This study concluded that when  $-5 \leq \beta \leq -1$  and  $-5 \leq \alpha \leq +5$ , modified procedure II is better than the two existing procedures. It is therefore recommended for use in probability proportional to size sampling without replacement.

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## ABBREVIATIONS AND NOTATIONS

The following are the list of abbreviations and notations used in this work. Other ones not listed here, if any, are explained in the text.

|                |                                                                                                                       |
|----------------|-----------------------------------------------------------------------------------------------------------------------|
| PPS            | Probability proportional to Size                                                                                      |
| PPSS           | Probability Proportional to Size Sampling                                                                             |
| PPSSWR         | Probability Proportional to Size Sampling with Replacement                                                            |
| PPSSWOR        | Probability Proportional to Size Sampling without Replacement                                                         |
| P I            | Procedure I                                                                                                           |
| P II           | Procedure II                                                                                                          |
| BSP            | Basit and Shahbaz Procedure                                                                                           |
| HT             | Horvitz & Thompson                                                                                                    |
| SYG            | Sen, Yates, and Grundy                                                                                                |
| SDRS           | Symmetrized Des Raj Strategy                                                                                          |
| RHC            | Rao, Hartley, and Cochran                                                                                             |
| DHS            | Demographic and Health Survey                                                                                         |
| $\hat{Y}_{HT}$ | Estimator of the total given by Horvitz & Thompson                                                                    |
| $Y_i$          | $i$ th variable of the population unit of the estimated variable of interest, $y_i$ is the corresponding sample value |
| Y              | Actual total of the auxiliary variable $= \sum_{i=1}^N Y_i$                                                           |
| $X_i$          | $i$ th variable of the auxiliary variable, $x_i$ is the corresponding auxiliary variable                              |
| X              | Actual total of the auxiliary variable $= \sum_{i=1}^N X_i$                                                           |
| $P_i$          | Probability of the selection of the $i$ th population unit in the sample                                              |
| $\pi_i$        | Probability of inclusion of $i$ th population unit in the sample                                                      |
| $\pi_{ij}$     | Joint Probability of inclusion of $i$ th and $j$ th population units in the sample                                    |

## **CHAPTER ONE**

### **1.0 Introduction**

Everitt (2006) in his Cambridge Dictionary of Statistics, defined sampling as “the process of selecting some parts of a population to observe, so as to estimate something of interest about the whole population”, while he defines sample survey as the study which deals with the collection of planned information from a sample of elements about their history, knowledge, attitudes or behaviour in order to estimate particular population characteristics. Sampling is the process of selecting units from population of interest so that by studying the sample, inference could be made on the population from which they are selected. In carrying out a survey, various sampling designs could be used, e.g., simple random sampling, stratified sampling, systematic sampling, cluster sampling, two – stage sampling, multi – stage sampling, and so on.

### **1.1 Definition of Some Related Terms**

#### **1.1.1 Non – Probability Sampling**

It refers to a method of obtaining samples from the population in a non – random manner. It makes use of samples that are readily available and convenient to use. Examples of non – probability sampling are; quota sampling, judgment sampling, purposive sampling, convenience sample, and so on. It is otherwise known as non – random sampling.

### 1.1.2 Probability Sampling

Cochran (1977) described probability sampling as one that meets the following:

- It is possible to select a set of different samples  $S_1, S_2, \dots, S_n$ . It is also possible to say specifically what sampling units belong to  $S_1, S_2, \dots, S_n$ .
- There is a known probability value, called probability of selection,  $P_i$  assigned to each possible sample.
- All  $S_i$  are selected by random process.
- The method of computation of the estimate from the sample must be clearly stated and must lead to a unique estimate for any specific sample.

Probability sampling is usually classified as:

- Equal Probability Sampling, and
- Unequal Probability Sampling or Probability Proportional to Size Sampling (PPSS).

Attention in this research work is on PPSS.

### 1.1.3 Equal Probability Sampling

This refers to a probability sampling in which every unit in the population has equal probability of being selected into the sample. A very good example is the simple random sampling.

#### **1.1.4 Probability Proportional to Size Sampling**

This refers to a probability sampling in which every unit in the population has different probabilities of being selected in the sample.

PPSS can be categorized into two namely;

- Probability Proportional to Size Sampling with Replacement (PPSSWR), and
- Probability Proportional to Size Sampling without Replacement (PPSSWOR).

#### **1.1.5 Probability Proportional to Size Sampling with Replacement**

This has to do with situations where units have different chances of being selected in the sample and are being returned back into the population after each selection is made. This means that a unit can be selected more than once.

#### **1.1.6 Probability Proportional to Size Sampling without Replacement**

This has to do with situations where units have different chances of being selected in the sample, and are not return back into the population after each selection is made. This means that a unit can only be selected once.

This research deals with PPSSWOR.

#### **1.1.7 Probability of Selection**

The probability of selection is defined as the non – zero known probability value assigned to each and every unit of the population at any specific draw. It is usually represented by  $P_i$ .

### 1.1.8 Probability of Inclusion

The probability of inclusion of a population unit in the sample is defined as “the total probability assigned to a population unit, for being included in the sample in all draws. Every element of the population is assigned inclusion probability denoted by  $\pi_i$ , which is the sum of the sample probability for all samples that contain element  $i$ .

### 1.1.9 Design Weights

Design weight on the other hand is defined as the inverse of the inclusion probability, denoted by  $w_i = \frac{1}{\pi_i}$ . This shows that elements that are assigned low inclusion probabilities receive high design weight, and elements that are assigned high inclusion probabilities receive low design weights. This means that elements that are sampled with unequal inclusion probability, an element that is very unlikely to be included in the sample (i.e. has a small value of  $\pi_i$ ) is weighted up, which makes it more important than an element that has a very high chance of being selected.

Therefore, inclusion probabilities and hence design weights depend on the sample design, the sample size, and the size of the population.

There are designs that assign non – constant inclusion probabilities to the elements. Most of the complex sample designs such as cluster sampling and multi – stage sampling belong to this class.

## **1.2 Statement of Problem**

Hansen & Hurwitz (1943) worked on unequal probability sampling with replacement, and stressed the need to work on unequal probability sampling without replacement. Horvitz & Thompson (1952) were the first to work on unequal probability sampling without replacement in accordance with the unified approach.

Hansen & Hurwitz (1943) and Horvitz & Thompson (1952) all worked on PPSSWOR, selecting samples in two stages. The appropriate procedure to be followed while selecting samples under PPSSWOR is of great concern. There are a number of literature on the various sampling schemes or procedures that can be employed. Most of these procedures are only appropriate for selecting samples of size two. When the need to draw samples of size greater than two arises, there may be the problem of how to calculate the probabilities of inclusion of units in the sample. This research work will address this.

## **1.3 Justification of the Study**

There is the need for this research work because often time, there are reasons to select samples of any size from a population without replacement of units already drawn. To have accurate measurement of population total from finite populations, a good sampling scheme is required and an improved strategy is highly solicited to bring this to full accomplishment.

#### **1.4 Aim and Objectives of the Study**

This research work aimed at modifying the Basit & Shahbaz (2007) and Alodat (2009) sample selection procedures in PPSSWOR to produce more efficient ones.

The underlying objectives are to:

- (i) obtain two modified sample selection procedures (P I and P II) in PPSSWOR for any sample size;
- (ii) derive the probability of inclusion of single unit  $\pi_i$  and the joint probability of inclusion of units  $\pi_{ij}$  in the sample for procedures I and II; and
- (iii) determine the most efficient sample selection procedure with minimum variance using both real life and simulated data.

## **CHAPTER TWO**

### **2.0 Literature Review**

#### **2.1 Probability proportional to size sampling with and without Replacement from finite population**

Hansen & Hurwitz (1943) worked on a stratified two-stage sampling design, for estimating population mean or total, in which the primary units within a given stratum are selected with probability proportional to their sizes and with replacement, while dealing with sampling with unequal probability from finite population. Then, simple random sampling is used to select second stage unit within selected primary units so that the design could be made to be self – weighting in each stratum or in all strata.

While determining the optimum probabilities of selection in the above two-stage design, for a given stratum where primary units are selected with varying probabilities and with replacement, Hansen & Hurwitz (1943) mentioned the need for developing the theory further when primary units are selected with varying probability and without replacement.

Madow (1949) was the first to introduce the concept of PPSSWOR, while Narain (1951) and Horvitz and Thompson (1952) worked independently to develop the theory further.

Narain (1951) gotten the variance of the two-stage sampling units in each of the selected primary units and also studied the conditions to make the estimator unbiased. Following the unified approach, Horvitz and Thompson (1952) formulated the problem and they defined three classes of linear estimators for the population total,

$$T = \sum_{i=1}^N Y_i \quad (1)$$

Rao (2004) described the three classes of linear estimators for the population above as:

- (i)  $\hat{T}_1 = \sum_{i=1}^n \alpha_i y_i$ , where  $\alpha_i$  is a constant to be used as a weight for the unit selected in the  $i^{th}$  draw,  $i = 1, 2, \dots, n$ .
- (ii)  $\hat{T}_2 = \sum_{i=1}^n \beta_i y_i$ , where  $\beta_i$  is a constant to be used as weight for the  $i^{th}$  unit whenever it is selected for the sample, and
- (iii)  $\hat{T}_3 = \gamma_{S_n(\sum y_i) s_n}$  where  $\gamma_{S_n}$  is a constant to be used as weight for the  $i^{th}$  unit whenever the  $S_n^{th}$  sample is selected.

The Horvitz and Thompson estimator is the best in the  $T_2$  – subclass. They considered the  $T_2$  – class of estimators and showed that the only unbiased linear estimator possible in this sub-class is the one with  $\beta_i = \frac{1}{\pi_i}$  where  $\pi_i$  is the probability of inclusion of the  $i^{th}$  unit in the sample.

## 2.2 Horvitz and Thompson Estimator

Horvitz & Thompson (1952) worked on the theoretical framework of unequal probability sampling without replacement. The population total,  $Y = \sum_{i=1}^N Y_i$ .

The estimator of the total given by Horvitz & Thompson (1952) is

$$\hat{Y}_{HT} = \sum_{i=1}^N \frac{Y_i}{\pi_i} \quad (2)$$

where  $\pi_i$  is the probability of inclusion of  $i$ th unit in the sample with corresponding variance given as:

$$V(\hat{Y}_{HT}) = \sum_{i=1}^N \left( \frac{1-\pi_i}{\pi_i} \right) Y_i^2 + \sum_{j=1} \sum_{i \neq j} \left( \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} \right) Y_i Y_j \quad (3)$$

with an unbiased variance estimator

$$\hat{V}(\hat{Y}_{HT}) = \sum_{i=1}^n \left( \frac{1-\pi_i}{\pi_i^2} \right) y_i^2 + \sum_{i=1}^n \sum_{i \neq j} \left( \frac{\pi_{ij} - \pi_i \pi_j}{\pi_i \pi_j} \right) \left( \frac{y_i y_j}{\pi_i \pi_j} \right) \quad (4)$$

## 2.3 Problems Arising From Horvitz and Thompson Estimator

The main problem of the Horvitz and Thompson estimator is that its variance usually gives negative values. In addition to this, there is the difficulty in the calculation of probabilities of inclusion of the population units in singles or in pairs, as the sample size increases.

## 2.4 Sen – Yates – Grundy Estimator

Sen (1953), and Yates & Grundy (1953) worked independently on the estimator given by Horvitz and Thompson.

The expression for variance of Horvitz – Thompson given independently by Sen (1953) and Yates & Grundy (1953) is given as:

$$V_{SYG}(\hat{Y}_{HT}) = \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N (\pi_i \pi_j - \pi_{ij}) \left( \frac{y_i}{\pi_i} - \frac{y_j}{\pi_j} \right)^2 \quad (5)$$

with an unbiased variance estimator

$$\hat{V}_{SYG}(\hat{Y}_{HT}) = \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n \left( \frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}} \right) \left( \frac{y_i}{\pi_i} - \frac{y_j}{\pi_j} \right)^2 \quad (6)$$

and showed that it is always positive under some sampling schemes.

Durbin (1953) showed that Sen – Yates – Grundy estimator of the variance also assumes negative values, but positive under some sampling schemes. Okafor (2002) stated that equation (6) above is unbiased.

## 2.5 Rao, Hartley and Cochran Estimator

In an attempt to solve the problems arising from Horvitz and Thompson estimator, Rao, Hartley and Cochran (RHC) (1962), suggested a slightly modified estimator based on different sampling scheme. Therefore, they proposed the selection procedure stated below, as described by Rueda *et al.* (2009):

Suppose a population consists of  $N$  units and it is desired to draw a sample of  $n$  units. We need to divide the  $N$  units randomly into  $n$  groups.

The first random group is obtained as follows:

- From  $N$  units, select  $N_1$  units by using simple random sampling without replacement,
- The second random group is obtained as follows: Out of the remaining  $(N - N_1)$  units, select  $N_2$  units by using simple random sampling without replacement,

This process is repeated in this manner until we have the desired  $n$  random groups, such that  $\sum_{i=1}^n N_i = N$ . The allocation of units to different groups is done randomly and one unit is selected from each of the  $n$  random groups with probability proportional to size. By doing this, a sample of size  $n$  is obtained. Let  $P_1, P_2, \dots, P_N$  be the probabilities associated with the  $N$  units in the population, and  $\sum_{i=1}^N P_i = 1$ . Suppose further that  $P_{ij}$  denotes the probability corresponding to the  $j$ th unit in the  $i$ th group  $G_i$ ;  $i = 1, 2, \dots, n$ . Let  $S$  be a sample of size  $n$  selected using the RHC scheme. The unbiased estimator of population total  $Y$  is given by

$$\hat{Y}_{RHC} = \sum_{i \in S} \frac{y_i}{P_i^*} \quad (7)$$

where  $P_i^* = \frac{P_i}{\tau_i}$

The variance of the estimator is given by:

$$V(\hat{Y}_{RHC}) = a \left[ \sum_{j=1}^N \frac{Y_j^2}{P_j} - Y^2 \right] \quad (8)$$

where  $a = \frac{\sum_{i \in S} N_i^2 - N}{N(N-1)}$  with an unbiased estimator of the variance  $V(\hat{Y}_{RHC})$  given by

$$\hat{V}(\hat{Y}_{RHC}) = \frac{(\sum_{i \in S} N_i^2 - N)}{(N^2 - \sum_{i \in S} N_i^2)} \left[ \sum \frac{y_i^2}{P_i^2/T_i} - Y_{RHC}^2 \right] \quad (9)$$

The estimator suggested by RHC (1962) usually have positive variance. Note that the RHC estimator makes use of the probability of selection,  $P_i$ , only rather than probability of inclusion,  $\pi_i$ .

## 2.6 Description of Some Sample Selection Procedures

Some sample selection procedures in probability proportional to size sampling are discussed below.

### 2.6.1 Sen – Midzuno Procedure

Horvitz & Thompson (1952) adopted this scheme and is applicable for sample of any size. It is as follows:

- the first unit is selected with probability  $P_i$ ,
- select a sample of size  $n - 1$  from remaining units with equal probability and without replacement.

The probabilities of inclusion of units in the sample are:

$$\pi_i = P_i + \frac{n-1}{N-1} (1 - P_i) \quad (10)$$

with  $\sum_{i=1}^N P_i = 1$

$$\pi_{ij} = \frac{n-1}{N-1} \left[ \frac{N-n}{N-2} (P_i + P_j) + \frac{n-2}{N-2} \right] \quad (11)$$

### 2.6.2 Yates – Grundy – Draw – By – Draw Procedure

Yates & Grundy (1953) developed the procedure stated as follows:

- select the first unit using probability proportional to size, and
- second unit is selected using probability proportional to size of remaining units.

The probabilities of inclusion of units in the sample are:

$$\pi_i = P_i \left[ 1 + \sum_{j=1}^N \frac{P_j}{1-P_j} - \frac{P_i}{1-P_i} \right] \quad (12)$$

$$\pi_{ij} = P_i P_j \left[ \frac{1}{1-P_i} + \frac{1}{1-P_j} \right] \quad (13)$$

### 2.6.3 Brewer Selection Procedure

Brewer (1963) developed this selection procedure for the use with PPSSWOR, stated as follows:

- first unit is selected using probability proportional to  $\frac{P_i(1-P_i)}{(1-2P_i)}$ , and
- selection of the second unit is done using probability proportional to size of the remaining units.

The probabilities inclusion are of single unit  $\pi_i$ , and joint probabilities of inclusion of units  $\pi_{ij}$ , for this selection procedure are given as:

$$\pi_i = 2P_i \quad (14)$$

$$\pi_{ij} = \frac{2P_i P_j}{k} \left[ \frac{1}{1-2P_i} + \frac{1}{1-2P_j} \right] \quad (15)$$

where  $k = 1 + \sum_{j=1}^N \frac{P_i}{1-2P_i}$

#### 2.6.4 Durbins – Draw – by – Draw Procedure

Durbin (1967) uses the idea of revised probabilities to develop this procedure, and strictly without replacement. It is as follows:

- first unit is selected using probability proportional to size, and
- second unit is selected using probability proportional to  $P_j \left[ \frac{1}{1-2P_i} + \frac{1}{1-2P_j} \right]$ .

The probabilities of inclusion of single unit and joint probabilities of inclusion of units for this procedure are the same as Brewer (1963) selection procedure.

#### 2.6.5 Rao – Sampford Selection Procedure

This selection procedure was developed independently by Rao (1963b) and Sampford (1967). It is as stated:

- select one element with a revised probability  $q_i$  and with replacement,
- select one unit with probability proportional to size, and
- repeat above two steps if same unit is selected twice.

This procedure also produces same probability of inclusion and joint probability of inclusion as the procedures of Brewer (1963) and Durbin (1967).

### 2.6.6 Deshpande, Prabu – Ajgonkar Selection Procedure

Deshpande, Prabu – Ajgonkar (1982) developed the procedure stated below which makes use of sampling with replacement at successive draws. It is stated as:

- the first unit is selected with PPSSWR, and
- select second unit using probability  $\frac{P_i(1-P_i)}{A(1-2P_i)}$ , where  $A$  is a normalizing constant,
- if same unit is selected twice, then select one more element, from the remaining population units, with probability proportional to size.

The probabilities of inclusion of single unit for this procedure is same as for Brewer (1963). The joint probability of inclusion is given as:

$$\pi_{ij} = \frac{P_i P_j}{A} \left[ \frac{1}{1-2P_i} + \frac{1}{1-2P_j} \right] \quad (16)$$

### 2.6.7 Durbin Selection Procedure

This selection procedure was developed by Durbin (1953) and is stated as:

- select two units with probabilities  $P_i$  and with replacement, and
- repeat first step if same elements are selected until two distinct elements turn up.

The inclusion probability of single unit, and that of the joint are as follows:

$$\pi_i = \frac{2P_i(1-P_i)}{1 - \sum_{i=1}^N P_i^2} \quad (17)$$

$$\pi_{ij} = \frac{2P_iP_j}{1 - \sum_{i=1}^N P_i^2} \quad (18)$$

Durbin used the estimator stated below for estimation of population total under this selection procedure:

$$y_D = \frac{1}{2} \left[ \frac{y_i}{P_i} + \frac{y_j}{P_j} \right] \quad (19)$$

The above estimator is a biased estimator of population total, but showed that the bias is generally negligible as compared to mean square error.

### 2.6.8 Yates – Grundy Procedure

Yates & Grundy (1953) developed this procedure, and can be applied to a sample of any size. For selecting a sample of size 2, this procedure is:

- two units with probability proportional to size and with replacement are selected, and
- repeat first stage if same unit is selected twice.

### 2.6.9 Shahbaz Selection Procedures

Shahbaz (2003) proposed three selection procedures stated as follows:

#### Procedure I

- first unit is selected with probability proportional to  $\frac{P_i}{1-P_i}$ , and
- second unit is selected with probability proportional to size of the remaining units.

For this procedure,

$$\pi_i = \frac{P_i}{d} \left[ \frac{1}{1-P_i} + \sum_{j=1}^N \frac{P_j}{(1-P_j)(1-2P_j)} \right] \quad (20)$$

$$\text{where } d = \sum_{i=1}^N \frac{P_i}{1-2P_i}$$

$$\pi_{ij} = \frac{P_i P_j}{d} \left[ \frac{1}{(1-P_i)(1-2P_i)} + \frac{1}{(1-P_j)(1-2P_j)} \right] \quad (21)$$

### Procedure II

- first unit is selected with probability proportional to  $\frac{2P_i(1-P_i)}{1-4P_i}$ , and
- second unit is selected with probability proportional to size of the remaining units.

For this procedure,

$$\pi_i = \frac{2P_i}{b} \left[ 1 + \frac{1}{1-4P_i} + 2 \sum_{j=1}^N \frac{P_j}{1-4P_j} \right] \quad \text{for } P_i, P_j < \frac{1}{4} \quad (22)$$

$$\text{where } b = 1 + 3 \sum_{i=1}^N \frac{P_i}{1-4P_i}$$

$$\pi_{ij} = \frac{4P_i P_j}{b} \left[ \frac{1}{1-4P_i} + \frac{1}{1-4P_j} \right] \quad \text{for } P_i, P_j < \frac{1}{4} \quad (23)$$

### Procedure III

- first unit is selected with probability proportional to  $\frac{P_i(1-P_i)}{(1-2P_i)}$ , and
- second unit  $j$  is selected with probability  $P_j \left[ \frac{1}{1-2P_i} + \frac{1}{1-2P_j} \right]$  from the remaining units.

For this procedure,

$$\pi_i = \frac{P_i}{k} \left[ 4 \left( \frac{1-P_i}{1-2P_i} \right) + \frac{1}{k} \left\{ \sum_{j=1}^N \frac{P_j}{(1-2P_j)^2} - \frac{P_i}{(1-2P_i)^2} \right\} \right] \quad (24)$$

$$\text{where } k = 1 + \sum_{i=1}^N \frac{P_i}{1-2P_i}$$

$$\pi_{ij} = \frac{P_i P_j}{k^2} \left[ \frac{1}{1-2P_i} + \frac{1}{1-2P_j} \right] \left[ 2 + \frac{1}{1-2P_i} + \frac{1}{1-2P_j} \right] \quad (25)$$

#### 2.6.10 Basit & Shahbaz Procedure

Basit & Shahbaz (2007) proposed a procedure for obtaining sample size  $n$  in PPSSWOR, stated as:

- first unit is selected with probability proportional to  $\frac{P_i^\alpha (1-P_i^\beta)}{(1-2P_i^\beta)}$  and without replacement, and
- a random sample of size  $(n-1)$  is selected from the remaining  $(N-1)$  units.

For this procedure,

$$\pi_i = \frac{1}{d} \left[ \frac{P_i^\alpha (1-P_i^\beta)}{(1-2P_i^\beta)} \left( \frac{N-n}{N-1} \right) + \frac{(n-1)}{N-1} d \right] \quad (26)$$

$$\text{where } d = \sum_{i=1}^N \frac{P_i^\alpha (1-P_i^\beta)}{(1-2P_i^\beta)}$$

$$\pi_{ij} = \frac{(n-1)(N-n)}{d(N-1)(N-2)} \left[ \frac{P_i^\alpha (1-P_i^\beta)}{(1-2P_i^\beta)} + \frac{P_j^\alpha (1-P_j^\beta)}{(1-2P_j^\beta)} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)} \quad (27)$$

### 2.6.11 Alodat Selection Procedure

Alodat (2009) proposed a procedure for selecting sample of size 2, stated as:

- first unit is selected with probability proportional to  $\frac{P_i}{2-P_i}$ , without replacement, and
- second unit is selected with probability  $\frac{P_i}{1-P_j}$ .

For this procedure,

$$\pi_i = \frac{P_i}{B} \left[ \frac{1-2P_i}{(2-P_i)(1-P_i)} + \sum_{j=1}^N \frac{P_j}{(2-P_j)(1-P_j)} \right] \quad (28)$$

$$\text{where } B = \sum_{i=1}^N \frac{P_i}{2-P_i}$$

$$\pi_{ij} = \frac{P_i P_j}{B} \left[ \frac{1}{(2-P_i)(1-P_i)} + \frac{1}{(2-P_j)(1-P_j)} \right] \quad (29)$$

## 2.7 Chronological Development

### 2.7.1 Early Period up to 1949

Hansen and Hurvitz (1943) first worked on the theory of PPSS. They limited their work to PPSSWR, and stressed the need to develop the theory further. Madow (1949) also worked on the PPSS using order systematic sampling.

### 2.7.2 From 1950 to 1959

Narain (1951) developed a selection procedure and introduced the idea of working probability but was not able to provide arduous theory for unequal probability sampling without replacement. Das (1951) gave a special estimator to be used for PPSSWOR, with the limitation that it can produce negative value in

some cases. Horvitz and Thompson (1952) worked on unequal probability sampling without replacement but their variance estimator sometimes gives negative values. This has generated the attention of many survey statisticians. Yates and Grundy (1953) and Sen (1953) independently worked on the Horvitz and Thompson estimator and gave another variance estimator that usually produces positive results for most sampling schemes. Durbin (1953) proposed a selection procedure that requires two units to be selected with replacement, and he made use of a different estimator of the population total. Yates and Grundy (1953) developed a selection procedure to be selected with probability proportional to size and with replacement, for sample size two. The procedure is repeated if same units are selected twice. Das (1951) and Raj (1956) gave some ordered estimators, that is, estimators which take into account the order in which the units are selected. Murthy (1957) takes a look at some ordered estimators, and showed that corresponding to any biased or unbiased ordered estimator, there exist an unordered estimator which is more efficient than the former, and he explained the technique of improving the ordered estimators by unordered ones. Raj and Khamis (1957) made some remarks on equal probability sampling with replacement from finite population.

### **2.7.3 From 1960 to 1969**

Pathak (1961) wrote that sampling with different probabilities with replacement, until a specified number of distinct units is obtained, is equivalent in some cases to sampling with different probabilities without replacement. Rao (1961a) worked on the estimate of the variance in unequal probability sampling. Rao (1961b)

developed different sampling procedures involving unequal probability selection and he examined the percentage gain in efficiency of the p due to stratification. Rao, *et al.* (1962) gave a sampling scheme for selecting sample of any size under PPSSWOR. They provided formula for estimating the population total and variance estimator based on probability of selection  $P_i$  alone and did not make use of the formula for probability of inclusion  $\pi_i$  of units in the sample. Rao (1963b) and Sampford (1967) independently developed a selection procedure which deals with revised probability and with replacement. Their procedure produces same probability of inclusion and also joint probability of inclusion as that of Brewer (1963) and Durbin (1967). Rao (1963a) wrote on systems of unequal probability sampling without replacement. Hajek (1964) also reported the Yates and Grundy sample selection procedure. Tikkiwal (1965) gave detailed discussion on the  $T_2$  – class estimators for sampling with unequal probabilities with or without replacement due to Prabu – Ajgaonkar and Tikkiwal. His paper only dealt with sample selection procedures for size two. Durbin (1967) developed two methods; first to select two first – stage units per stratum without replacement, with probability proportional to size and known joint probability of inclusion, in such a way to allow variances to be estimated as if the units had been selected with replacement. Pathak (1967a) worked on Des Raj’s estimator and explained its simplicity in terms of computation. He further examined why Des Raj’s estimator has not really attracted much attention due to lack of information on its efficiency and non – availability of an explicit expression for the population variance  $V(\bar{t}_{k(d)})$ . He derived an expression for  $V(\bar{t}_{k(d)})$  in a closed form and compared

this to other well – known strategies of similar kind. Also, Pathak (1967b) also worked on the Rao – Blackwellization of Des Raj’s (1956) strategy which is called the symmetrized Des Raj strategy (SDRS). He showed that the SDRS has smaller sampling variance than Des Raj’s strategy. Prabhu – Ajgoankar (1967) derived an expression for the variance of  $T_1$  class.

#### **2.7.4 From 1970 to 1979**

Basu (1971) worked on unequal probability sampling without replacement using the idea of ratio estimator. Royall (1971) worked on linear regression model in sample survey design and focus on how to obtain better efficiencies. Deshpande and Prabhu – Ajgaonkar (1977) worked on multitrial sampling method in survey, where a set of units is drawn from the population by a predetermine procedure and that set is subjected to a criterion either to accept or reject it. Singh (1978) gave a selection procedure for sample size two. Choudhry (1979) proposed procedure for selecting a sample of any size with PPS, but with replacement. Chaudhuri and Arnad (1979) made comparison on the efficiency of some strategies under super population model. Chromy (1979) also wrote on a sample selection procedure which uses the idea of conditional probability for any unit to be selected into the sample.

#### **2.7.5 From 1980 to 1989**

Deshpande and Prabhu – Ajgonkar (1980) made comparison of simple random sampling both with and without replacement. They used the Horvitz and Thompson estimator for sampling with replacement. Deshpande and Prabhu –

Ajgoankar (1982) developed a selection procedure which uses sampling with replacement at successive draws. This procedure is for sample size two.

Chaudhuri (1981) and (1985) also gave some sample selection procedures under PPSSWOR. Stehman and Overton (1987) wrote that variance estimator given by Yates and Grundy, and also Sen has gained favour in the statistical literature, but they stressed that computation of the true pairwise inclusion probabilities is often impractical.

#### **2.7.6 From 1990 to 1999**

Beg and Hanif (1991) worked on PPSS both with and without replacements and they made necessary comparison. Tille (1996) gave detailed explanation that the search for a sampling design with varying probabilities was an open problem. He thoroughly explained nine properties that a good sampling design with unequal probabilities should have. Bedi and Rao (1996) contributed in the aspect of efficient PPS method of estimation with transformed auxiliary variate in a case where there is negative correlation between the auxiliary variable and the variable of interest.

#### **2.7.7 From 2000 to 2009**

Lê et al (2001) described the Durbin selection method that deals with the selection of primary sampling units with replacement, where variance estimates can be computed using simple methods that are functions of primary sampling units only. They also claimed that sampling with replacement is not always practical or efficient. Okafor (2002) stated that the variance estimator of the

Horvitz & Thompson estimator, as giving by Sen, Yates & Grundy was unbiased. Chaudhary and Sen (2002) stressed that when unequal probability sampling without replacement is used in single and multiple stage sample designs, the general linear estimator of the population estimate of interest may not assume normal distribution. Shahbaz (2003) worked on three different sample selection procedures in PPSSWOR for sample size two. Rao (2004) considered the Horvitz and Thompson estimator over the last five decades and reviewed some of its optimality properties. He stressed that Horvitz and Thompson estimator is of great importance and will continue to play an important role in sampling theory and applications. Asparouhov (2004) worked on weighting for unequal probability of selection in multilevel modeling. Asparouhov (2006) worked on multilevel models that are used for analyzing data from cluster sampling designs that often use unequal probability of selection at the cluster and individual levels. He further explained how sampling weight are assigned, and that ignoring sampling weights may lead to biased parameter estimates. Rabe – Hesketh and Skrongdal (2006) also worked on multilevel modeling of complex survey data involving multistage sampling, unequal sampling probabilities and stratification. Chantala and Suchindran (2006) considered five different software packages for analyzing both individuals and clusters with unequal probabilities. Rabe – Hesketh and Skrongdal (2007) considered both model and designed – based inferences in relation to multilevel modelling of complex survey data. They estimated finite population mean of the Horvitz and Thompson estimator in the design – based inference. Sahoo *et al.* (2007) proposed an inclusion probability proportional to size

sampling scheme for sample of two units. They were able to show that proposed scheme compares well with other popularized schemes in terms of efficiency as well as stability of the estimated variance. Özdemir and Gökpinar (2007) worked on generalized formula for probabilities of inclusion for selecting sample of any size under ranked set sampling and compared their results with results of simple random sampling. Christ *et al.* (2007) explained the appropriate software packages that could handle multilevel modeling. The software packages are; MLWin and HLM. Basit & Shahbaz (2007) worked on a sample selection procedure for selecting sample of size  $n$ . They introduced two constants  $\alpha$  and  $\beta$  in form of exponential, and then carried out empirical studies in order to get the best values for  $\alpha$  and  $\beta$ . Murphy (2008) worked on grouping ultimate sampling units into primary sampling units and suggested that the only feasible way to do this is to make use of geographic entities already defined for other purposes; such as states, municipalities, etc. Asparouhov and Muthen (2009) considered estimation of two – level models when the data is obtained from a multistage stratified sampling design and the sampling units at each sampling level are selected with unequal probabilities. Machlin *et al.* (2009) examined different PPS sample design options with application to medical expenditure panel survey. Rueda *et al.* (2009) worked on the RHC scheme in the presence of random non – response to it. They claimed that no one has paid attention to this aspect. They considered a situation when some of the respondents selected using the RHC scheme either fail to respond or are unavailable in a completely random way called missing completely at random. Chromy (2009) wrote on the generalization

of the Horvitz and Thompson estimator in the following ways; allowing units to be drawn more than once, using expectations instead of probabilities, explaining how the Horvitz – Thompson method can be applied to the with replacement designs of Hansen and Hurwitz, and the simplifications to the variance estimation problem and how they can be justified under probability proportional to size systematic and probability proportional to size sequential sampling designs. Alodat (2009) proposed a sample selection procedure to obtaining sample of size two and he stressed the need to extend his proposed method to sample of any size.

#### **2.7.8 From 2010 till date**

Beaumont *et al.* (2010) worked on the usefulness of conditional bias in the derivation of robust estimators in finite population sampling. Tille (2010) considered different sampling algorithm that could be used with the Horvitz and Thompson estimator. He also worked on unequal probability exponential design. Dawodu *et al.* (2011) compared three sample selection procedures in PPSSWOR; Alodat procedure, Sen – Mudzino procedure, and Yates – Grundy procedure. They realized that of the three procedures under consideration, Alodat procedure produced the lowest variance estimates. Nafiu *et al.* (2011) worked on equal and unequal probability sampling without replacement. They considered one – stage sampling and two – stage sampling and showed that two – stage sampling performed better under unequal probability sampling without replacement. Shahbaz *et al.* (2011) proposed a selection procedure for sample size two, in PPSS without replacement that can be used with Horvitz & Thompson estimator. They presented an open problem that the procedure can be extended to sample of

any size. Grafström *et al.* (2012) stressed that the already available methods used to draw unequal probability samples with fixed sample size, require the sum of the inclusion probabilities to be integer number. They said that, there are cases where the sum of desired inclusion probabilities is not an integer. They also presented two methods to overcome the problem of sample selection with unequal inclusion probabilities when their sum is not an integer and the sample size cannot be fixed. They achieved this using what is called Splitting method. Dawodu (2013) worked on selection procedure proposed by Basit & Shahbaz. Here, a sample size  $n = 2$  was selected and empirical studies were carried out by varying the values of constant  $\beta$  while fixing the value of  $\alpha$  to be equal to 1. It was discovered that the least variances were obtained when  $\beta$  was fixed to be 1 or  $-1$ . Tiwari and Chilwa (2013) proposed a sample selection procedure in PPSSWOR with restriction to sample size 2. Ahmad and Hanif (2013) stressed that the expression of the joint probabilities of inclusion for most of the unequal probability sampling design is difficult. Therefore, they classified the methods into three groups and compare their performance by calculating their values and their results were obtained.

## CHAPTER THREE

### 3.0 Methodology

#### 3.1 Modified Sample Selection Procedures

##### 3.1.1 Modified Procedure I

This modified procedure uses the ideas of Basit & Shahbaz (2007) and also Alodat (2009) procedures in PPSSWOR. It is used for the selection of sample of any size. It states that:

- selecte first unit with probability proportional to  $\frac{P_i}{2-P_i}$  without replacement,
- select a random sample of size  $n - 1$  from the remaining  $N - 1$  units.

Therefore,

$$\pi_i = \frac{\frac{P_i}{2-P_i}}{\sum_{i=1}^N \frac{P_i}{2-P_i}} + \frac{n-1}{N-1} \sum_{\substack{j=1 \\ j \neq i}}^N \frac{\frac{P_j}{2-P_j}}{\sum_{j=1}^N \frac{P_j}{2-P_j}}$$

$$\pi_i = \frac{1}{\sum_{i=1}^N \frac{P_i}{2-P_i}} \left[ \frac{P_i}{2-P_i} + \frac{n-1}{N-1} \sum_{\substack{j=1 \\ j \neq i}}^N \frac{P_j}{2-P_j} \right]$$

$$\text{Let } K = \sum_{i=1}^N \frac{P_i}{2-P_i}$$

$$\pi_i = \frac{1}{K} \left[ \frac{P_i}{2-P_i} + \frac{n-1}{N-1} \sum_{\substack{j=1 \\ j \neq i}}^N \frac{P_j}{2-P_j} \right]$$

$$\text{But } \sum_{\substack{j=1 \\ j \neq i}}^N \frac{P_j}{2-P_j} = \sum_{j=1}^N \frac{P_j}{2-P_j} - \frac{P_i}{2-P_i}$$

$$\pi_i = \frac{1}{K} \left[ \frac{P_i}{2-P_i} + \frac{n-1}{N-1} \left( \sum_{j=1}^N \frac{P_j}{2-P_j} - \frac{P_i}{2-P_i} \right) \right]$$

$$\pi_i = \frac{1}{K} \left[ \frac{P_i}{2-P_i} - \frac{n-1}{N-1} \frac{P_i}{2-P_i} + \frac{n-1}{N-1} \sum_{j=1}^N \frac{P_j}{2-P_j} \right]$$

$$\pi_i = \frac{1}{K} \left[ \frac{P_i}{2-P_i} \left( 1 - \frac{n-1}{N-1} \right) + \frac{n-1}{N-1} \sum_{j=1}^N \frac{P_j}{2-P_j} \right]$$

$$\pi_i = \frac{1}{K} \left[ \frac{P_i}{2-P_i} \left( \frac{N-n}{N-1} \right) + \frac{n-1}{N-1} \sum_{j=1}^N \frac{P_j}{2-P_j} \right]$$

$$\pi_i = \frac{1}{K} \frac{P_i}{2-P_i} \frac{N-n}{N-1} + \frac{1}{K} \frac{n-1}{N-1} \sum_{j=1}^N \frac{P_j}{2-P_j}$$

$$\text{but } K = \sum_{i=1}^N \frac{P_i}{2-P_i}$$

$$\text{where } \sum_{i=1}^N \frac{P_i}{2-P_i} = \sum_{j=1}^N \frac{P_j}{2-P_j} = K$$

$$\pi_i = \frac{1}{K} \frac{N-n}{N-1} \frac{P_i}{2-P_i} + \frac{n-1}{N-1} \quad (30)$$

Tikkiwal (1965) shows that:

$\pi_{ij} =$  (Probability of the  $i$ th unit or  $j$ th unit occurring at the first draw and the remaining units occurring at a subsequent draw) + (probability of  $i$ th and  $j$ th units occurring after the first draw).

$$\pi_{ij} = (P_i + P_j) \frac{n-1}{N-1} + (1 - P_i - P_j) \frac{(n-1)(n-2)}{(N-1)(N-2)}$$

Since  $\pi_i = \frac{n}{N}$  and  $\pi_{ij} = \frac{n(n-1)}{N(N-1)}$  for simple random sampling without

replacement,

hence,

$$\pi_{ij} = \frac{\frac{P_i}{2-P_i}}{\sum_{i=1}^N \frac{P_i}{2-P_i}} \frac{n-1}{N-1} + \frac{\frac{P_j}{2-P_j}}{\sum_{j=1}^N \frac{P_j}{2-P_j}} \frac{n-1}{N-1} + \left( 1 - \frac{\frac{P_i}{2-P_i}}{\sum_{i=1}^N \frac{P_i}{2-P_i}} - \frac{\frac{P_j}{2-P_j}}{\sum_{j=1}^N \frac{P_j}{2-P_j}} \right) \left( \frac{n-1}{N-1} \right) \left( \frac{n-2}{N-2} \right)$$

$$\pi_{ij} = \frac{(n-1)(N-n)}{k(N-1)(N-2)} \left[ \frac{P_i}{2-P_i} + \frac{P_j}{2-P_j} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)} \quad (31)$$

### 3.1.2 Modified Procedure II

This modified procedure is an improvement on P I above which deals with, and it is for selecting sample of any size. It states that:

- select first unit with probability proportional to  $\frac{P_i^\alpha}{2-P_i^\beta}$  without replacement,
- select a random sample of size  $n - 1$  from the remaining  $N - 1$  units.

Note that  $\alpha$  and  $\beta$  can assume integer values which optimum values would be determined through empirical studies.

Therefore,

$$\pi_i = \frac{\frac{P_i^\alpha}{2-P_i^\beta}}{\sum_{i=1}^N \frac{P_i^\alpha}{2-P_i^\beta}} + \frac{n-1}{N-1} \sum_{j=1, j \neq i}^N \frac{\frac{P_j^\alpha}{2-P_j^\beta}}{\sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta}}$$

$$\pi_i = \frac{1}{\sum_{i=1}^N \frac{P_i^\alpha}{2-P_i^\beta}} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1} \sum_{j=1, j \neq i}^N \frac{P_j^\alpha}{2-P_j^\beta} \right]$$

$$\text{Let } R = \sum_{i=1}^N \frac{P_i^\alpha}{2-P_i^\beta}$$

$$\pi_i = \frac{1}{R} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1} \sum_{j \neq i}^N \frac{P_j^\alpha}{2-P_j^\beta} \right]$$

$$\text{But } \sum_{j \neq i}^N \frac{P_j^\alpha}{2-P_j^\beta} = \sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta} - \frac{P_i^\alpha}{2-P_i^\beta}$$

$$\pi_i = \frac{1}{R} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1} \left( \sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta} - \frac{P_i^\alpha}{2-P_i^\beta} \right) \right]$$

$$\pi_i = \frac{1}{R} \left[ \frac{P_i^\alpha}{2-P_i^\beta} - \frac{n-1}{N-1} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1} \sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta} \right]$$

$$\pi_i = \frac{1}{R} \left[ \frac{P_i^\alpha}{2-P_i^\beta} \left( 1 - \frac{n-1}{N-1} \right) + \frac{n-1}{N-1} \sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta} \right]$$

$$\pi_i = \frac{1}{R} \left[ \frac{P_i^\alpha}{2-P_i^\beta} \left( \frac{N-n}{N-1} \right) + \frac{n-1}{N-1} \sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta} \right]$$

$$\pi_i = \frac{1}{R} \frac{P_i^\alpha}{2-P_i^\beta} \frac{N-n}{N-1} + \frac{1}{R} \frac{n-1}{N-1} \sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta}$$

$$\text{but } R = \sum_{i=1}^N \frac{P_i^\alpha}{2-P_i^\beta}$$

$$\text{where } \sum_{i=1}^N \frac{P_i^\alpha}{2-P_i^\beta} = \sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta} = R$$

$$\pi_i = \frac{1}{R} \frac{N-n}{N-1} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1} \quad (32)$$

Since  $\pi_i = \frac{n}{N}$  and  $\pi_{ij} = \frac{n(n-1)}{N(N-1)}$  for simple random sampling without replacement,

hence,

$$\pi_{ij} = \frac{\frac{P_i^\alpha}{2-P_i^\beta}}{\sum_{i=1}^N \frac{P_i^\alpha}{2-P_i^\beta}} \frac{n-1}{N-1} + \frac{\frac{P_j^\alpha}{2-P_j^\beta}}{\sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta}} \frac{n-1}{N-1} + \left( 1 - \frac{\frac{P_i^\alpha}{2-P_i^\beta}}{\sum_{i=1}^N \frac{P_i^\alpha}{2-P_i^\beta}} - \frac{\frac{P_j^\alpha}{2-P_j^\beta}}{\sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta}} \right) \left( \frac{n-1}{N-1} \right) \left( \frac{n-2}{N-2} \right)$$

$$\pi_{ij} = \frac{(n-1)(N-n)}{R(N-1)(N-2)} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{P_j^\alpha}{2-P_j^\beta} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)} \quad (33)$$

### 3.2 Some Results for the Modified Procedures

#### 3.2.1 Results for Procedure I

Result 1:  $\sum_{i=1}^N \pi_i = n$

Recall that  $\pi_i = \frac{1}{K} \frac{N-n}{N-1} \frac{P_i}{2-P_i} + \frac{n-1}{N-1}$

Summing both sides give

$$\sum_{i=1}^N \pi_i = \frac{1}{K} \frac{N-n}{N-1} \sum_{i=1}^N \frac{P_i}{2-P_i} + N \frac{n-1}{N-1}$$

$$\sum_{i=1}^N \pi_i = \frac{N-n}{N-1} + \frac{N(n-1)}{N-1}$$

$$\sum_{i=1}^N \pi_i = \frac{N-n+N(n-1)}{N-1} = \frac{N-n+Nn-N}{N-1} = \frac{Nn-n}{N-1} = \frac{n(N-1)}{N-1}$$

$$\sum_{i=1}^N \pi_i = n \quad (34)$$

Result 2:  $\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1)\pi_i$

Recall that  $\pi_{ij} = \frac{(n-1)(N-n)}{K(N-1)(N-2)} \left[ \frac{P_i}{2-P_i} + \frac{P_j}{2-P_j} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)}$

$$\pi_{ij} = \frac{(n-1)(N-n)}{K(N-1)(N-2)} \frac{P_i}{2-P_i} + \frac{(n-1)(N-n)}{K(N-1)(N-2)} \frac{P_j}{2-P_j} + \frac{(n-1)(n-2)}{(N-1)(N-2)}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) \sum_{\substack{j=1 \\ j \neq i}}^N \left[ \frac{(N-n)}{K(N-1)(N-2)} \frac{P_i}{2-P_i} + \frac{(N-n)}{K(N-1)(N-2)} \frac{P_j}{2-P_j} + \frac{(n-2)}{(N-1)(N-2)} \right]$$

$$\begin{aligned} \sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) & \left\{ \left[ \sum_{\substack{j=1 \\ j \neq i}}^N \frac{(N-n)}{K(N-1)(N-2)} \frac{P_i}{2-P_i} \right] + \left[ \sum_{\substack{j=1 \\ j \neq i}}^N \frac{(N-n)}{K(N-1)(N-2)} \frac{P_j}{2-P_j} \right] + \right. \\ & \left. \left[ \sum_{\substack{j=1 \\ j \neq i}}^N \frac{(n-2)}{(N-1)(N-2)} \right] \right\} \end{aligned}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) \left[ \frac{1}{K} \frac{(N-n)(N-2)}{(N-1)(N-2)} \frac{P_i}{2-P_i} + \frac{1}{K} \frac{(N-n)}{(N-1)(N-2)} \sum_{j=1}^N \frac{P_j}{2-P_j} + \frac{(N-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) \left[ \frac{1}{K} \frac{(N-n)}{(N-1)} \frac{P_i}{2-P_i} + \frac{(N-n)}{(N-1)(N-2)} + \frac{(n-2)}{(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) \left[ \frac{1}{K} \frac{(N-n)}{(N-1)} \frac{P_i}{2-P_i} + \frac{(N-n)+(n-2)(N-1)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) \left[ \frac{1}{K} \frac{(N-n)}{(N-1)} \frac{P_i}{2-P_i} + \frac{N-n+Nn-n-2N+2}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) \left[ \frac{1}{K} \frac{(N-n)}{(N-1)} \frac{P_i}{2-P_i} + \frac{Nn-N-2n+2}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) \left[ \frac{1}{K} \frac{(N-n)}{(N-1)} \frac{P_i}{2-P_i} + \frac{N(n-1)-2(n-1)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) \left[ \frac{1}{K} \frac{(N-n)}{(N-1)} \frac{P_i}{2-P_i} + \frac{(n-1)(N-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1) \left[ \frac{1}{K} \frac{(N-n)}{(N-1)} \frac{P_i}{2-P_i} + \frac{(n-1)}{(N-1)} \right] \pi_i = \frac{1}{K} \frac{N-n}{N-1} \frac{P_i}{2-P_i} + \frac{n-1}{N-1}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1)\pi_i \quad (35)$$

$$\text{Result 3:} \quad \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = n(n-1)$$

$$\text{Recall that } \pi_{ij} = \frac{(n-1)(N-n)}{K(N-1)(N-2)} \left[ \frac{P_i}{2-P_i} + \frac{P_j}{2-P_j} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \left[ \frac{(n-1)(N-n)}{K(N-1)(N-2)} \left[ \frac{P_i}{2-P_i} + \frac{P_j}{2-P_j} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^N \left[ \sum_{i=1}^N \left( \frac{(n-1)(N-n)}{K(N-1)(N-2)} \right) \left[ \frac{P_i}{2-P_i} + \frac{P_j}{2-P_j} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^N \left[ \frac{(n-1)(N-n)}{K(N-1)(N-2)} (K + K) + \frac{N(n-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^N \left[ \frac{2K(n-1)(N-n)}{K(N-1)(N-2)} + \frac{N(n-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \frac{2K(n-1)(N-n)(N-1)}{K(N-1)(N-2)} + \frac{N(n-1)(n-2)(N-1)}{(N-1)(N-2)}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \frac{2(n-1)(N-n)}{(N-2)} + \frac{N(n-1)(n-2)}{(N-2)}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \frac{2(n-1)(N-n) + N(n-1)(n-2)}{N-2}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = (n-1) \left[ \frac{2(N-n) + N(n-2)}{N-2} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = (n-1) \left[ \frac{2N - 2n + Nn - 2N}{N-2} \right]$$

$$\begin{aligned}
\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} &= (n-1) \left[ \frac{Nn-2n}{N-2} \right] \\
\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} &= (n-1) \left[ \frac{n(N-2)}{N-2} \right] \\
\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} &= n(n-1)
\end{aligned} \tag{36}$$

Result 4: The Sen – Yates – Grundy variance estimator is always non – negative under this procedure.

Proof: Equation (6) will be positive if  $\pi_i \pi_j - \pi_{ij} \geq 0$

$$\text{Recall that } \pi_i = \frac{1}{K} \frac{N-n}{N-1} \frac{P_i}{2-P_i} + \frac{n-1}{N-1}$$

$$\pi_j = \frac{1}{K} \frac{N-n}{N-1} \frac{P_j}{2-P_j} + \frac{n-1}{N-1}$$

$$\pi_{ij} = \frac{(n-1)(N-n)}{K(N-1)(N-2)} \left[ \frac{P_i}{2-P_i} + \frac{P_j}{2-P_j} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)}$$

$$\begin{aligned}
\pi_i \pi_j - \pi_{ij} &= \left( \frac{1}{K} \frac{N-n}{N-1} \frac{P_i}{2-P_i} + \frac{n-1}{N-1} \right) \left( \frac{1}{K} \frac{N-n}{N-1} \frac{P_j}{2-P_j} + \frac{n-1}{N-1} \right) - \left[ \frac{(n-1)(N-n)}{K(N-1)(N-2)} \left[ \frac{P_i}{2-P_i} + \right. \right. \\
&\quad \left. \left. \frac{P_j}{2-P_j} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)} \right]
\end{aligned}$$

$$\text{Let } C = \frac{1}{K} \frac{N-n}{N-1} \frac{P_i}{2-P_i} \text{ and } D = \frac{1}{K} \frac{N-n}{N-1} \frac{P_j}{2-P_j}$$

$$\begin{aligned}
\pi_i \pi_j - \pi_{ij} &= \left( C + \frac{n-1}{N-1} \right) \left( D + \frac{n-1}{N-1} \right) - \frac{(n-1)(N-n)}{K(N-1)(N-2)} \frac{P_i}{2-P_i} - \frac{(n-1)(N-n)}{K(N-1)(N-2)} \frac{P_j}{2-P_j} - \\
&\quad \frac{(n-1)(n-2)}{(N-1)(N-2)}
\end{aligned}$$

$$\pi_i \pi_j - \pi_{ij} = \left[ CD + \frac{C(n-1)}{N-1} + \frac{D(n-1)}{N-1} - \frac{C(n-1)}{N-2} - \frac{D(n-1)}{N-2} + \frac{n-1}{N-1} \frac{n-1}{N-1} \right] - \frac{(n-1)(n-2)}{(N-1)(N-2)}$$

Items in the big bracket is always  $\geq \frac{(n-1)(n-2)}{(N-1)(N-2)}$ , so equation (6) remains non – negative for all  $P_i$  and  $P_j$ .

### 3.2.2 Results for Procedure II

$$\text{Result 1: } \sum_{i=1}^N \pi_i = n$$

Recall that

$$\pi_i = \frac{1}{R} \frac{N-n}{N-1} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1}$$

Summing both sides give

$$\sum_{i=1}^N \pi_i = \frac{1}{R} \frac{N-n}{N-1} \sum_{i=1}^N \frac{P_i^\alpha}{2-P_i^\beta} + N \frac{n-1}{N-1}$$

$$\sum_{i=1}^N \pi_i = \frac{N-n}{N-1} + \frac{N(n-1)}{N-1}$$

$$\sum_{i=1}^N \pi_i = \frac{N-n+N(n-1)}{N-1} = \frac{N-n+Nn-N}{N-1} = \frac{Nn-n}{N-1} = \frac{n(N-1)}{N-1}$$

$$\sum_{i=1}^N \pi_i = n \tag{37}$$

$$\text{Result 2: } \sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = (n-1)\pi_i$$

$$\text{Recall that } \pi_{ij} = \frac{(n-1)(N-n)}{R(N-1)(N-2)} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{P_j^\alpha}{2-P_j^\beta} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)}$$

$$\pi_{ij} = \frac{(n-1)(N-n)}{R(N-1)(N-2)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{(n-1)(N-n)}{R(N-1)(N-2)} \frac{P_j^\alpha}{2-P_j^\beta} + \frac{(n-1)(n-2)}{(N-1)(N-2)}$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \sum_{j=1, j \neq i}^N \left[ \frac{(N-n)}{R(N-1)(N-2)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{(N-n)}{R(N-1)(N-2)} \frac{P_j^\alpha}{2-P_j^\beta} + \frac{(n-2)}{(N-1)(N-2)} \right]$$

$$\begin{aligned} \sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) & \left\{ \left[ \sum_{j=1, j \neq i}^N \frac{(N-n)}{R(N-1)(N-2)} \frac{P_i^\alpha}{2-P_i^\beta} \right] + \left[ \sum_{j=1, j \neq i}^N \frac{(N-n)}{R(N-1)(N-2)} \frac{P_j^\alpha}{2-P_j^\beta} \right] + \right. \\ & \left. \left[ \sum_{j=1, j \neq i}^N \frac{(n-2)}{(N-1)(N-2)} \right] \right\} \end{aligned}$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \left[ \frac{1}{R} \frac{(N-n)(N-2)}{(N-1)(N-2)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{1}{R} \frac{(N-n)}{(N-1)(N-2)} \sum_{j=1}^N \frac{P_j^\alpha}{2-P_j^\beta} + \frac{(N-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \left[ \frac{1}{R} \frac{(N-n)}{(N-1)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{(N-n)}{(N-1)(N-2)} + \frac{(n-2)}{(N-2)} \right]$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \left[ \frac{1}{R} \frac{(N-n)}{(N-1)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{(N-n)+(n-2)(N-1)}{(N-1)(N-2)} \right]$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \left[ \frac{1}{R} \frac{(N-n)}{(N-1)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{N-n+Nn-n-2N+2}{(N-1)(N-2)} \right]$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \left[ \frac{1}{R} \frac{(N-n)}{(N-1)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{Nn-N-2n+2}{(N-1)(N-2)} \right]$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \left[ \frac{1}{R} \frac{(N-n)}{(N-1)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{N(n-1)-2(n-1)}{(N-1)(N-2)} \right]$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \left[ \frac{1}{R} \frac{(N-n)}{(N-1)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{(n-1)(N-2)}{(N-1)(N-2)} \right]$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \left[ \frac{1}{R} \frac{(N-n)}{(N-1)} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{(n-1)}{(N-1)} \right] \pi_i = \frac{1}{R} \frac{N-n}{N-1} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1}$$

$$\sum_{j=1, j \neq i}^N \pi_{ij} = (n-1) \pi_i \tag{38}$$

Result 3:  $\sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \pi_{ij} = n(n-1)$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = n(n-1)$$

Recall that  $\pi_{ij} = \frac{(n-1)(N-n)}{R(N-1)(N-2)} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{P_j^\alpha}{2-P_j^\beta} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)}$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \left[ \frac{(n-1)(N-n)}{R(N-1)(N-2)} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{P_j^\alpha}{2-P_j^\beta} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^N \left[ \sum_{i=1}^N \left( \frac{(n-1)(N-n)}{R(N-1)(N-2)} \right) \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{P_j^\alpha}{2-P_j^\beta} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^N \left[ \frac{(n-1)(N-n)}{R(N-1)(N-2)} (R + R) + \frac{N(n-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^N \left[ \frac{2R(n-1)(N-n)}{R(N-1)(N-2)} + \frac{N(n-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \frac{2R(n-1)(N-n)(N-1)}{R(N-1)(N-2)} + \frac{N(n-1)(n-2)(N-1)}{(N-1)(N-2)}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \frac{2(n-1)(N-n)}{(N-2)} + \frac{N(n-1)(n-2)}{(N-2)}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = \frac{2(n-1)(N-n) + N(n-1)(n-2)}{N-2}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = (n-1) \left[ \frac{2(N-n) + N(n-2)}{N-2} \right]$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} = (n-1) \left[ \frac{2N-2n+Nn-2N}{N-2} \right]$$

$$\begin{aligned}
\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} &= (n-1) \left[ \frac{Nn-2n}{N-2} \right] \\
\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} &= (n-1) \left[ \frac{Nn-2n}{N-2} \right] \\
\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} &= (n-1) \left[ \frac{n(N-2)}{N-2} \right] \\
\sum_{\substack{j=1 \\ j \neq i}}^N \sum_{i=1}^N \pi_{ij} &= n(n-1) \tag{39}
\end{aligned}$$

Result 4: The Sen – Yates – Grundy variance estimator is always non – negative under this procedure.

Proof: The Sen – Yates – Grundy variance estimator given in equation (6) will be positive if  $\pi_i \pi_j - \pi_{ij} \geq 0$

$$\text{Recall that } \pi_i = \frac{1}{R} \frac{N-n}{N-1} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1}$$

$$\pi_j = \frac{1}{R} \frac{N-n}{N-1} \frac{P_j^\alpha}{2-P_j^\beta} + \frac{n-1}{N-1}$$

$$\pi_{ij} = \frac{(n-1)(N-n)}{R(N-1)(N-2)} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{P_j^\alpha}{2-P_j^\beta} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)}$$

$$\pi_i \pi_j - \pi_{ij} = \left( \frac{1}{R} \frac{N-n}{N-1} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1} \right) \left( \frac{1}{R} \frac{N-n}{N-1} \frac{P_j^\alpha}{2-P_j^\beta} + \frac{n-1}{N-1} \right) -$$

$$\left[ \frac{(n-1)(N-n)}{R(N-1)(N-2)} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{P_j^\alpha}{2-P_j^\beta} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)} \right]$$

$$\text{Let } A = \frac{1}{R} \frac{N-n}{N-1} \frac{P_i^\alpha}{2-P_i^\beta} \text{ and } B = \frac{1}{R} \frac{N-n}{N-1} \frac{P_j^\alpha}{2-P_j^\beta}$$

$$\pi_i \pi_j - \pi_{ij} = \left( A + \frac{n-1}{N-1} \right) \left( B + \frac{n-1}{N-1} \right) - \frac{(n-1)(N-n)}{R(N-1)(N-2)} \frac{P_i^\alpha}{2-P_i^\beta} - \frac{(n-1)(N-n)}{R(N-1)(N-2)} \frac{P_j^\alpha}{2-P_j^\beta} -$$

$$\frac{(n-1)(n-2)}{(N-1)(N-2)}$$

$$\pi_i \pi_j - \pi_{ij} = \left[ AB + \frac{A(n-1)}{N-1} + \frac{B(n-1)}{N-1} - \frac{A(n-1)}{N-2} - \frac{B(n-1)}{N-2} + \frac{n-1}{N-1} \frac{n-1}{N-1} \right] - \frac{(n-1)(n-2)}{(N-1)(N-2)}$$

Items in the big bracket is always  $\geq \frac{(n-1)(n-2)}{(N-1)(N-2)}$ , so equation (6) remains non –

negative for all  $P_i^\alpha, P_j^\alpha, P_i^\beta$ , and  $P_j^\alpha$ .

Table 3.1: Summary of the derived probability of inclusion  $\pi_i$  and the joint probability of inclusion  $\pi_{ij}$  for the two modified sample selection procedures under unequal probability sampling without replacement

| Serial Number | Probability of inclusion ( $\pi_i$ )                                                   | Joint probability of inclusion ( $\pi_{ij}$ )                                                                                                                                                                                        |
|---------------|----------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Procedure I   | $\pi_i = \frac{1}{K} \frac{N-n}{N-1} \frac{P_i}{2-P_i} + \frac{n-1}{N-1}$              | $\pi_{ij} = \frac{(n-1)(N-n)}{K(N-1)(N-2)} \left[ \frac{P_i}{2-P_i} + \frac{P_j}{2-P_j} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)}$ <p>where <math>K = \sum_{i=1}^N \frac{P_i}{2-P_i}</math></p>                                        |
| Procedure II  | $\pi_i = \frac{1}{R} \frac{N-n}{N-1} \frac{P_i^\alpha}{2-P_i^\beta} + \frac{n-1}{N-1}$ | $\pi_{ij} = \frac{(n-1)(N-n)}{R(N-1)(N-2)} \left[ \frac{P_i^\alpha}{2-P_i^\beta} + \frac{P_j^\alpha}{2-P_j^\beta} \right] + \frac{(n-1)(n-2)}{(N-1)(N-2)}$ <p>where <math>R = \sum_{i=1}^N \frac{P_i^\alpha}{2-P_i^\beta}</math></p> |

### 3.2.3 Flaw of Alodat Sample Selection Procedure

This sample selection procedure is restricted to the selection of sample of size 2.

### 3.2.4 Flaw of Basit & Shahbaz Sample Selection Procedure

It can be used to select sample of any size, but has large variance.

### 3.2.5 Assumptions of the Modified Procedures I and II

- (i). There must be auxiliary variables  $X_i$ , that will be used to compute the probability of selection,  $P_i$ .
- (ii). There must be variables of interest  $Y_i$  and

- (iii). There must be positive and reasonable correlation between the auxiliary variables  $X_i$  and the variables of interest  $Y_i$ .

### 3.2.6 Instrumentation and Data Gathering

Two sets of data would be used here. The first is simulated data generated using statistical program *R*. The second is real life data, which is secondary data, where data on Childs' weight at birth are extracted from the Demographic and Health Survey of five African countries and also census data of the respective countries.

The variance estimator given by Sen – Yates – Grundy shown below would be used to calculate the variance.

$$\hat{V}_{SYG}(\hat{Y}_{HT}) = \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n \left( \frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}} \right) \left( \frac{y_i}{\pi_i} - \frac{y_j}{\pi_j} \right)^2$$

## CHAPTER FOUR

### 4.0 Application of Modified Sample Selection Procedures to Simulated Data and Real Life Data

Here, auxiliary variables are used to compute the probabilities of selection  $P_i$ . For simulated data, the auxiliary variables  $X_i$  and the variables of interest  $Y_i$  are generated using statistical program  $R$  as shown in the appendix, and ensuring that there exists reasonable and positive correlation between them. The Pearson correlation coefficients between the variables of interest and the auxiliary variables for the six simulated datasets under consideration are; 0.9104, 0.9358, 0.9112, 0.9822, 0.8318, and 0.8701 respectively.

Next is the application to real life data. Generally speaking, any child with birth weight less than 2.5 kilograms is considered small in size. A child's birth weight is a very important indicator of his susceptibility to the risk of childhood illnesses and the chances of survival. A child with birth weight of less than 2.5 kilograms has a higher risk of early childhood death, according to Nigerian Demographic and Health Survey, 2013. Data on the variable of interest  $Y_i$  (Childs' weight at birth from different zones / provinces of each of the five countries) were obtained from Demographic and Health Survey of five African countries. Data on Childs' weight at birth for five African countries were extracted from the following: Final Report of 2006 – 07 Swaziland Demographic and Health Survey, Final Report of 2013 Liberia Demographic and Health Survey, Final Report of 2013 Nigeria National Demographic and Health Survey, Final Report of 2013 Gambia Demographic and Health Survey, Final Report of 2003 South Africa Demographic and Health Survey.

Also, the auxiliary variables, based on the population of the different zones / provinces of each of the five countries, are extracted from: Swaziland 2012 Census Data, Liberia 2008 Census Data, Nigeria 2006 Population and Housing Data, Preliminary Results of The Gambia 2013 Population and Housing Census, South Africa 2011 Census Data. The Pearson correlation coefficients between the variables of interest and the auxiliary variables for the five life data under consideration are; 0.8497, 0.9832, 0.7808, 0.9639, and 0.6094 respectively.

#### 4.1 Application to Simulated Data

##### 4.1.1 Presentation of Simulated Data

Table 4.1: Distribution of the auxiliary variable and variable of interest: Data I (N=4)

| Units | Auxiliary variable, $X_i$ | Variable of interest, $Y_i$ |
|-------|---------------------------|-----------------------------|
| 1     | 224                       | 122                         |
| 2     | 284                       | 176                         |
| 3     | 243                       | 112                         |
| 4     | 274                       | 159                         |

Source: Simulation

Table 4.2: Distribution of the auxiliary variable and variable of interest: Data II (N=5)

| Units | Auxiliary variable, $X_i$ | Variable of interest, $Y_i$ |
|-------|---------------------------|-----------------------------|
| 1     | 227                       | 124                         |
| 2     | 284                       | 182                         |
| 3     | 271                       | 169                         |
| 4     | 202                       | 122                         |
| 5     | 242                       | 162                         |

Source: Simulation

Table 4.3: Distribution of the auxiliary variable and variable of interest: Data III (N=6)

| Units | Auxiliary variable, $X_i$ | Variable of interest, $Y_i$ |
|-------|---------------------------|-----------------------------|
| 1     | 269                       | 138                         |
| 2     | 205                       | 100                         |
| 3     | 232                       | 117                         |
| 4     | 203                       | 109                         |
| 5     | 277                       | 177                         |
| 6     | 292                       | 158                         |

Source: Simulation

Table 4.4: Distribution of the auxiliary variable and variable of interest: Data IV  
(N=7)

| Units | Auxiliary variable, $X_i$ | Variable of interest, $Y_i$ |
|-------|---------------------------|-----------------------------|
| 1     | 201                       | 48                          |
| 2     | 218                       | 60                          |
| 3     | 230                       | 75                          |
| 4     | 220                       | 65                          |
| 5     | 237                       | 80                          |
| 6     | 240                       | 92                          |
| 7     | 250                       | 102                         |

Source: Simulation

Table 4.5: Distribution of the auxiliary variable and variable of interest: Data V  
(N=8)

| Units | Auxiliary variable, $X_i$ | Variable of interest, $Y_i$ |
|-------|---------------------------|-----------------------------|
| 1     | 267                       | 162                         |
| 2     | 242                       | 110                         |
| 3     | 215                       | 125                         |
| 4     | 275                       | 198                         |
| 5     | 287                       | 185                         |
| 6     | 259                       | 140                         |
| 7     | 235                       | 101                         |
| 8     | 246                       | 117                         |

Source: Simulation

Table 4.6: Distribution of the auxiliary variable and variable of interest: Data VI  
(N=9)

| Units | Auxiliary variable, $X_i$ | Variable of interest, $Y_i$ |
|-------|---------------------------|-----------------------------|
| 1     | 269                       | 181                         |
| 2     | 231                       | 135                         |
| 3     | 247                       | 155                         |
| 4     | 269                       | 160                         |
| 5     | 217                       | 107                         |
| 6     | 292                       | 178                         |
| 7     | 239                       | 133                         |
| 8     | 286                       | 179                         |
| 9     | 291                       | 157                         |

Source: Simulation

#### 4.1.2 Analysis of Simulated Data

Analysis was carried out on the six simulated data for number of units,

$N = 4, 5, 6, 7, 8$  and 9 using P I and P II, and BSP. Sample sizes

$n = 3, 4, 5, 6, 7$ , and 8 was selected for each of the data under consideration.

##### 4.1.2.1 Analysis of Data I

Variance using P I = 2133 (Data I:  $N=4$ )

Table 4.7: Comparison of variances of P II and BSP (Data I:  $N = 4$ )

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | -5       |      | -4   |      | -3   |      | -2   |      | -1   |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 2437     | 3896 | 2167 | 3615 | 1919 | 3322 | 1695 | 3023 | 1497 | 2727 |
|         | -4 | 2717     | 3897 | 2431 | 3617 | 2161 | 3323 | 1913 | 3025 | 1690 | 2729 |
|         | -3 | 2993     | 3902 | 2698 | 3621 | 2413 | 3328 | 2144 | 3030 | 1898 | 2733 |
|         | -2 | 3236     | 3914 | 2937 | 3634 | 2643 | 3341 | 2360 | 3043 | 2096 | 2746 |
|         | -1 | 3321     | 3946 | 3022 | 3669 | 2725 | 3377 | 2438 | 3079 | 2167 | 2782 |
|         | 1  | 3856     | 3709 | 3573 | 3419 | 3279 | 3121 | 2980 | 2823 | 2685 | 2532 |
|         | 2  | 3877     | 3853 | 3596 | 3570 | 3302 | 3276 | 3003 | 2977 | 2707 | 2682 |
|         | 3  | 3889     | 3882 | 3607 | 3600 | 3314 | 3306 | 3015 | 3008 | 2719 | 2712 |
|         | 4  | 3893     | 3891 | 3612 | 3610 | 3319 | 3316 | 3020 | 3018 | 2724 | 2721 |
|         | 5  | 3894     | 3894 | 3614 | 3613 | 3320 | 3320 | 3022 | 3021 | 2725 | 2725 |

Source: Researcher's computation

Table 4.7 continues

|         |    | $\alpha$ |      |      |      |      |      |      |      |            |             |
|---------|----|----------|------|------|------|------|------|------|------|------------|-------------|
|         |    | 1        |      | 2    |      | 3    |      | 4    |      | 5          |             |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II       | BSP         |
|         | -5 | 1178     | 2170 | 1054 | 1922 | 949  | 1698 | 862  | 1500 | <b>790</b> | 1328        |
|         | -4 | 1322     | 2172 | 1175 | 1923 | 1051 | 1699 | 947  | 1501 | 860        | 1329        |
|         | -3 | 1481     | 2176 | 1311 | 1927 | 1166 | 1702 | 1043 | 1504 | 941        | 1331        |
|         | -2 | 1637     | 2188 | 1447 | 1938 | 1282 | 1712 | 1141 | 1512 | 1022       | 1338        |
|         | -1 | 1694     | 2220 | 1496 | 1967 | 1324 | 1738 | 1177 | 1535 | 1052       | 1358        |
|         | 1  | 2133     | 1999 | 1887 | 1766 | 1667 | 1560 | 1473 | 1379 | 1304       | <b>1224</b> |
|         | 2  | 2153     | 2130 | 1906 | 1885 | 1683 | 1665 | 1487 | 1471 | 1317       | 1303        |
|         | 3  | 2164     | 2157 | 1915 | 1909 | 1692 | 1687 | 1495 | 1490 | 1323       | 1319        |
|         | 4  | 2168     | 2166 | 1919 | 1917 | 1695 | 1694 | 1498 | 1496 | 1326       | 1325        |
|         | 5  | 2169     | 2168 | 1921 | 1920 | 1697 | 1696 | 1499 | 1498 | 1327       | 1326        |

Source: Researcher's computation

Table 4.8: Probabilities of inclusion,  $\pi_i$  (Data I:  $N = 4$ )

| Units           | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|-----------------|-----------------------------------|---------------------------------|
| 1               | 0.6821505                         | 0.7017452                       |
| 2               | 0.8332263                         | 0.8008933                       |
| 3               | 0.7016354                         | 0.7217467                       |
| 4               | 0.7829878                         | 0.7756147                       |
| Sample size (n) | 3                                 | 3                               |

Source: Researcher's computation

Table 4.9: Joint Probabilities of inclusion,  $\pi_{ij}$  (Data I:  $N = 4$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 0.5153768                         | 0.5026386                       |
| 1, 3          | 0.3837859                         | 0.423492                        |
| 1, 4          | 0.4651383                         | 0.4773599                       |
| 2, 3          | 0.5348617                         | 0.5226401                       |
| 2, 4          | 0.6162141                         | 0.576508                        |
| 3, 4          | 0.4846232                         | 0.4973614                       |
| Total         | 3                                 | 3                               |

Source: Researcher's computation

Table 4.10: Least variance estimates (Data I:  $N = 4$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 107.8459                          | 248.9357                        |
| 1, 3          | 91.27361                          | 68.33119                        |
| 1, 4          | 87.00588                          | 136.0041                        |
| 2, 3          | 247.7043                          | 442.0376                        |
| 2, 4          | 3.909626                          | 16.87359                        |
| 3, 4          | 252.134                           | 311.5753                        |
| Total         | 790                               | 1224                            |

Source: Researcher's Computation

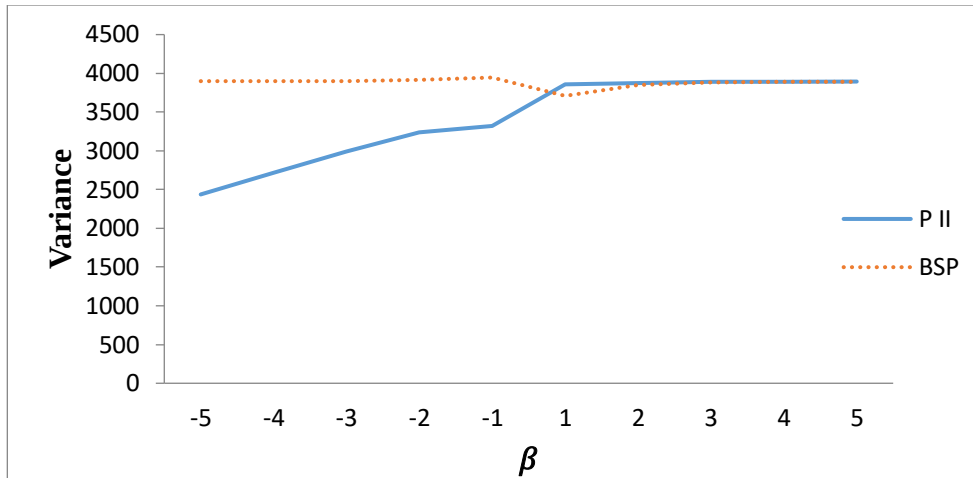


Figure 4.1: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data I)

Interpretation of Figure 4.1:

Figure 4.1 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

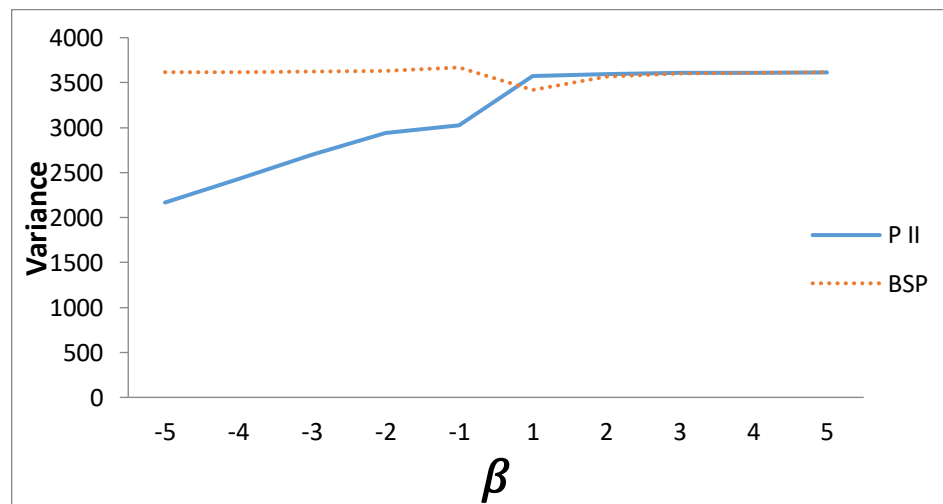


Figure 4.2: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data I)

Interpretation of Figure 4.2:

Figure 4.2 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

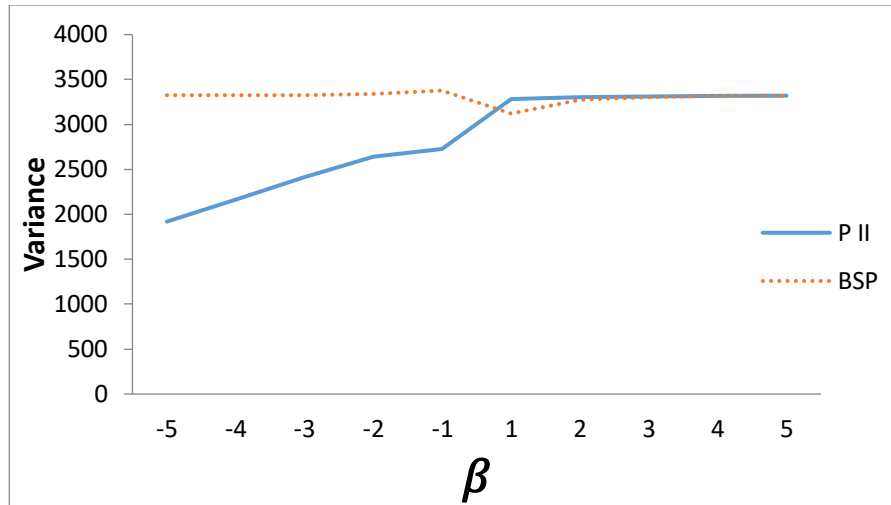


Figure 4.3: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data I)

Interpretation of Figure 4.3:

Figure 4.3 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

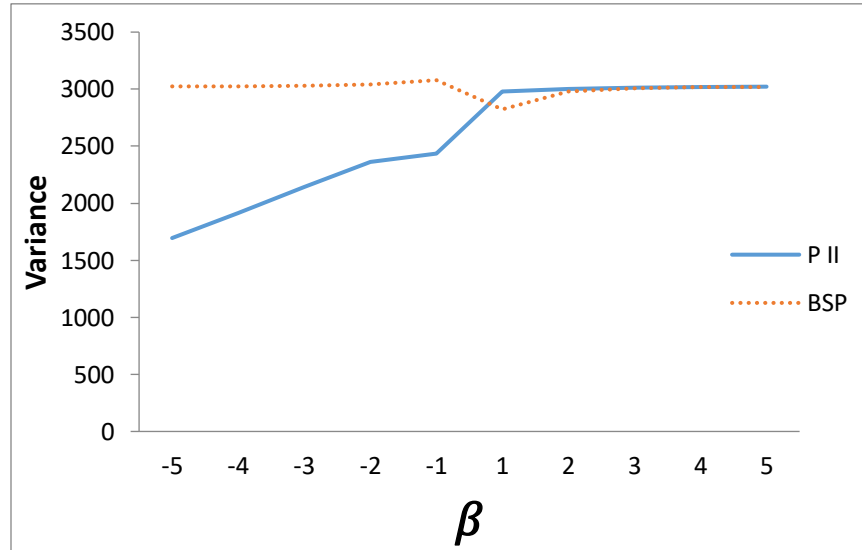


Figure 4.4: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data I)

Interpretation of Figure 4.4:

Figure 4.4 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

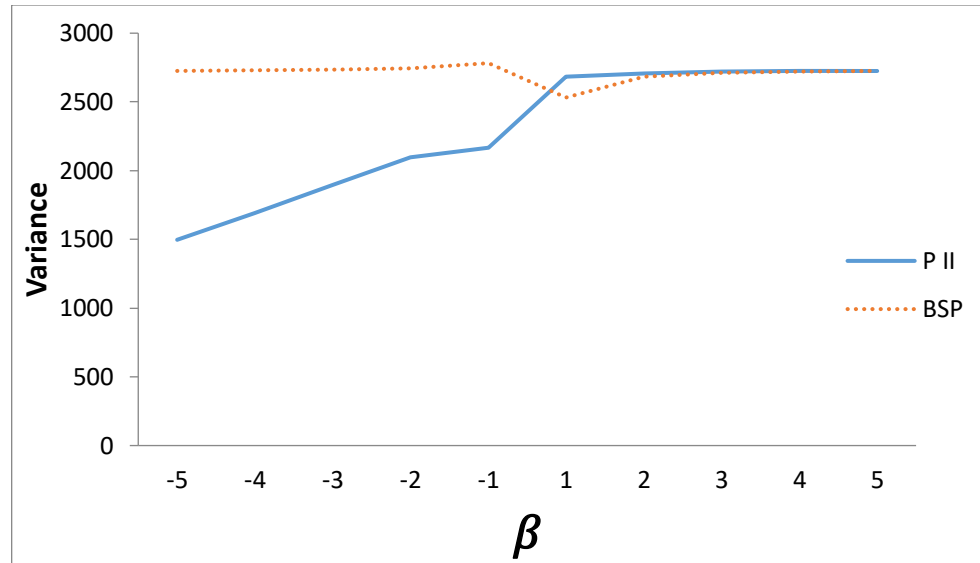


Figure 4.5: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data I)

Interpretation of Figure 4.5:

Figure 4.5 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

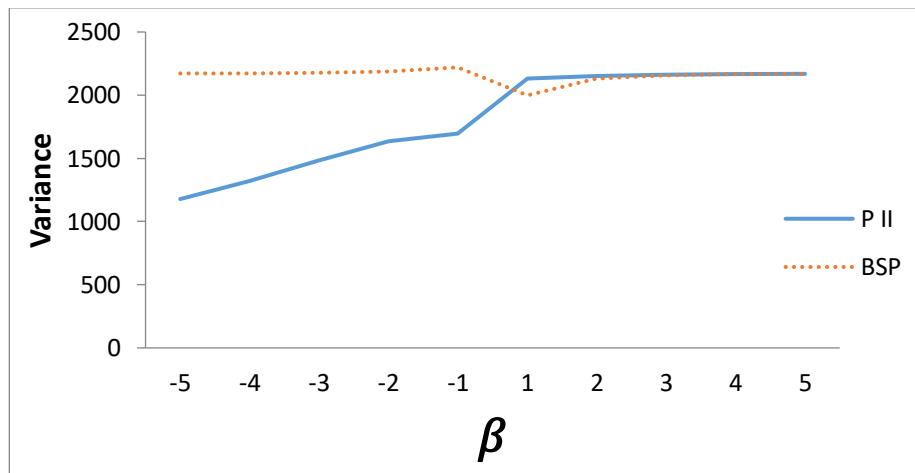


Figure 4.6: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data I)

Interpretation of Figure 4.6:

Figure 4.6 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

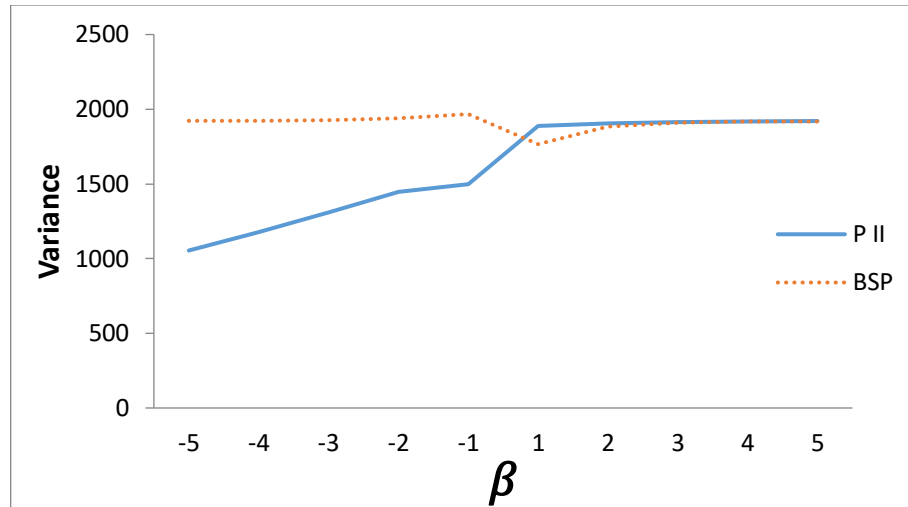


Figure 4.7: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data I)

Interpretation of Figure 4.7:

Figure 4.7 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

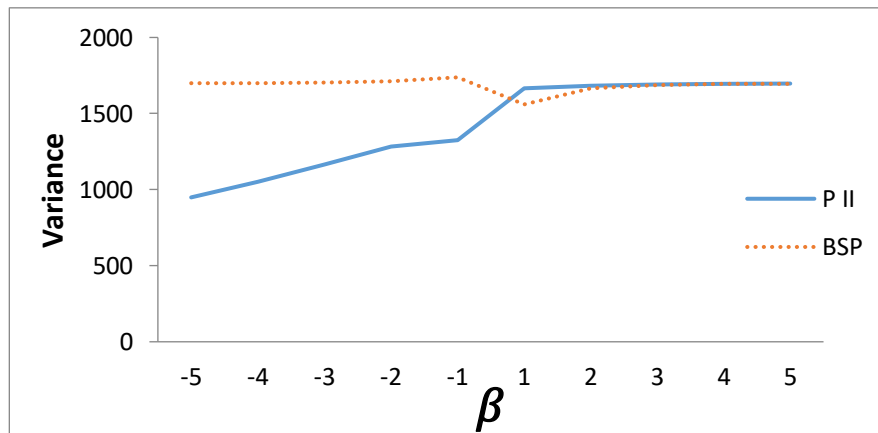


Figure 4.8: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data I)

Interpretation of Figure 4.8:

Figure 4.8 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

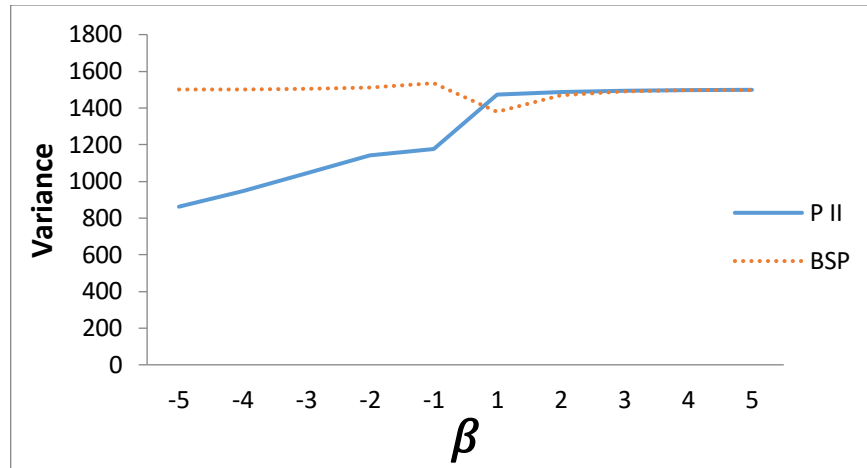


Figure 4.9: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data I)

Interpretation of Figure 4.9:

Figure 4.9 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 4$ .

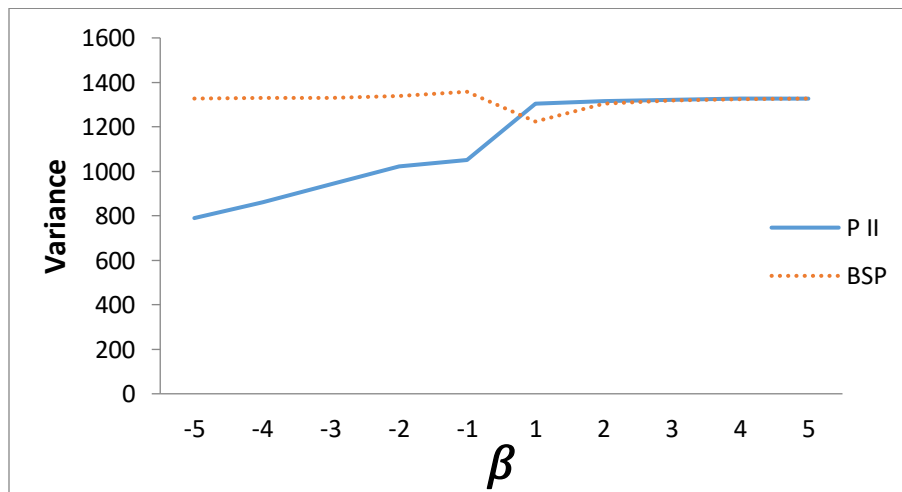


Figure 4.10: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data I)

Interpretation of Figure 4.10:

Figure 4.10 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

#### 4.1.2.2 Analysis of Data II

Variance using P I = 1411 (Data II: N=5)

Table 4.11: Comparison of variances of P II and BSP (Data II:  $N = 5$ )

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | -5       |      | -4   |      | -3   |      | -2   |      | -1   |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 1548     | 2102 | 1424 | 2018 | 1309 | 1916 | 1206 | 1800 | 1117 | 1676 |
|         | -4 | 1674     | 2103 | 1547 | 2018 | 1423 | 1916 | 1308 | 1800 | 1205 | 1676 |
|         | -3 | 1794     | 2103 | 1669 | 2019 | 1542 | 1917 | 1418 | 1801 | 1303 | 1677 |
|         | -2 | 1896     | 2106 | 1779 | 2022 | 1653 | 1920 | 1526 | 1805 | 1403 | 1681 |
|         | -1 | 1951     | 2113 | 1839 | 2031 | 1717 | 1931 | 1590 | 1817 | 1464 | 1693 |
|         | 1  | 2094     | 2068 | 2007 | 1977 | 1903 | 1868 | 1786 | 1748 | 1661 | 1622 |
|         | 2  | 2099     | 2095 | 2014 | 2009 | 1911 | 1906 | 1795 | 1789 | 1670 | 1664 |
|         | 3  | 2101     | 2100 | 2017 | 2015 | 1914 | 1913 | 1798 | 1797 | 1674 | 1672 |
|         | 4  | 2102     | 2102 | 2017 | 2017 | 1915 | 1915 | 1800 | 1799 | 1675 | 1675 |
|         | 5  | 2102     | 2102 | 2018 | 2018 | 1916 | 1916 | 1800 | 1800 | 1675 | 1675 |

Source: Researcher's computation

Table 4.11 continues

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | 1        |      | 2    |      | 3    |      | 4    |      | 5    |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 987      | 1425 | 943  | 1309 | 912  | 1206 | 892  | 1117 | 880  | 1045 |
|         | -4 | 1044     | 1425 | 986  | 1309 | 943  | 1206 | 912  | 1118 | 892  | 1045 |
|         | -3 | 1113     | 1426 | 1041 | 1310 | 984  | 1207 | 941  | 1118 | 911  | 1045 |
|         | -2 | 1189     | 1430 | 1103 | 1314 | 1033 | 1121 | 978  | 1047 | 937  | 1047 |
|         | -1 | 1238     | 1441 | 1144 | 1324 | 1066 | 1219 | 1004 | 1129 | 956  | 1054 |
|         | 1  | 1411     | 1374 | 1297 | 1264 | 1195 | 1166 | 1108 | 1085 | 1037 | 1018 |
|         | 2  | 1419     | 1413 | 1304 | 1299 | 1202 | 1197 | 1114 | 1110 | 1042 | 1038 |
|         | 3  | 1423     | 1421 | 1308 | 1306 | 1205 | 1203 | 1116 | 1115 | 1044 | 1043 |
|         | 4  | 1424     | 1424 | 1309 | 1308 | 1206 | 1205 | 1117 | 1117 | 1044 | 1044 |
|         | 5  | 1424     | 1424 | 1309 | 1309 | 1206 | 1206 | 1117 | 1117 | 1044 | 1044 |

Source: Researcher's computation

Table 4.12: Probabilities of inclusion,  $\pi_i$  (Data II:  $N = 5$ )

| Units           | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|-----------------|-----------------------------------|---------------------------------|
| 1               | 0.7635162                         | 0.7779145                       |
| 2               | 0.8771071                         | 0.8446604                       |
| 3               | 0.829535                          | 0.8230374                       |
| 4               | 0.7542075                         | 0.7649947                       |
| 5               | 0.7756342                         | 0.7893931                       |
| Sample size (n) | 4                                 | 4                               |

Source: Researcher's computation

Table 4.13: Joint Probabilities of inclusion  $\pi_{ij}$  (Data II:  $N = 5$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 0.6406233                         | 0.6225749                       |
| 1, 3          | 0.5930512                         | 0.6009519                       |
| 1, 4          | 0.5177237                         | 0.5429091                       |
| 1, 5          | 0.5391504                         | 0.5673075                       |
| 2, 3          | 0.706642                          | 0.6676978                       |
| 2, 4          | 0.6313146                         | 0.6096551                       |
| 2, 5          | 0.6527413                         | 0.6340535                       |
| 3, 4          | 0.5837425                         | 0.5880321                       |
| 3, 5          | 0.6051692                         | 0.6124305                       |
| 4, 5          | 0.5298418                         | 0.5543877                       |
| Total         | 6                                 | 6                               |

Source: Researcher's computation

Table 4.14: Least variance estimates (Data II:  $N = 5$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 92.24852                          | 174.2136                        |
| 1, 3          | 116.0671                          | 137.999                         |
| 1, 4          | 0.04704414                        | 0.0005800334                    |
| 1, 5          | 212.378                           | 173.098                         |
| 2, 3          | 0.4217362                         | 4.228311                        |
| 2, 4          | 100.1067                          | 187.7339                        |
| 2, 5          | 0.07824635                        | 5.421231                        |
| 3, 4          | 126.4295                          | 148.7312                        |
| 3, 5          | 1.664988                          | 0.000818873                     |
| 4, 5          | 230.9193                          | 186.8017                        |
| Total         | 880                               | 1018                            |

Source: Researcher's computation

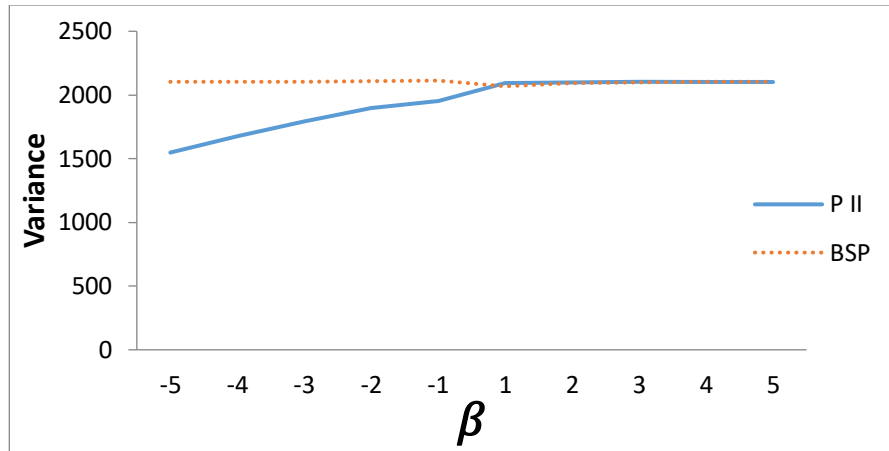


Figure 4.11: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data II)

Interpretation of Figure 4.11:

Figure 4.11 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

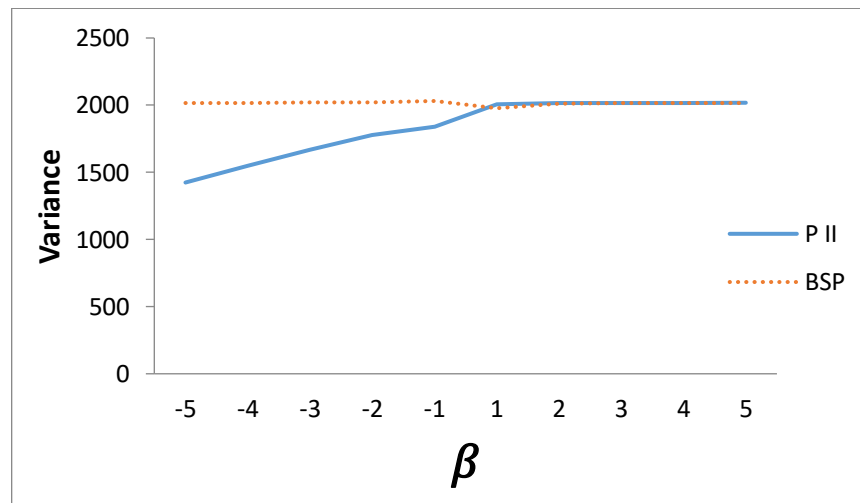


Figure 4.12: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data II)

Interpretation of Figure 4.12:

Figure 4.12 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

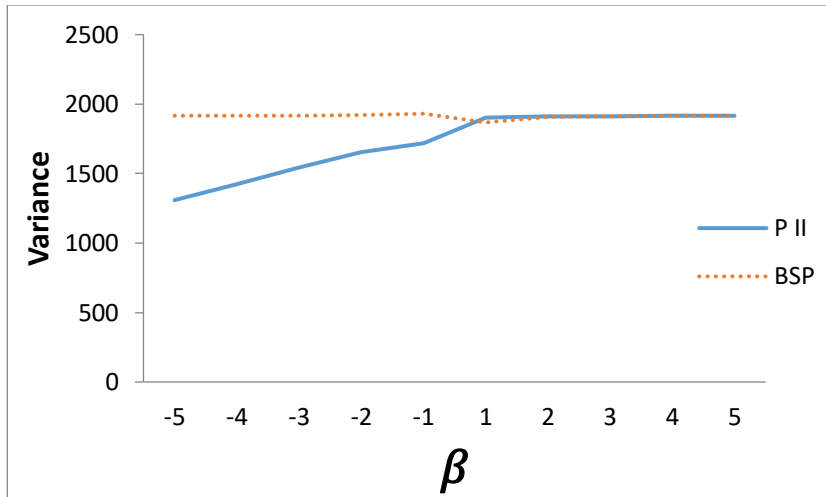


Figure 4.13: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data II)

Interpretation of Figure 4.13:

Figure 4.13 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

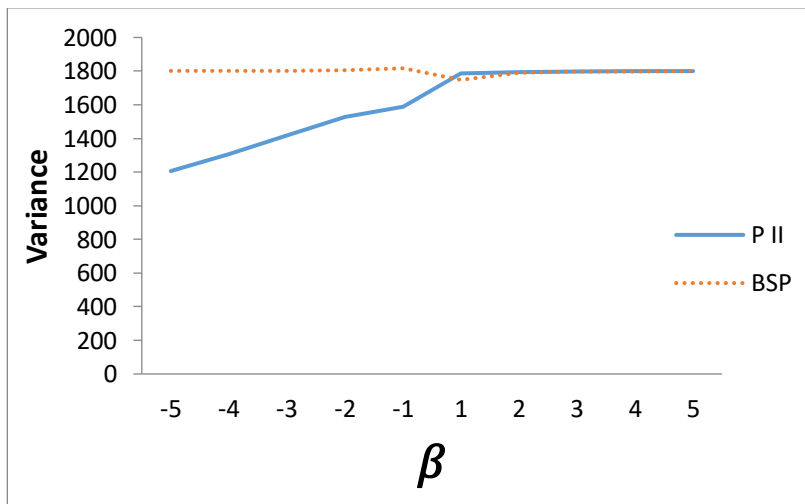


Figure 4.14: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data II)

Interpretation of Figure 4.14:

Figure 4.14 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

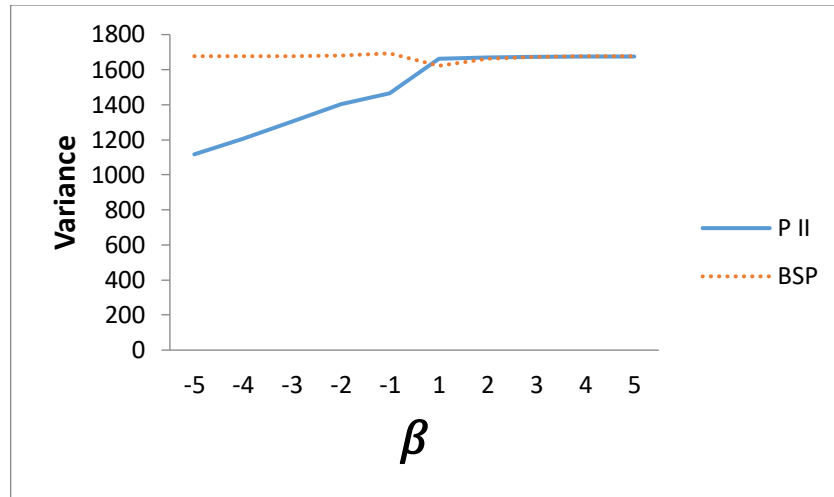


Figure 4.15: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data II)

Interpretation of Figure 4.15:

Figure 4.15 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

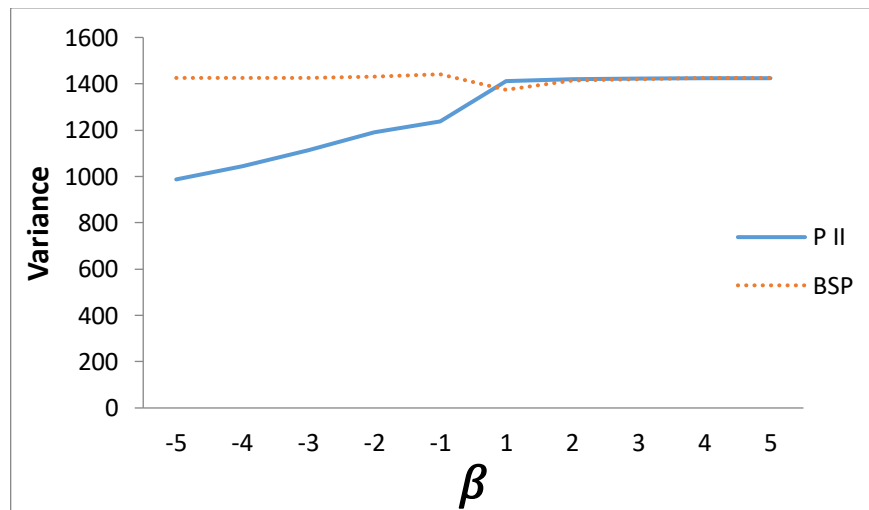


Figure 4.16: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data II)

Interpretation of Figure 4.16:

Figure 4.16 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

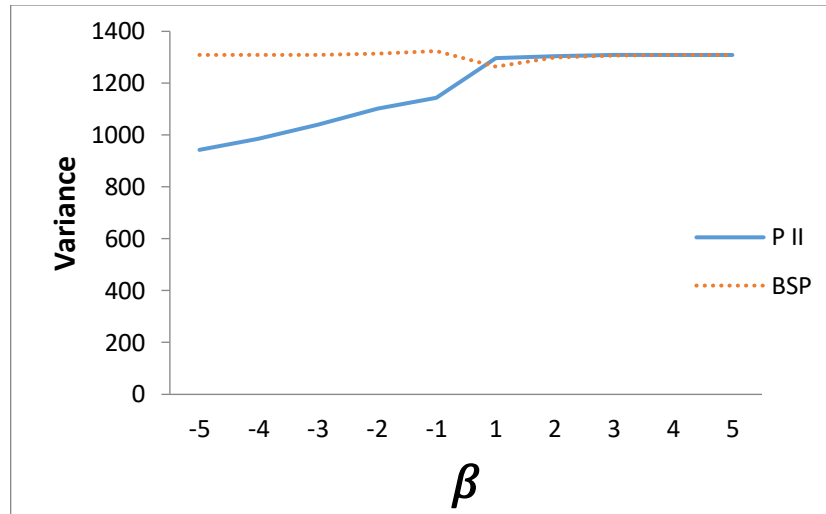


Figure 4.17: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data II)

Interpretation of Figure 4.17:

Figure 4.17 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

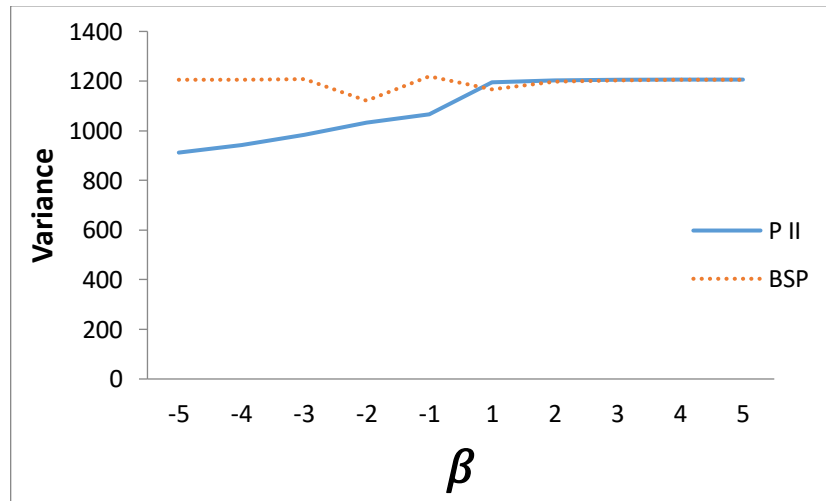


Figure 4.18: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data II)

Interpretation of Figure 4.18:

Figure 4.18 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

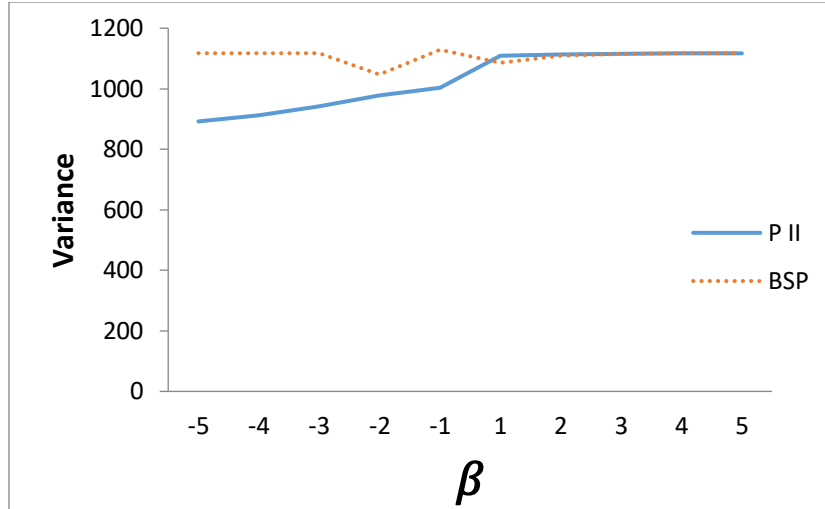


Figure 4.19: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data II)

Interpretation of Figure 4.19:

Figure 4.19 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 4$ .

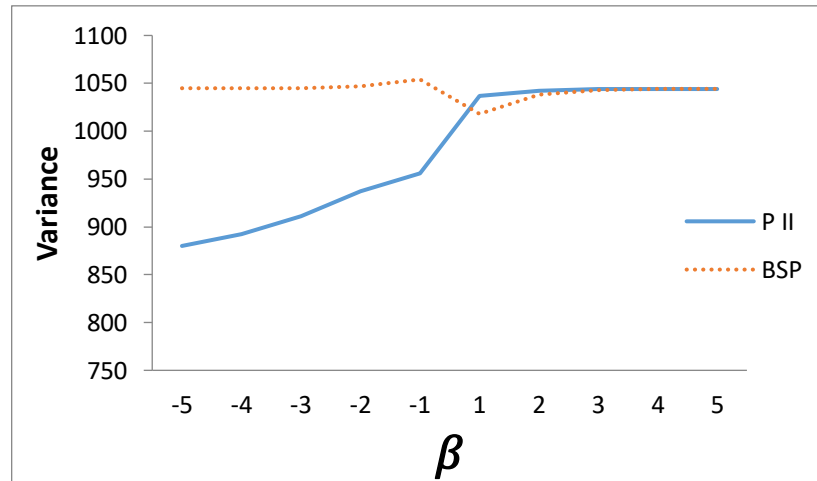


Figure 4.20: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data II)

Interpretation of Figure 4.20:

Figure 4.20 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

#### 4.1.2.3 Analysis of Data III

Variance using P I = 1527 (Data III: N=6)

Table 4.15: Comparison of variances of P II and BSP (Data III:  $N = 6$ )

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | -5       |      | -4   |      | -3   |      | -2   |      | -1   |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 1622     | 1992 | 1535 | 1937 | 1452 | 1871 | 1376 | 1794 | 1311 | 1710 |
|         | -4 | 1710     | 1992 | 1622 | 1937 | 1535 | 1871 | 1452 | 1794 | 1376 | 1710 |
|         | -3 | 1792     | 1993 | 1708 | 1938 | 1620 | 1871 | 1533 | 1795 | 1450 | 1711 |
|         | -2 | 1862     | 1994 | 1785 | 1939 | 1700 | 1873 | 1612 | 1797 | 1525 | 1713 |
|         | -1 | 1905     | 1998 | 1834 | 1944 | 1753 | 1878 | 1667 | 1803 | 1578 | 1720 |
|         | 1  | 1988     | 1977 | 1932 | 1918 | 1864 | 1849 | 1787 | 1770 | 1702 | 1684 |
|         | 2  | 1991     | 1989 | 1935 | 1933 | 1869 | 1866 | 1792 | 1789 | 1708 | 1705 |
|         | 3  | 1992     | 1992 | 1937 | 1936 | 1870 | 1870 | 1794 | 1793 | 1710 | 1709 |
|         | 4  | 1992     | 1992 | 1937 | 1937 | 1871 | 1870 | 1794 | 1794 | 1710 | 1710 |
|         | 5  | 1992     | 1992 | 1937 | 1937 | 1871 | 1871 | 1794 | 1794 | 1710 | 1710 |

Source: Researcher's computation

Table 4.15 continues

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | 1        |      | 2    |      | 3    |      | 4    |      | 5    |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 1210     | 1535 | 1175 | 1452 | 1148 | 1377 | 1129 | 1311 | 1117 | 1255 |
|         | -4 | 1255     | 1535 | 1210 | 1452 | 1175 | 1377 | 1148 | 1311 | 1129 | 1255 |
|         | -3 | 1309     | 1536 | 1254 | 1453 | 1209 | 1377 | 1174 | 1311 | 1148 | 1256 |
|         | -2 | 1368     | 1538 | 1304 | 1454 | 1249 | 1379 | 1205 | 1312 | 1171 | 1257 |
|         | -1 | 1413     | 1544 | 1342 | 1461 | 1282 | 1384 | 1231 | 1317 | 1192 | 1261 |
|         | 1  | 1527     | 1510 | 1445 | 1428 | 1370 | 1356 | 1305 | 1293 | 1251 | 1241 |
|         | 2  | 1533     | 1530 | 1450 | 1447 | 1375 | 1372 | 1309 | 1307 | 1254 | 1252 |
|         | 3  | 1534     | 1534 | 1452 | 1451 | 1376 | 1376 | 1310 | 1310 | 1255 | 1254 |
|         | 4  | 1535     | 1535 | 1452 | 1452 | 1376 | 1376 | 1311 | 1310 | 1255 | 1255 |
|         | 5  | 1452     | 1535 | 1377 | 1452 | 1311 | 1377 | 1255 | 1311 | 1255 | 1255 |

Source: Researcher's computation

Table 4.16: Probabilities of inclusion,  $\pi_i$  (Data III:  $N = 6$ )

| Units           | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|-----------------|-----------------------------------|---------------------------------|
| 1               | 0.8402729                         | 0.8430567                       |
| 2               | 0.8026601                         | 0.8102567                       |
| 3               | 0.8091681                         | 0.8196291                       |
| 4               | 0.8024117                         | 0.8097449                       |
| 5               | 0.8539902                         | 0.8503792                       |
| 6               | 0.891497                          | 0.8669334                       |
| Sample size (n) | 5                                 | 5                               |

Source: Researcher's computation

Table 4.17: Joint Probabilities of inclusion,  $\pi_{ij}$  (Data III:  $N = 6$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 0.642933                          | 0.6533134                       |
| 1, 3          | 0.649441                          | 0.6626859                       |
| 1, 4          | 0.6426846                         | 0.6528016                       |
| 1, 5          | 0.6942631                         | 0.6934359                       |
| 1, 6          | 0.7317699                         | 0.7099901                       |
| 2, 3          | 0.6118282                         | 0.6298858                       |
| 2, 4          | 0.6050718                         | 0.6200016                       |
| 2, 5          | 0.6566503                         | 0.6606359                       |
| 2, 6          | 0.6941571                         | 0.6771901                       |
| 3, 4          | 0.6115798                         | 0.629374                        |
| 3, 5          | 0.6631583                         | 0.6700083                       |
| 3, 6          | 0.7006651                         | 0.6865625                       |
| 4, 5          | 0.6564019                         | 0.6601241                       |
| 4, 6          | 0.6939086                         | 0.6766782                       |
| 5, 6          | 0.7454872                         | 0.7173125                       |
| Total         | 10                                | 10                              |

Source: Researcher's computation

Table 4.18: Least variance estimates (Data III:  $N = 6$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 77.06197                          | 73.92682                        |
| 1, 3          | 18.10284                          | 18.73532                        |
| 1, 4          | 39.58492                          | 38.67936                        |
| 1, 5          | 62.19828                          | 66.91426                        |
| 1, 6          | 4.001056                          | 10.13417                        |
| 2, 3          | 24.63823                          | 20.30141                        |
| 2, 4          | 8.162849                          | 7.29413                         |
| 2, 5          | 299.9354                          | 308.4729                        |
| 2, 6          | 85.48718                          | 129.0568                        |
| 3, 4          | 4.723007                          | 3.610296                        |
| 3, 5          | 165.0161                          | 172.253                         |
| 3, 6          | 31.47778                          | 54.55572                        |
| 4, 5          | 224.1998                          | 233.1617                        |
| 4, 6          | 52.92747                          | 84.91623                        |
| 5, 6          | 19.16733                          | 18.60565                        |
| Total         | 1117                              | 1241                            |

Source: Researcher's computation

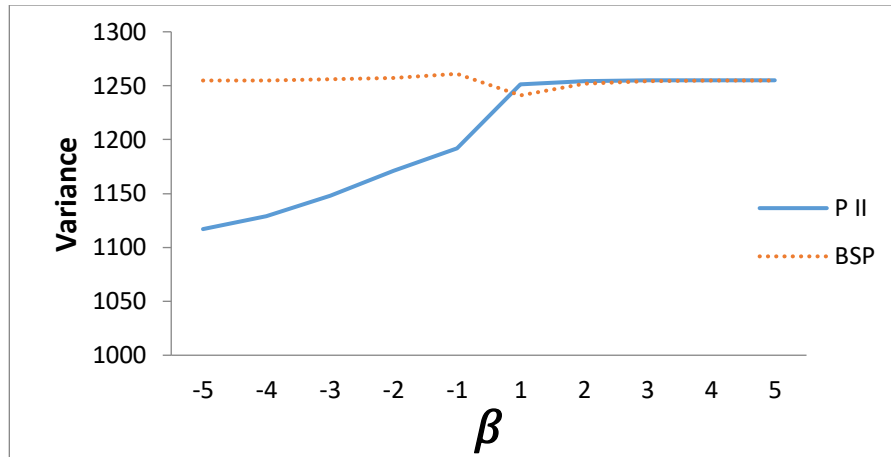


Figure 4.21: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data III)

Interpretation of Figure 4.21:

Figure 4.21 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

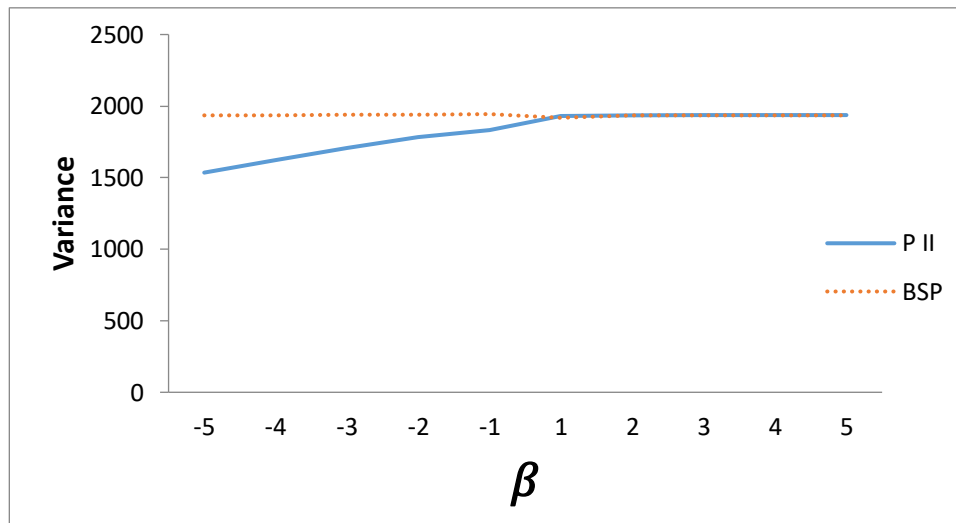


Figure 4.22: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data III)

Interpretation of Figure 4.22:

Figure 4.22 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 4$ .

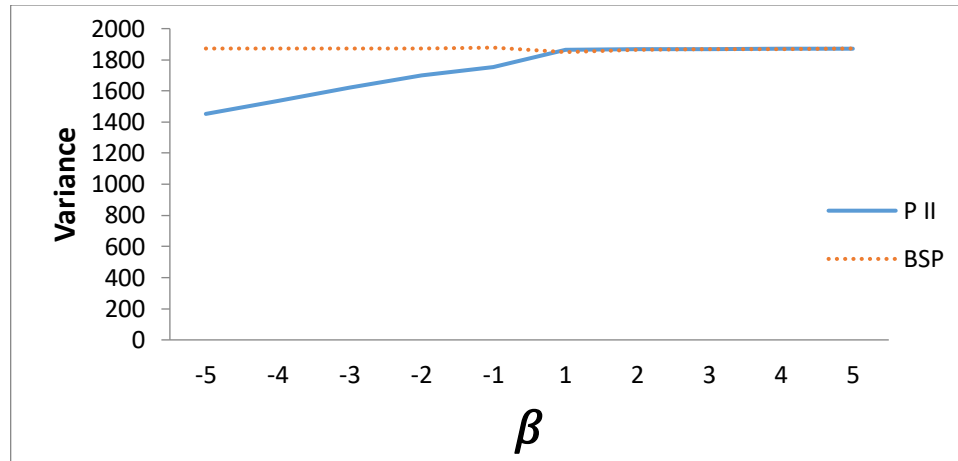


Figure 4.23: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data III)

Interpretation of Figure 4.23:

Figure 4.23 shows that P II has lower variances than BSP when  $\beta - 5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

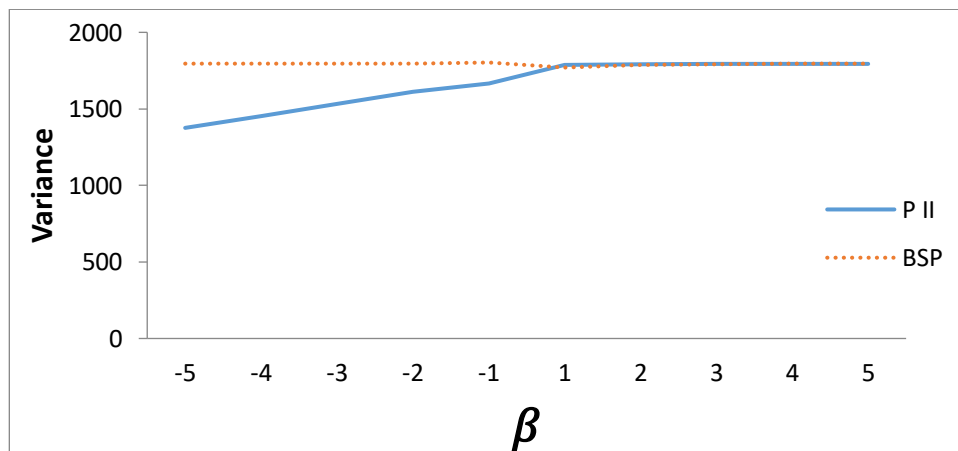


Figure 4.24: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data III)

Interpretation of Figure 4.24:

Figure 4.24 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

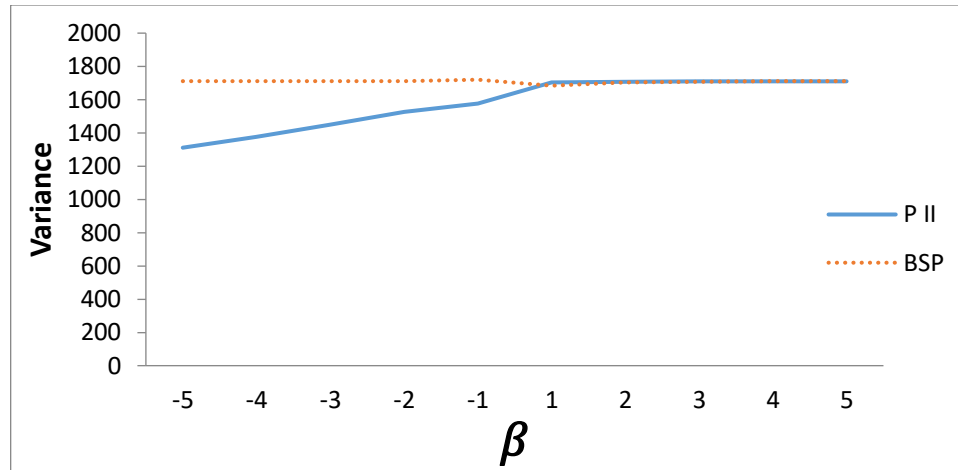


Figure 4.25: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data III)

Interpretation of Figure 4.25:

Figure 4.25 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

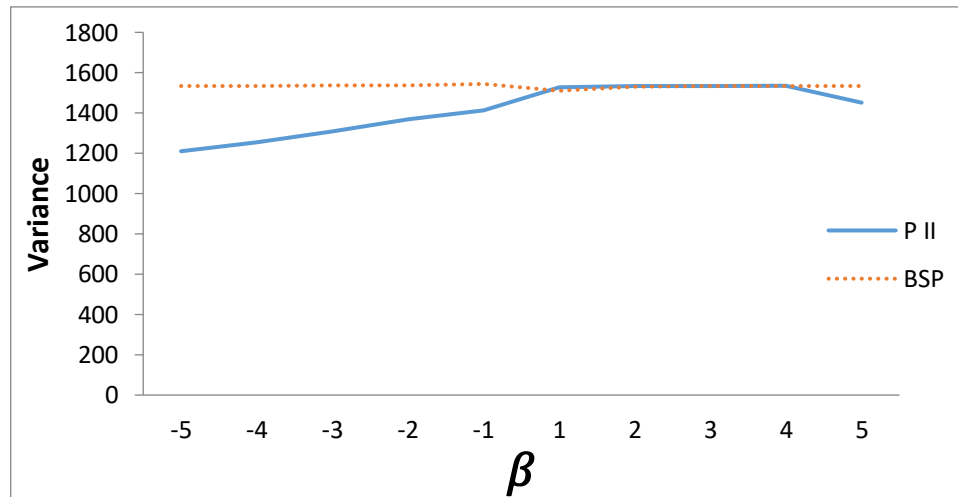


Figure 4.26: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data III)

Interpretation of Figure 4.26:

Figure 4.26 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, while the variance of P II reduces further, for  $\alpha = 1$ .

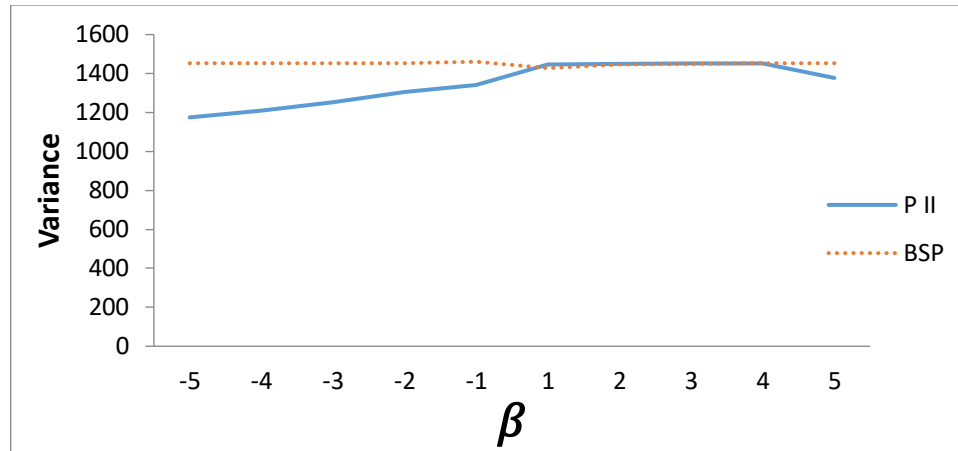


Figure 4.27: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data III)

Interpretation of Figure 4.27:

Figure 4.27 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, while the variance of P II reduces further, for  $\alpha = 2$ .

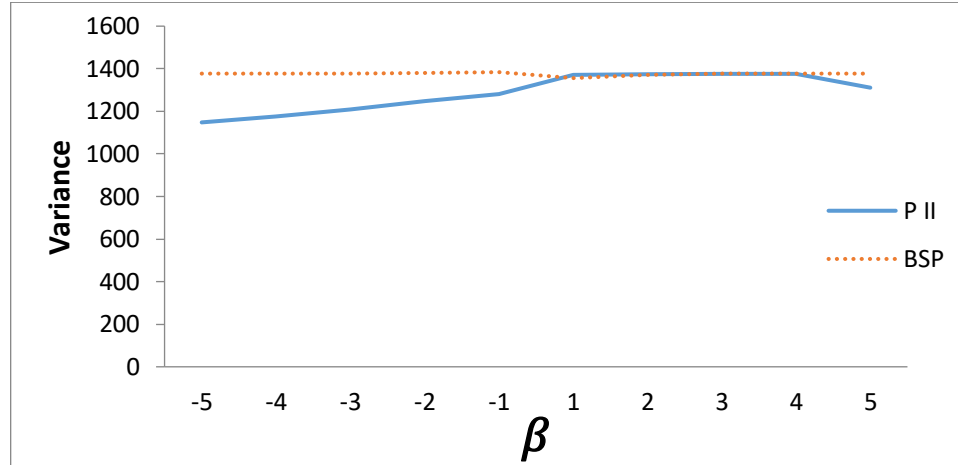


Figure 4.28: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data III)

Interpretation of Figure 4.28:

Figure 4.28 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, while the variance of P II reduces further, for  $\alpha = 3$ .

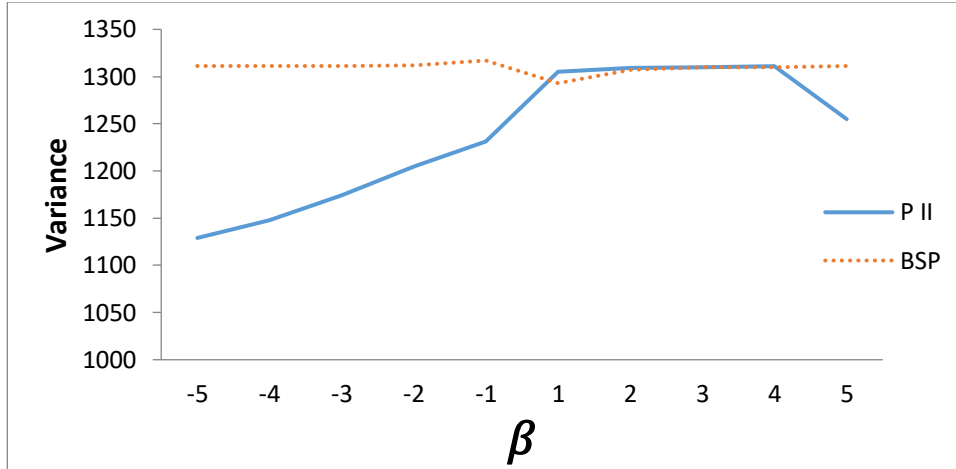


Figure 4.29: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data III)

Interpretation of Figure 4.29:

Figure 4.29 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, while the variance of P II reduces further, for  $\alpha = 4$ .

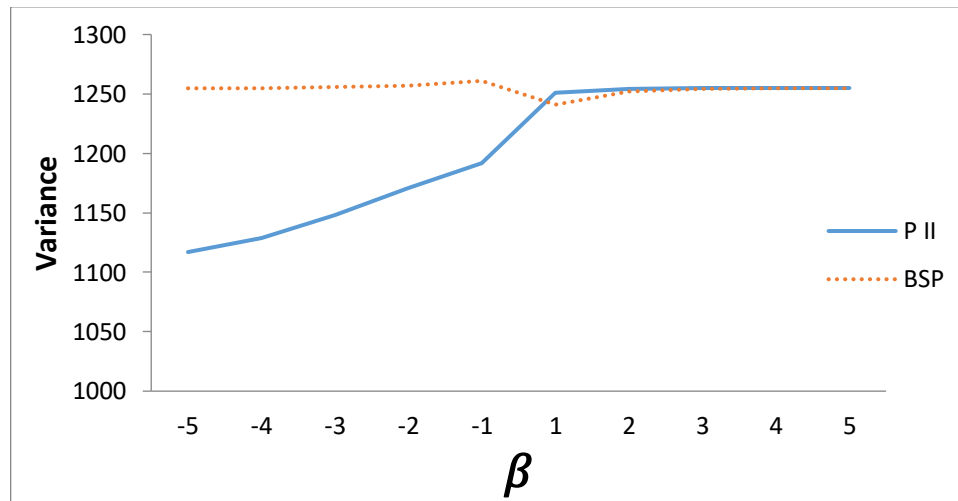


Figure 4.30: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data III)

Interpretation of Figure 4.30:

Figure 4.30 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

#### 4.1.2.4 Analysis of Data IV

Variance using P I = 561 (Data IV: N=7)

Table 4.19: Comparison of variances of P II and BSP (Data IV:  $N = 7$ )

|         |    | $\alpha$ |     |      |     |      |     |      |     |      |     |
|---------|----|----------|-----|------|-----|------|-----|------|-----|------|-----|
|         |    | -5       |     | -4   |     | -3   |     | -2   |     | -1   |     |
| $\beta$ |    | P II     | BSP | P II | BSP | P II | BSP | P II | BSP | P II | BSP |
|         | -5 | 570      | 603 | 562  | 598 | 553  | 592 | 544  | 586 | 535  | 578 |
|         | -4 | 578      | 603 | 570  | 598 | 562  | 592 | 553  | 586 | 544  | 578 |
|         | -3 | 586      | 603 | 578  | 598 | 570  | 592 | 562  | 586 | 553  | 578 |
|         | -2 | 592      | 603 | 585  | 598 | 578  | 593 | 570  | 586 | 561  | 579 |
|         | -1 | 596      | 603 | 590  | 599 | 583  | 593 | 575  | 586 | 567  | 579 |
|         | 1  | 603      | 602 | 598  | 597 | 592  | 591 | 585  | 584 | 578  | 577 |
|         | 2  | 603      | 603 | 598  | 598 | 592  | 592 | 586  | 585 | 578  | 578 |
|         | 3  | 603      | 603 | 598  | 598 | 592  | 592 | 586  | 586 | 578  | 578 |
|         | 4  | 603      | 603 | 598  | 598 | 592  | 592 | 586  | 586 | 578  | 578 |
|         | 5  | 603      | 603 | 598  | 598 | 592  | 592 | 586  | 586 | 578  | 578 |

Source: Researcher's computation

Table 4.19 continues

|         |    | $\alpha$ |     |      |     |      |     |      |     |      |     |
|---------|----|----------|-----|------|-----|------|-----|------|-----|------|-----|
|         |    | 1        |     | 2    |     | 3    |     | 4    |     | 5    |     |
| $\beta$ |    | P II     | BSP | P II | BSP | P II | BSP | P II | BSP | P II | BSP |
|         | -5 | 516      | 562 | 506  | 553 | 497  | 544 | 488  | 535 | 479  | 525 |
|         | -4 | 525      | 562 | 516  | 553 | 506  | 544 | 497  | 535 | 488  | 525 |
|         | -3 | 534      | 562 | 525  | 553 | 515  | 544 | 506  | 535 | 497  | 525 |
|         | -2 | 543      | 562 | 534  | 553 | 524  | 544 | 515  | 535 | 505  | 525 |
|         | -1 | 549      | 563 | 540  | 554 | 531  | 545 | 521  | 535 | 512  | 526 |
|         | 1  | 561      | 560 | 552  | 551 | 543  | 542 | 534  | 532 | 524  | 523 |
|         | 2  | 562      | 562 | 553  | 553 | 544  | 544 | 534  | 534 | 525  | 525 |
|         | 3  | 562      | 562 | 553  | 553 | 544  | 544 | 535  | 534 | 525  | 525 |
|         | 4  | 562      | 562 | 553  | 553 | 544  | 544 | 535  | 535 | 525  | 525 |
|         | 5  | 562      | 562 | 553  | 553 | 544  | 544 | 535  | 535 | 525  | 525 |

Source: Researcher's computation

Table 4.20: Probability of inclusion,  $\pi_i$  (Data IV)

| Units           | P II( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|-----------------|----------------------------------|---------------------------------|
| 1               | 0.83898                          | 0.8451027                       |
| 2               | 0.8460511                        | 0.8512921                       |
| 3               | 0.8550673                        | 0.857097                        |
| 4               | 0.8472673                        | 0.8521688                       |
| 5               | 0.8626661                        | 0.8611414                       |
| 6               | 0.8665982                        | 0.86304                         |
| 7               | 0.88337                          | 0.8701579                       |
| Sample size (n) | 6                                | 6                               |

Source: Researcher's computation

Table 4.21: Joint Probability of inclusion  $\pi_{ij}$  (Data IV)

| Units Sampled | P II( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|----------------------------------|---------------------------------|
| 1, 2          | 0.6850312                        | 0.6963948                       |
| 1, 3          | 0.6940473                        | 0.7021997                       |
| 1, 4          | 0.6862474                        | 0.6972715                       |
| 1, 5          | 0.7016461                        | 0.7062441                       |
| 1, 6          | 0.7055782                        | 0.7081427                       |
| 1, 7          | 0.72235                          | 0.7152606                       |
| 2, 3          | 0.7011184                        | 0.7083891                       |
| 2, 4          | 0.6933185                        | 0.7034609                       |
| 2, 5          | 0.7087172                        | 0.7124335                       |
| 2, 6          | 0.7126493                        | 0.7143321                       |
| 2, 7          | 0.7294211                        | 0.72145                         |
| 3, 4          | 0.7023346                        | 0.7092659                       |
| 3, 5          | 0.7177333                        | 0.7182385                       |
| 3, 6          | 0.7216654                        | 0.7201371                       |
| 3, 7          | 0.7384373                        | 0.7272549                       |
| 4, 5          | 0.7099334                        | 0.7133102                       |
| 4, 6          | 0.7138655                        | 0.7152089                       |
| 4, 7          | 0.7306373                        | 0.7223267                       |
| 5, 6          | 0.7292642                        | 0.7241815                       |
| 5, 7          | 0.746036                         | 0.7312993                       |
| 6, 7          | 0.7499681                        | 0.7331979                       |
| Total         | 15                               | 15                              |

Source: Researcher's computation

Table 4.22: Least variance estimates (Data IV)

| Units Sampled | P II( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|----------------------------------|---------------------------------|
| 1, 2          | 6.79716                          | 6.193011                        |
| 1, 3          | 31.27943                         | 29.72315                        |
| 1, 4          | 13.63384                         | 12.45956                        |
| 1, 5          | 39.77128                         | 39.69422                        |
| 1, 6          | 72.94578                         | 74.30393                        |
| 1, 7          | 88.22722                         | 102.6572                        |
| 2, 3          | 8.976271                         | 8.69371                         |
| 2, 4          | 1.140672                         | 1.049416                        |
| 2, 5          | 14.20084                         | 14.56765                        |
| 2, 6          | 35.79695                         | 37.19587                        |
| 2, 7          | 48.85269                         | 58.46588                        |
| 3, 4          | 3.810292                         | 3.755415                        |
| 3, 5          | 0.7332039                        | 0.8042168                       |
| 3, 6          | 9.119637                         | 9.909919                        |
| 3, 7          | 17.63317                         | 22.5286                         |
| 4, 5          | 7.581205                         | 7.952997                        |
| 4, 6          | 24.74576                         | 26.03146                        |
| 4, 7          | 36.60806                         | 44.54834                        |
| 5, 6          | 4.52875                          | 4.928989                        |
| 5, 7          | 11.09356                         | 14.58229                        |
| 6, 7          | 1.796104                         | 2.735598                        |
| Total         | 479                              | 523                             |

Source: Researcher's computation

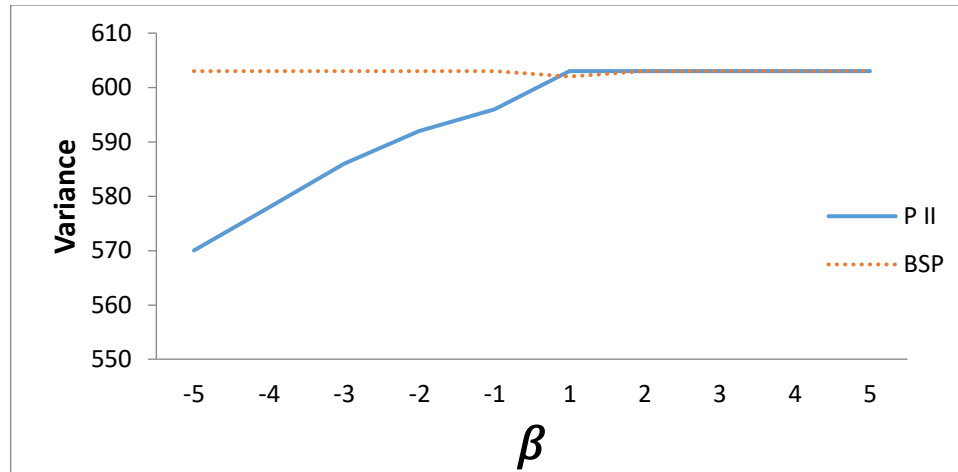


Figure 4.31: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data IV)

Interpretation of Figure 4.31:

Figure 4.31 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

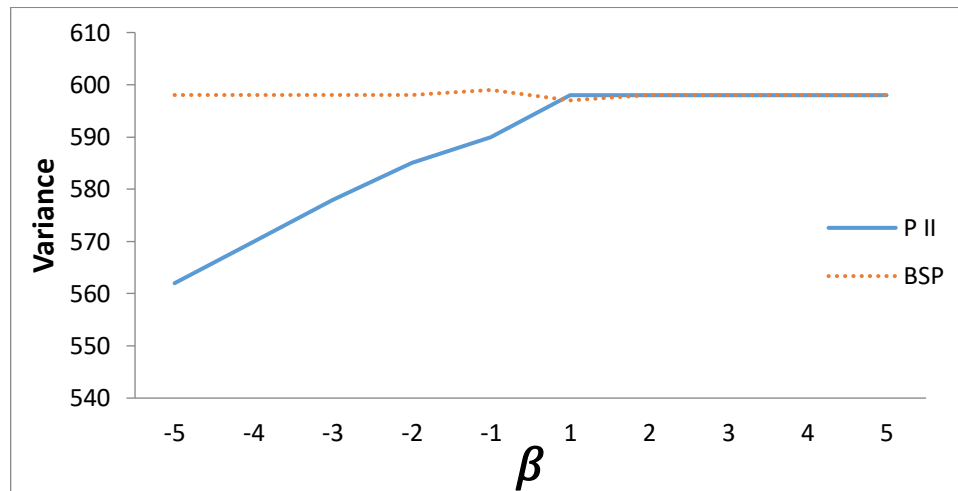


Figure 4.32: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data VI)

Interpretation of Figure 4.32:

Figure 4.32 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

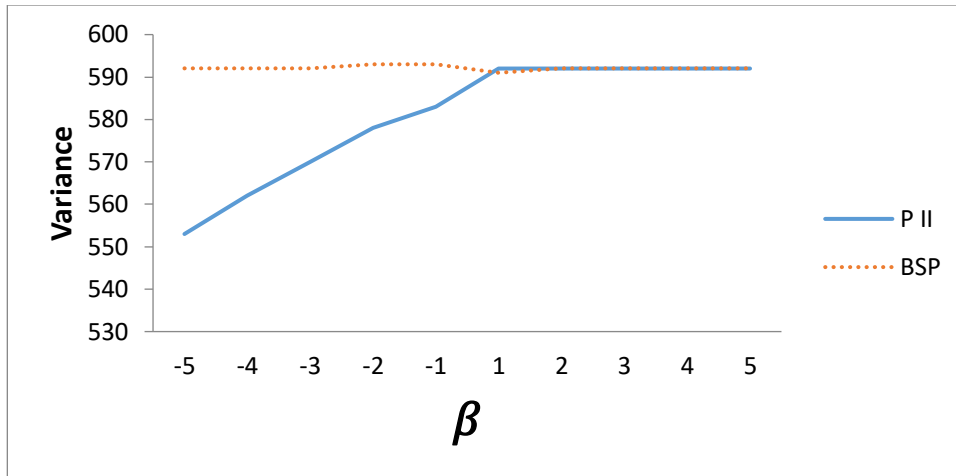


Figure 4.33: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data IV)

Interpretation of Figure 4.33:

Figure 4.33 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

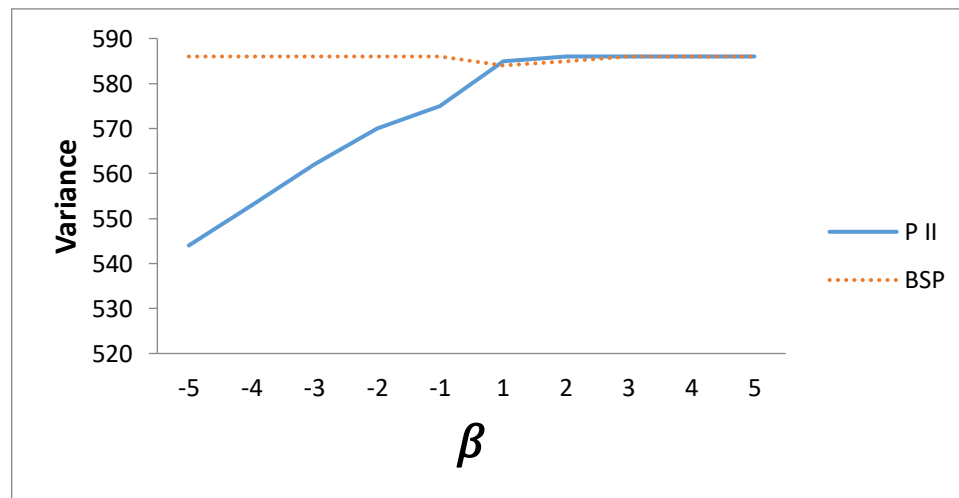


Figure 4.34: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data IV)

Interpretation of Figure 4.34:

Figure 4.34 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

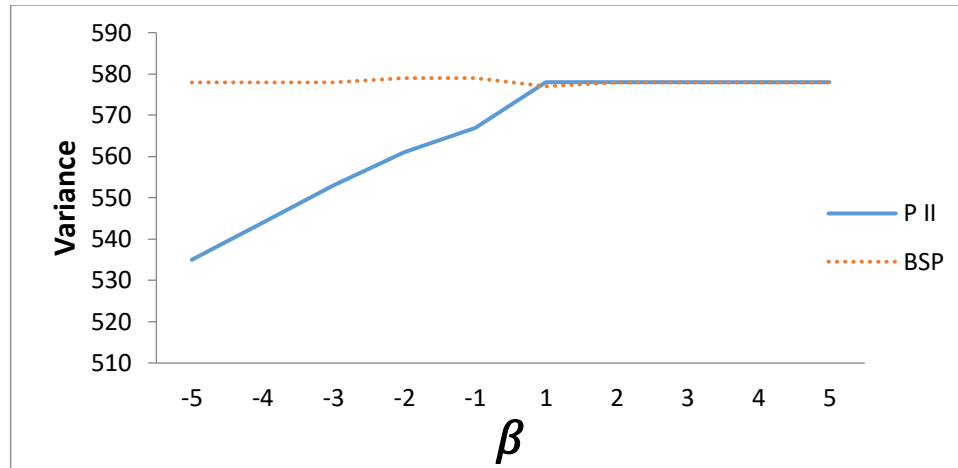


Figure 4.35: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data IV)

Interpretation of Figure 4.35:

Figure 4.35 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

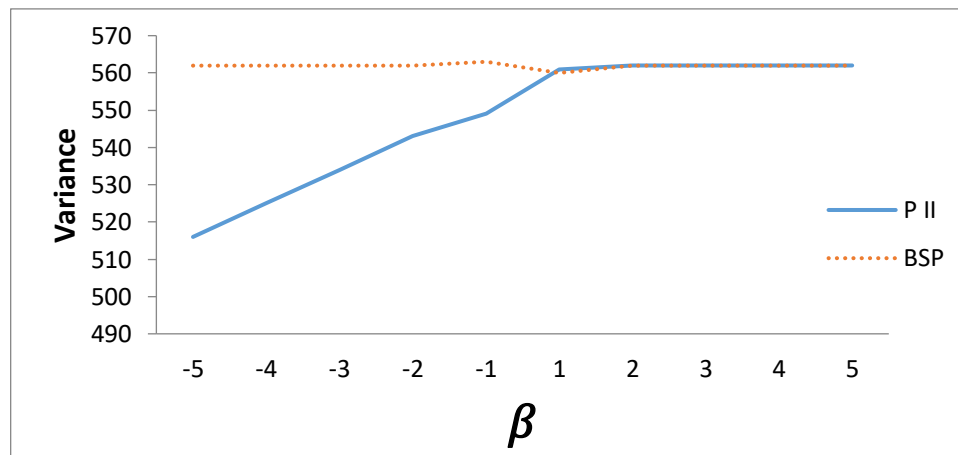


Figure 4.36: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data IV)

Interpretation of Figure 4.36:

Figure 4.36 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

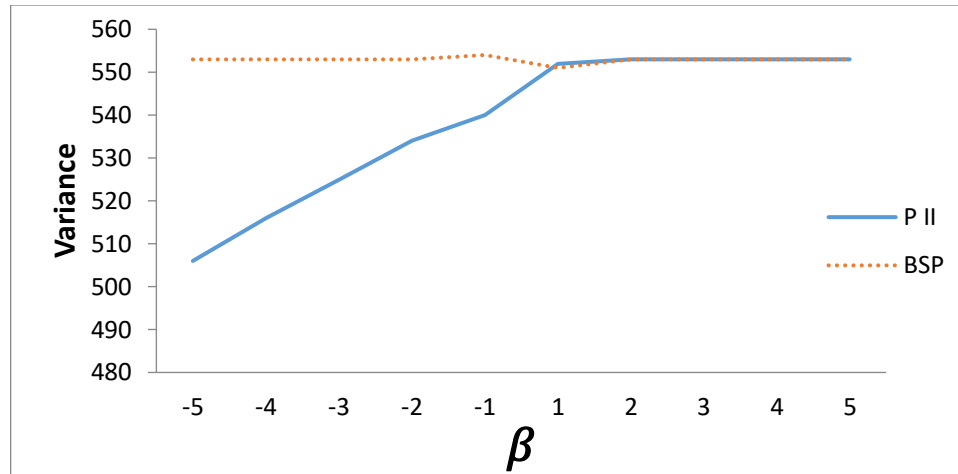


Figure 4.37: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data IV)

Interpretation of Figure 4.37:

Figure 4.37 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

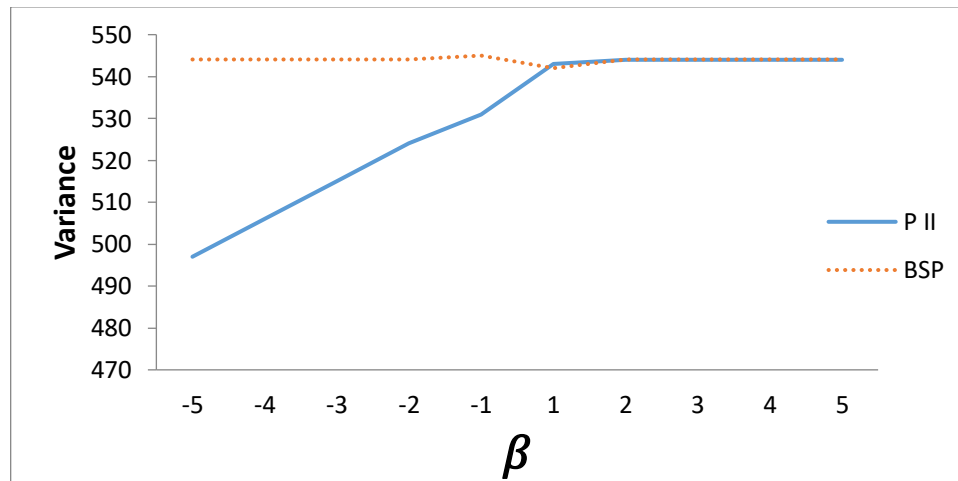


Figure 4.38: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data IV)

Interpretation of Figure 4.38:

Figure 4.38 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

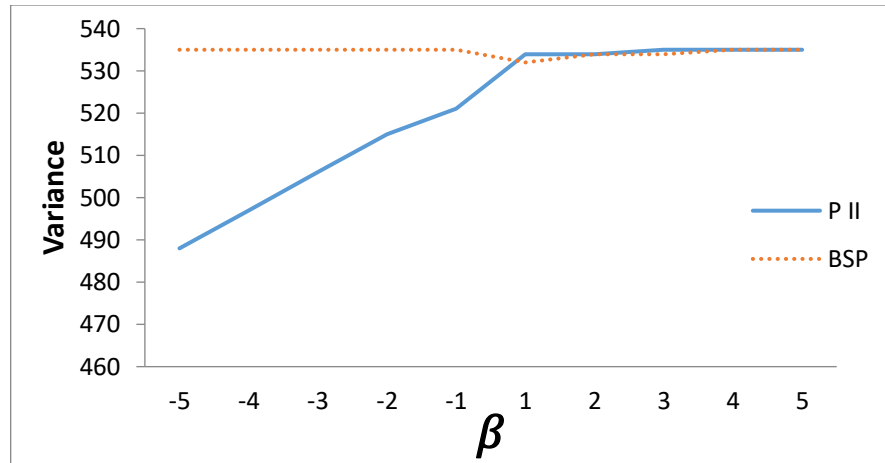


Figure 4.39: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data IV)

Interpretation of Figure 4.39:

Figure 4.39 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 4$ .

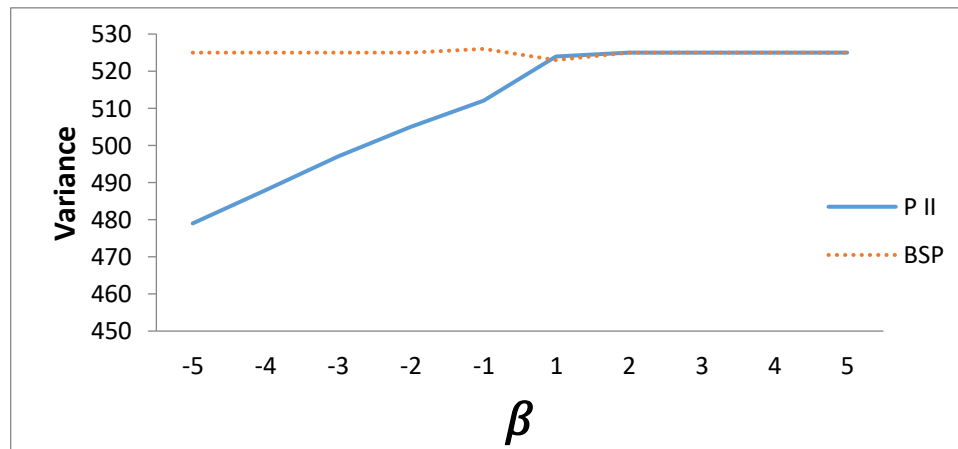


Figure 4.40: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data IV)

Interpretation of Figure 4.40:

Figure 4.40 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

#### 4.1.2.5 Analysis of Data V

Variance using P I = 1920 (Data V: N=8)

Table 4.23: Comparison of variances of PP II and BSP (Data V:  $N = 8$ )

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | -5       |      | -4   |      | -3   |      | -2   |      | -1   |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 1961     | 2181 | 1923 | 2149 | 1885 | 2115 | 1848 | 2080 | 1811 | 2042 |
|         | -4 | 1998     | 2172 | 1961 | 2141 | 1923 | 2107 | 1885 | 2071 | 1848 | 2034 |
|         | -3 | 2034     | 2163 | 1998 | 2132 | 1960 | 2098 | 1923 | 2063 | 1885 | 2026 |
|         | -2 | 2068     | 2155 | 2032 | 2123 | 1996 | 2090 | 1958 | 2054 | 1921 | 2018 |
|         | -1 | 2093     | 2147 | 2058 | 2116 | 2023 | 2083 | 1986 | 2048 | 1948 | 2011 |
|         | 1  | 2134     | 2132 | 2102 | 2100 | 2068 | 2066 | 2032 | 2031 | 1996 | 1994 |
|         | 2  | 2135     | 2134 | 2103 | 2102 | 2069 | 2069 | 2034 | 2034 | 1998 | 1997 |
|         | 3  | 2136     | 2134 | 2104 | 2103 | 2070 | 2069 | 2035 | 2034 | 1998 | 1998 |
|         | 4  | 2136     | 2134 | 2104 | 2103 | 2070 | 2069 | 2035 | 2034 | 1998 | 1998 |
|         | 5  | 2136     | 2134 | 2104 | 2103 | 2070 | 2069 | 2035 | 2034 | 1998 | 1998 |

Source: Researcher's computation

Table 4.23 continues

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | 1        |      | 2    |      | 3    |      | 4    |      | 5    |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 1741     | 1964 | 1709 | 1924 | 1678 | 1885 | 1649 | 1846 | 1623 | 1808 |
|         | -4 | 1775     | 1957 | 1741 | 1917 | 1709 | 1878 | 1678 | 1839 | 1649 | 1802 |
|         | -3 | 1811     | 1949 | 1775 | 1910 | 1741 | 1871 | 1708 | 1833 | 1678 | 1796 |
|         | -2 | 1845     | 1941 | 1809 | 1903 | 1773 | 1864 | 1739 | 1826 | 1707 | 1790 |
|         | -1 | 1872     | 1936 | 1835 | 1897 | 1799 | 1859 | 1764 | 1822 | 1730 | 1785 |
|         | 1  | 1920     | 1919 | 1883 | 1881 | 1845 | 1843 | 1809 | 1806 | 1773 | 1770 |
|         | 2  | 1922     | 1922 | 1885 | 1885 | 1847 | 1847 | 1810 | 1810 | 1775 | 1774 |
|         | 3  | 1923     | 1923 | 1885 | 1885 | 1848 | 1848 | 1811 | 1811 | 1775 | 1775 |
|         | 4  | 1923     | 1923 | 1885 | 1885 | 1848 | 1848 | 1811 | 1811 | 1775 | 1775 |
|         | 5  | 1923     | 1923 | 1885 | 1885 | 1848 | 1848 | 1811 | 1811 | 1775 | 1775 |

Source: Researcher's computation

Table 4.24: Probabilities of inclusion,  $\pi_i$  (Data V:  $N = 8$ )

| Units           | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|-----------------|-----------------------------------|---------------------------------|
| 1               | 0.8796437                         | 0.8788769                       |
| 2               | 0.8655612                         | 0.8701888                       |
| 3               | 0.8597218                         | 0.864225                        |
| 4               | 0.8873716                         | 0.8824913                       |
| 5               | 0.9034791                         | 0.8888262                       |
| 6               | 0.8737418                         | 0.8756962                       |
| 7               | 0.8634198                         | 0.8683505                       |
| 8               | 0.8670609                         | 0.8713451                       |
| Sample size (n) | 7                                 | 7                               |

Source: Researcher's computation

Table 4.25: Joint Probabilities of inclusion,  $\pi_{ij}$  (Data V:  $N = 8$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 0.745205                          | 0.7490657                       |
| 1, 3          | 0.7393656                         | 0.743102                        |
| 1, 4          | 0.7670153                         | 0.7613682                       |
| 1, 5          | 0.7831229                         | 0.7677031                       |
| 1, 6          | 0.7533855                         | 0.7545731                       |
| 1, 7          | 0.7430636                         | 0.7472274                       |
| 1, 8          | 0.7467047                         | 0.7502221                       |
| 2, 3          | 0.7252831                         | 0.7344138                       |
| 2, 4          | 0.7529328                         | 0.7526801                       |
| 2, 5          | 0.7690403                         | 0.7590149                       |
| 2, 6          | 0.739303                          | 0.745885                        |
| 2, 7          | 0.728981                          | 0.7385393                       |
| 2, 8          | 0.7326221                         | 0.7415339                       |
| 3, 4          | 0.7470934                         | 0.7467163                       |
| 3, 5          | 0.763201                          | 0.7530512                       |
| 3, 6          | 0.7334636                         | 0.7399212                       |
| 3, 7          | 0.7231417                         | 0.7325755                       |
| 3, 8          | 0.7267828                         | 0.7355702                       |
| 4, 5          | 0.7908507                         | 0.7713174                       |
| 4, 6          | 0.7611133                         | 0.7581875                       |
| 4, 7          | 0.7507914                         | 0.7508418                       |
| 4, 8          | 0.7544325                         | 0.7536356                       |
| 5, 6          | 0.7772209                         | 0.7645223                       |
| 5, 7          | 0.7668989                         | 0.7571766                       |
| 5, 8          | 0.77054                           | 0.7601713                       |
| 6, 7          | 0.7371616                         | 0.7440467                       |
| 6, 8          | 0.7408027                         | 0.7470413                       |
| 7, 8          | 0.7304807                         | 0.7396956                       |
| Total         | 21                                | 21                              |

Source: Researcher's Computation

Table 4.26: Least variance estimates (Data V:  $N = 8$ )

| Units Sampled | PP II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|------------------------------------|---------------------------------|
| 1, 2          | 70.74403                           | 70.409                          |
| 1, 3          | 34.32276                           | 34.85894                        |
| 1, 4          | 26.83313                           | 29.96826                        |
| 1, 5          | 6.294086                           | 9.946818                        |
| 1, 6          | 11.55527                           | 11.93127                        |
| 1, 7          | 99.86739                           | 98.71558                        |
| 1, 8          | 51.92501                           | 52.03417                        |
| 2, 3          | 8.71799                            | 7.974663                        |
| 2, 4          | 185.5121                           | 194.4595                        |
| 2, 5          | 101.8127                           | 127.008                         |
| 2, 6          | 25.22341                           | 24.22513                        |
| 2, 7          | 2.573757                           | 2.359018                        |
| 2, 8          | 1.504586                           | 1.393456                        |
| 3, 4          | 127.7899                           | 135.8127                        |
| 3, 5          | 64.03868                           | 80.82858                        |
| 3, 6          | 5.313974                           | 5.293921                        |
| 3, 7          | 21.39819                           | 19.57718                        |
| 3, 8          | 2.805905                           | 2.550361                        |
| 4, 5          | 4.637092                           | 4.458778                        |
| 4, 6          | 73.92074                           | 80.12902                        |
| 4, 7          | 230.8823                           | 240.5522                        |
| 4, 8          | 154.3624                           | 164.9739                        |
| 5, 6          | 31.09653                           | 42.111                          |
| 5, 7          | 132.4751                           | 162.9924                        |
| 5, 8          | 81.19053                           | 102.6572                        |
| 6, 7          | 43.76544                           | 41.73357                        |
| 6, 8          | 14.49336                           | 14.02704                        |
| 7, 8          | 8.01931                            | 7.388123                        |
| Total         | 1623.076                           | 1770.37                         |

Source: Researcher's computation

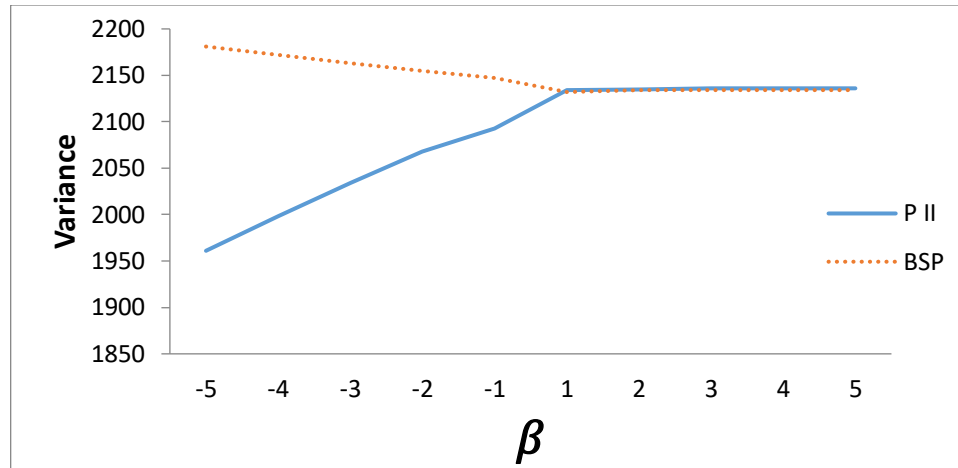


Figure 4.41: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data V)

Interpretation of Figure 4.41:

Figure 4.41 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

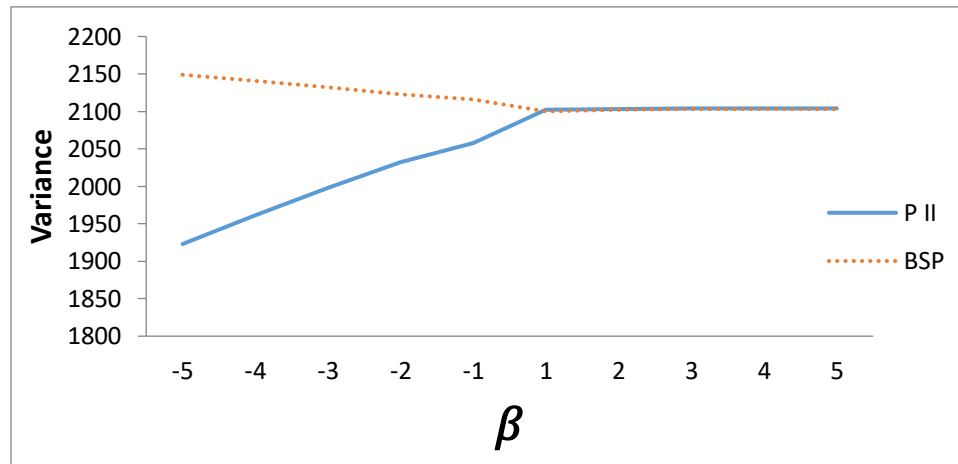


Figure 4.42: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data V)

Interpretation of Figure 4.42:

Figure 4.42 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

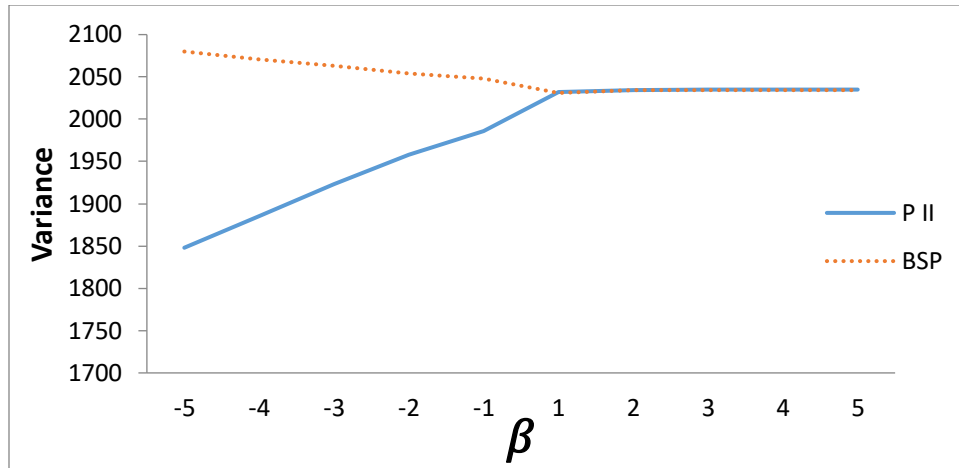


Figure 4.43: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data V)

Interpretation of Figure 4.43:

Figure 4.43 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

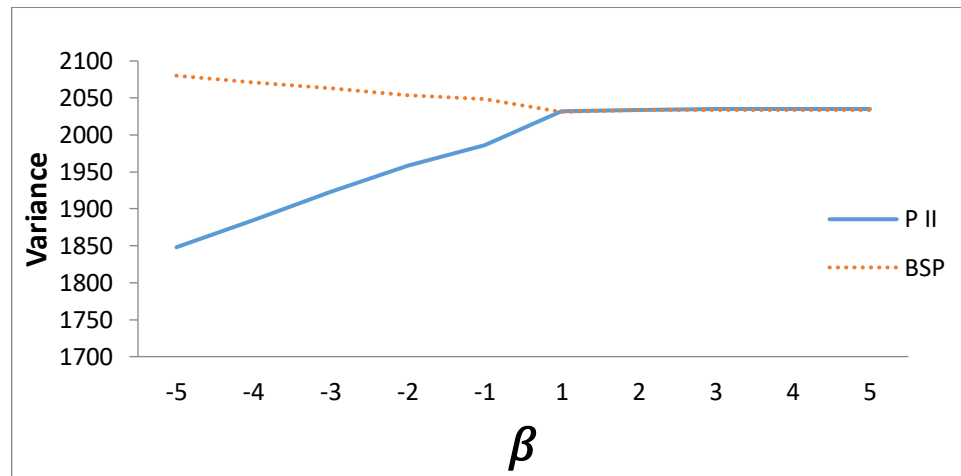


Figure 4.44: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data V)

Interpretation of Figure 4.44:

Figure 4.44 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

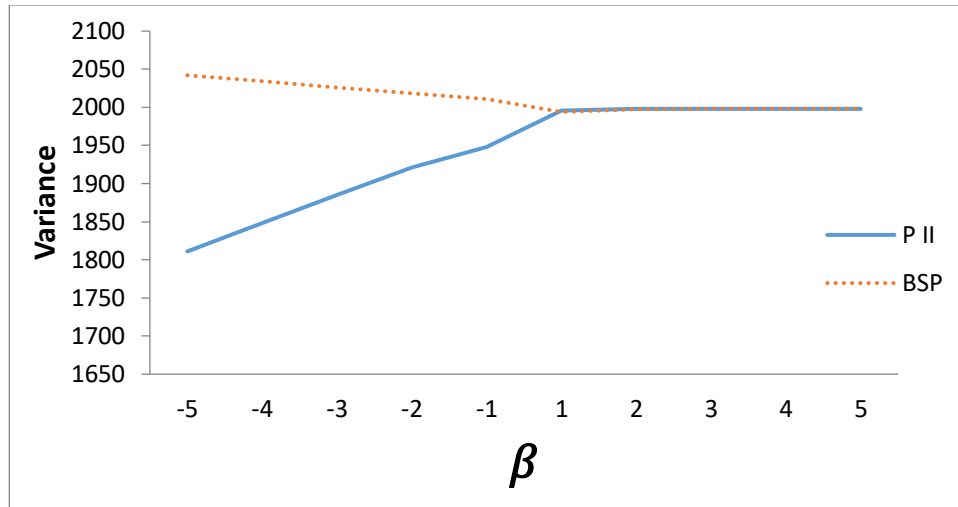


Figure 4.45: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data V)

Interpretation of Figure 4.45:

Figure 4.45 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

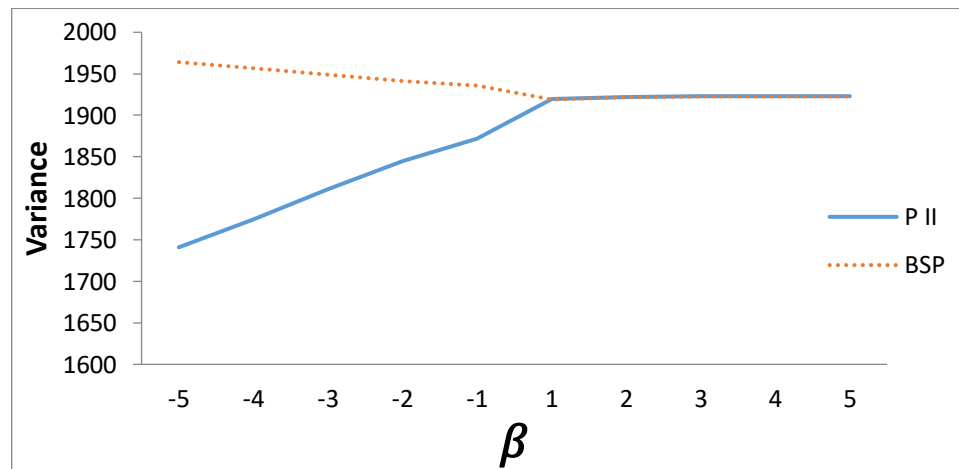


Figure 4.46: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data V)

Interpretation of Figure 4.46:

Figure 4.46 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

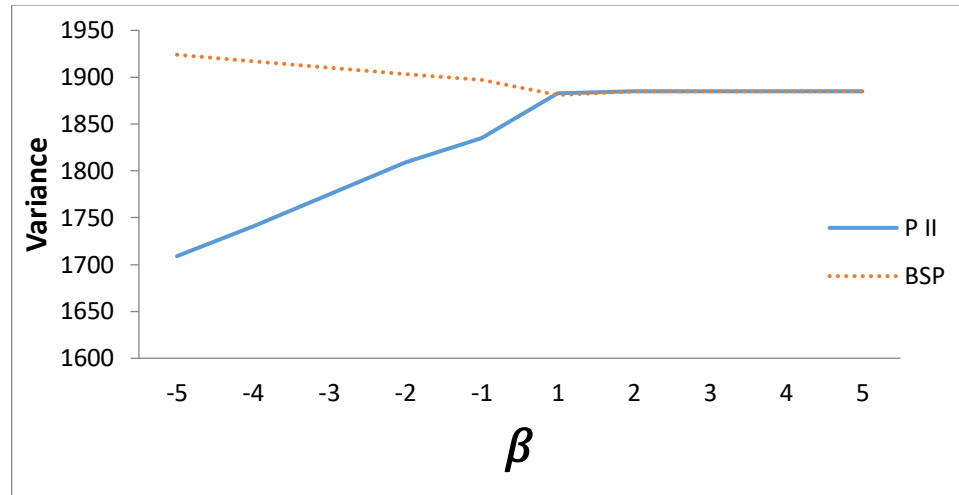


Figure 4.47: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data V)

Interpretation of Figure 4.47:

Figure 4.47 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

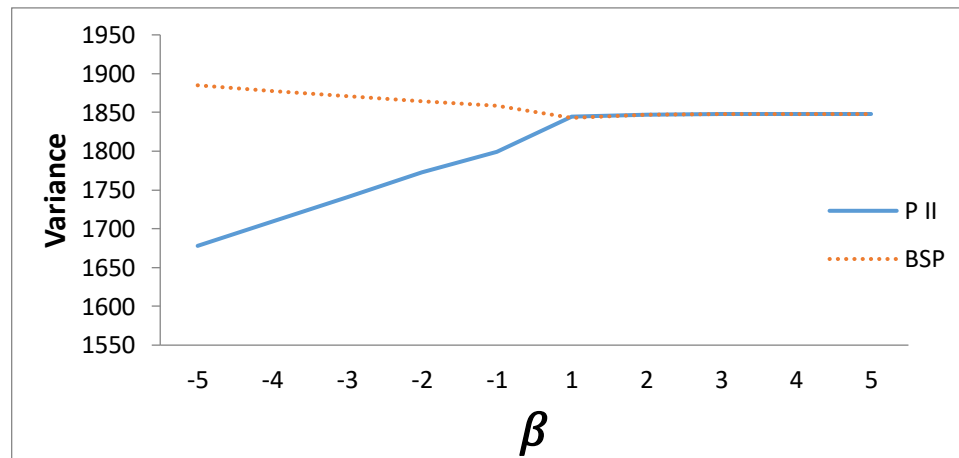


Figure 4.48: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data V)

Interpretation of Figure 4.48:

Figure 4.48 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

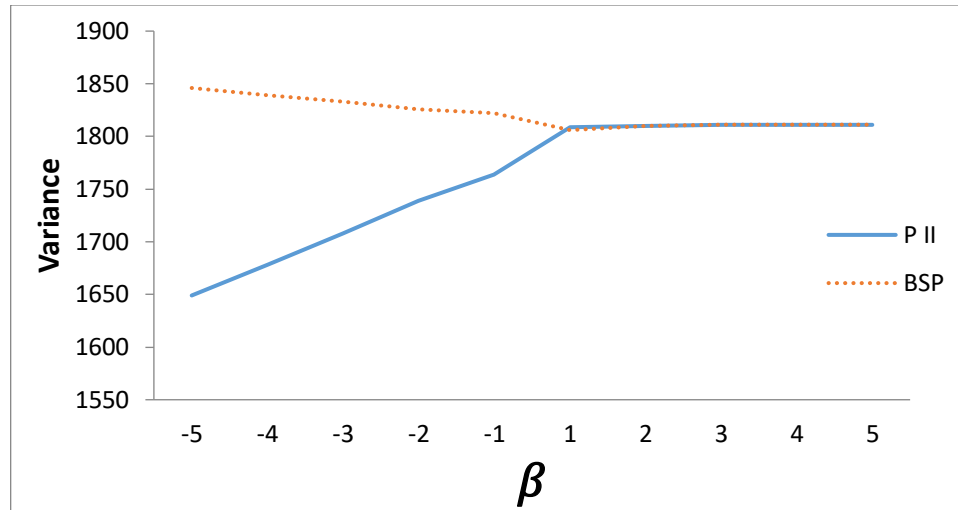


Figure 4.49: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data V)

Interpretation of Figure 4.49:

Figure 4.49 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 4$ .

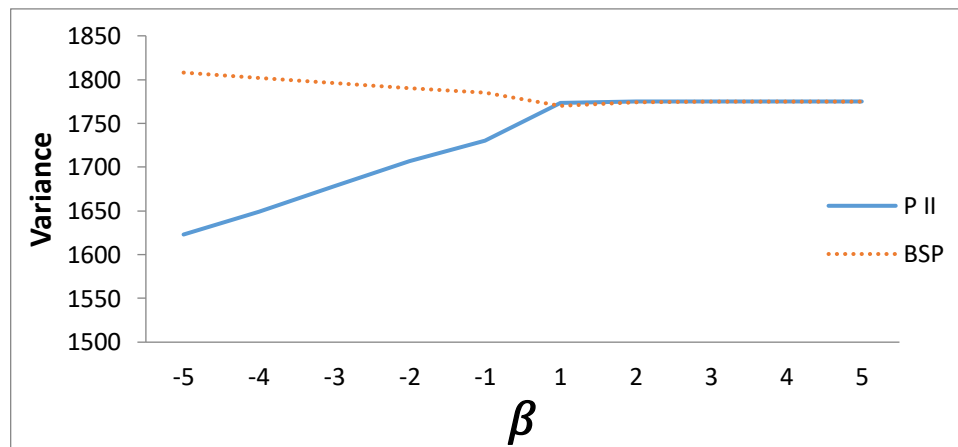


Figure 4.50: Comparison of variances using PP II and BSP for  $\alpha = 5$  (Data V)

Interpretation of Figure 4.50:

Figure 4.50 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

#### 4.1.2.6 Analysis of Data VI

Variance using P I = 894 (Data VI: N=9)

Table 4.27: Comparison of variances of P II and BSP (Data VI:  $N = 9$ )

|         |    | $\alpha$ |     |      |     |      |     |      |     |      |     |
|---------|----|----------|-----|------|-----|------|-----|------|-----|------|-----|
|         |    | -5       |     | -4   |     | -3   |     | -2   |     | -1   |     |
| $\beta$ |    | P II     | BSP | P II | BSP | P II | BSP | P II | BSP | P II | BSP |
|         | -5 | 902      | 926 | 894  | 924 | 886  | 920 | 879  | 915 | 872  | 909 |
|         | -4 | 909      | 926 | 902  | 924 | 894  | 920 | 886  | 915 | 879  | 909 |
|         | -3 | 915      | 926 | 909  | 924 | 902  | 920 | 894  | 915 | 886  | 909 |
|         | -2 | 920      | 926 | 915  | 924 | 908  | 920 | 901  | 915 | 894  | 909 |
|         | -1 | 923      | 926 | 919  | 924 | 913  | 920 | 907  | 915 | 900  | 909 |
|         | 1  | 926      | 925 | 923  | 923 | 920  | 919 | 915  | 914 | 908  | 908 |
|         | 2  | 926      | 926 | 924  | 924 | 920  | 920 | 915  | 915 | 909  | 909 |
|         | 3  | 926      | 926 | 924  | 924 | 920  | 920 | 915  | 915 | 909  | 909 |
|         | 4  | 926      | 926 | 924  | 924 | 920  | 920 | 915  | 915 | 909  | 909 |
|         | 5  | 926      | 926 | 924  | 924 | 920  | 920 | 915  | 915 | 909  | 909 |

Source: Researcher's computation

Table 4.27 continues

|         |    | $\alpha$ |     |      |     |      |     |      |     |      |     |
|---------|----|----------|-----|------|-----|------|-----|------|-----|------|-----|
|         |    | 1        |     | 2    |     | 3    |     | 4    |     | 5    |     |
| $\beta$ |    | P II     | BSP | P II | BSP | P II | BSP | P II | BSP | P II | BSP |
|         | -5 | 860      | 894 | 854  | 886 | 850  | 879 | 847  | 872 | 844  | 865 |
|         | -4 | 865      | 894 | 860  | 886 | 854  | 879 | 850  | 872 | 847  | 865 |
|         | -3 | 872      | 894 | 865  | 886 | 859  | 879 | 854  | 872 | 850  | 865 |
|         | -2 | 879      | 894 | 872  | 887 | 865  | 879 | 859  | 872 | 854  | 865 |
|         | -1 | 884      | 895 | 877  | 887 | 870  | 879 | 864  | 872 | 858  | 866 |
|         | 1  | 894      | 893 | 886  | 885 | 879  | 893 | 871  | 885 | 865  | 878 |
|         | 2  | 894      | 894 | 886  | 886 | 879  | 879 | 872  | 872 | 865  | 865 |
|         | 3  | 894      | 894 | 886  | 886 | 879  | 879 | 872  | 872 | 865  | 865 |
|         | 4  | 894      | 894 | 886  | 886 | 879  | 879 | 872  | 872 | 865  | 865 |
|         | 5  | 894      | 894 | 886  | 886 | 879  | 879 | 872  | 872 | 865  | 865 |

Source: Researcher's computation

Table 4.28: Probabilities of inclusion,  $\pi_i$  (Data VI)

| Units           | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 2$ ) |
|-----------------|-----------------------------------|---------------------------------|
| 1               | 0.8883272                         | 0.8899424                       |
| 2               | 0.8779062                         | 0.8819528                       |
| 3               | 0.8806778                         | 0.8847322                       |
| 4               | 0.8883272                         | 0.8899424                       |
| 5               | 0.8765553                         | 0.8800803                       |
| 6               | 0.9052728                         | 0.8975757                       |
| 7               | 0.8790849                         | 0.8832489                       |
| 8               | 0.8995969                         | 0.8953362                       |
| 9               | 0.9042518                         | 0.8971893                       |
| Sample size (n) | 8                                 | 8                               |

Source: Researcher's computation

Table 4.29: Joint Probabilities of inclusion,  $\pi_{ij}$  (Data VI)

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 2$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 0.7662334                         | 0.7718952                       |
| 1, 3          | 0.7690049                         | 0.7746745                       |
| 1, 4          | 0.7766543                         | 0.7798848                       |
| 1, 5          | 0.7648824                         | 0.7700227                       |
| 1, 6          | 0.7935999                         | 0.7875181                       |
| 1, 7          | 0.7674121                         | 0.7731913                       |
| 1, 8          | 0.7879241                         | 0.7852785                       |
| 1, 9          | 0.792579                          | 0.7871317                       |
| 2, 3          | 0.758584                          | 0.766685                        |
| 2, 4          | 0.7662334                         | 0.7718952                       |
| 2, 5          | 0.7544614                         | 0.7620331                       |
| 2, 6          | 0.783179                          | 0.7795285                       |
| 2, 7          | 0.7569911                         | 0.7652017                       |
| 2, 8          | 0.7775031                         | 0.7772889                       |
| 2, 9          | 0.782158                          | 0.7791421                       |
| 3, 4          | 0.7690049                         | 0.7746745                       |
| 3, 5          | 0.757233                          | 0.7648124                       |
| 3, 6          | 0.7859505                         | 0.7823078                       |
| 3, 7          | 0.7597627                         | 0.7679811                       |
| 3, 8          | 0.7802747                         | 0.7800683                       |
| 3, 9          | 0.7849296                         | 0.7819214                       |
| 4, 5          | 0.7648824                         | 0.7700227                       |
| 4, 6          | 0.7935999                         | 0.7875181                       |
| 4, 7          | 0.7674121                         | 0.7731913                       |
| 4, 8          | 0.7879241                         | 0.7852785                       |
| 4, 9          | 0.792579                          | 0.7871317                       |
| 5, 6          | 0.781828                          | 0.777656                        |
| 5, 7          | 0.7556402                         | 0.7633292                       |
| 5, 8          | 0.7761522                         | 0.7754164                       |
| 5, 9          | 0.7808071                         | 0.7772695                       |
| 6, 7          | 0.7843577                         | 0.7808246                       |
| 6, 8          | 0.8048697                         | 0.7929118                       |
| 6, 9          | 0.8095246                         | 0.7947649                       |
| 7, 8          | 0.7786819                         | 0.7785851                       |
| 7, 9          | 0.7833368                         | 0.7804382                       |
| 8, 9          | 0.8038488                         | 0.7925254                       |
| Total         | 28                                | 28                              |

Source: Researcher's computation

Table 4.30: Least variance estimates (Data VI)

| Units Sampled | P II( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 2$ ) |
|---------------|----------------------------------|---------------------------------|
| 1, 2          | 44.44794                         | 42.60931                        |
| 1, 3          | 13.34622                         | 13.01336                        |
| 1, 4          | 8.973464                         | 8.648178                        |
| 1, 5          | 120.257                          | 114.6985                        |
| 1, 6          | 0.67725                          | 0.3682295                       |
| 1, 7          | 48.42361                         | 46.33615                        |
| 1, 8          | 0.3245606                        | 0.1755114                       |
| 1, 9          | 12.24676                         | 11.58868                        |
| 2, 3          | 9.486971                         | 8.687751                        |
| 2, 4          | 12.34451                         | 12.0146                         |
| 2, 5          | 20.08252                         | 18.42073                        |
| 2, 6          | 27.11605                         | 31.74834                        |
| 2, 7          | 0.1200707                        | 0.1115799                       |
| 2, 8          | 32.21613                         | 34.89723                        |
| 2, 9          | 5.888667                         | 7.485472                        |
| 3, 4          | 0.2931301                        | 0.3454118                       |
| 3, 5          | 55.91483                         | 51.47591                        |
| 3, 6          | 6.117719                         | 8.065347                        |
| 3, 7          | 11.59226                         | 10.61639                        |
| 3, 8          | 8.106149                         | 9.458954                        |
| 3, 9          | 0.08221214                       | 0.0006265052                    |
| 4, 5          | 60.72343                         | 58.07099                        |
| 4, 6          | 3.634295                         | 4.912261                        |
| 4, 7          | 14.61475                         | 14.17597                        |
| 4, 8          | 5.063908                         | 5.948744                        |
| 4, 9          | 0.5681649                        | 0.3306472                       |
| 5, 6          | 83.14077                         | 92.99529                        |
| 5, 7          | 16.87117                         | 15.42603                        |
| 5, 8          | 94.45606                         | 99.35184                        |
| 5, 9          | 40.23543                         | 45.25024                        |
| 6, 7          | 30.00914                         | 34.89165                        |
| 6, 8          | 0.06537867                       | 0.03517318                      |
| 6, 9          | 5.927783                         | 7.206062                        |
| 7, 8          | 35.45017                         | 38.21455                        |
| 7, 9          | 7.369872                         | 9.164628                        |
| 8, 9          | 7.687571                         | 8.441207                        |
| Total         | 843.8759                         | 865.1815                        |

Source: Researcher's computation

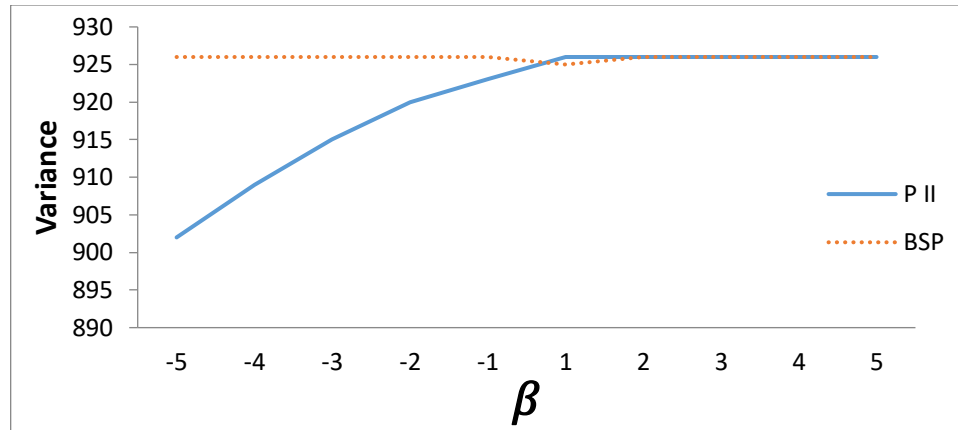


Figure 4.51: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data VI)

Interpretation of Figure 4.51:

Figure 4.51 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

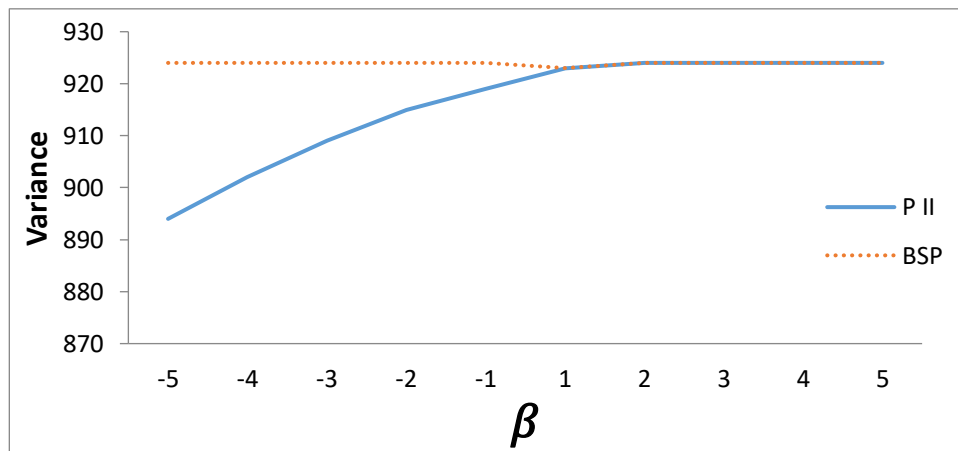


Figure 4.52: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data VI)

Interpretation of Figure 4.52:

Figure 4.52 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

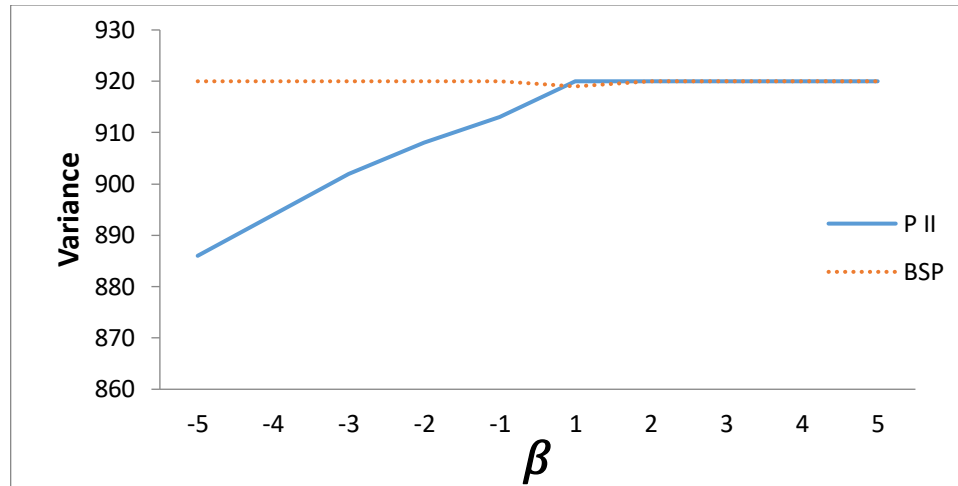


Figure 4.53: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data VI)

Interpretation of Figure 4.53:

Figure 4.53 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

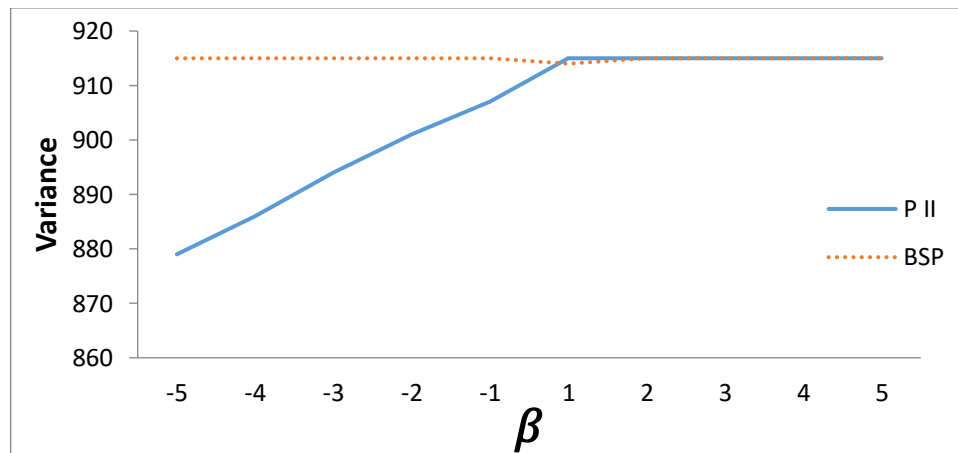


Figure 4.54: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data VI)

Interpretation of Figure 4.54:

Figure 4.54 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

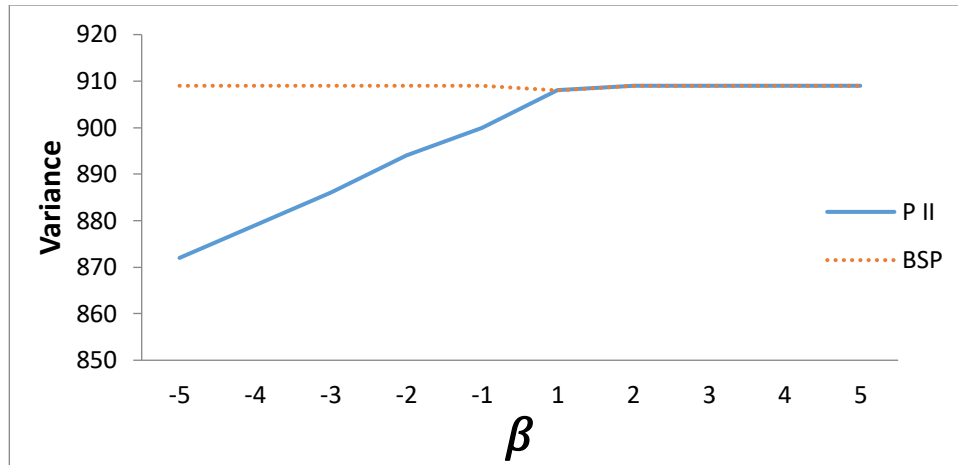


Figure 4.55: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data VI)

Interpretation of Figure 4.55:

Figure 4.55 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

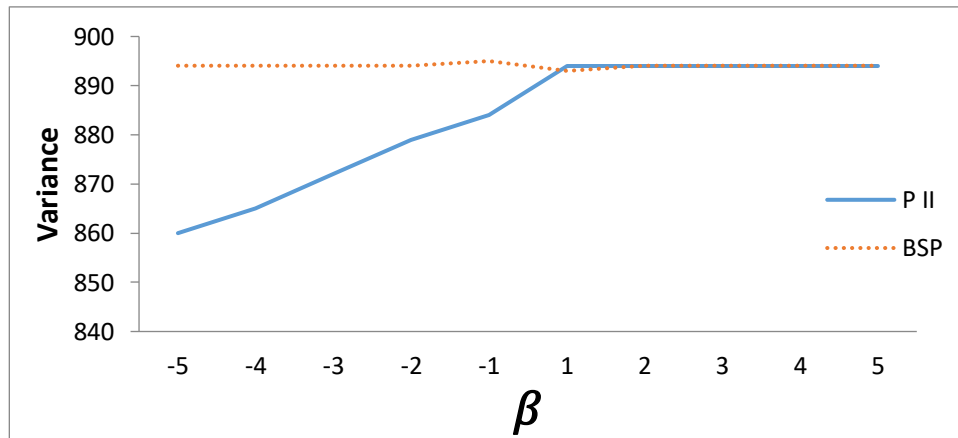


Figure 4.56: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data VI)

Interpretation of Figure 4.56:

Figure 4.56 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

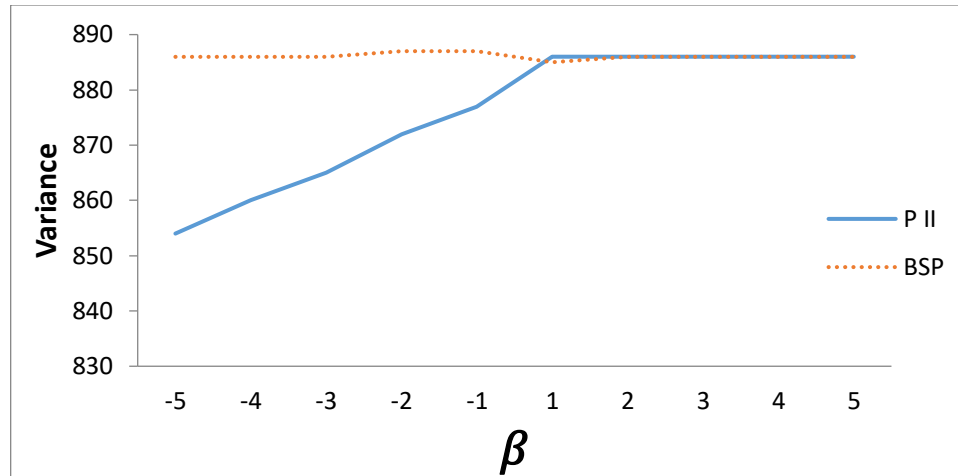


Figure 4.57: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data VI)

Interpretation of Figure 4.57:

Figure 4.57 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

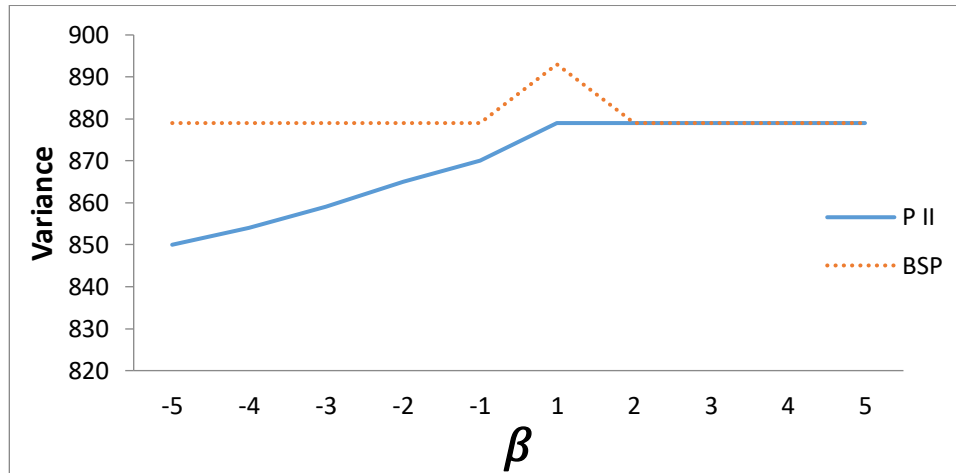


Figure 4.58: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data VI)

Interpretation of Figure 4.58:

Figure 4.58 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

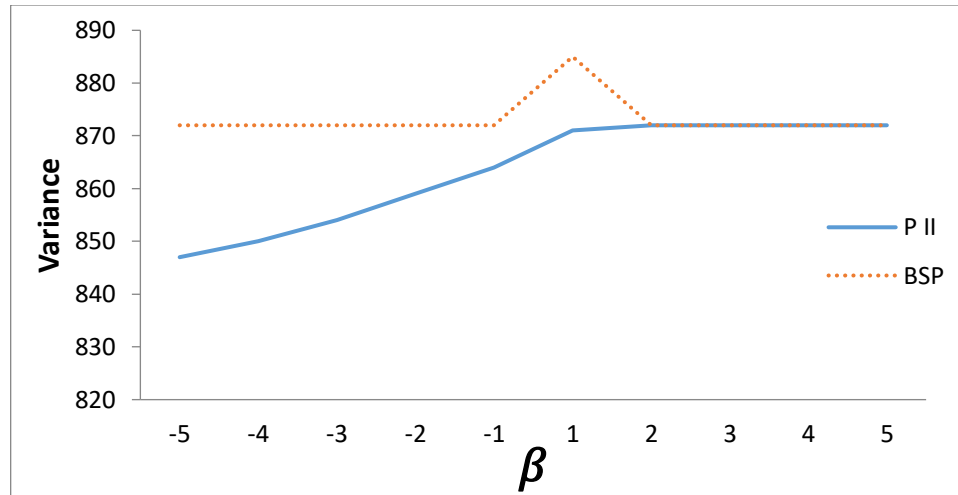


Figure 4.59: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data VI)

Interpretation of Figure 4.59:

Figure 4.59 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 4$ .

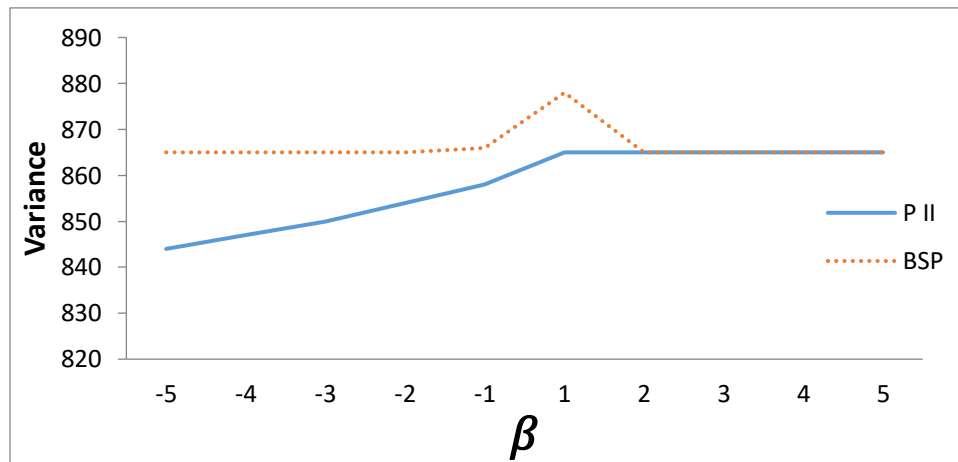


Figure 4.60: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data VI)

Interpretation of Figure 4.60:

Figure 4.60 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

#### 4.1.3 Optimum values of the constants and the least variance obtained for the six simulated data using PP II and BSP

Tables 4.31 and 4.32 show the optimum values of the constants  $\alpha$  and  $\beta$  for the six simulated data in which the least variances are obtained using P II and BSP.

Table 4.31: The optimum values of  $\alpha$  and  $\beta$  at which the PP II and BSP have least variance for all the six simulated data using P II and BSP.

| Units | Simulated Data | P II     |         | BSP      |         |
|-------|----------------|----------|---------|----------|---------|
|       |                | $\alpha$ | $\beta$ | $\alpha$ | $\beta$ |
| 1     | Data I         | 5        | -5      | 5        | 1       |
| 2     | Data II        | 5        | -5      | 5        | 1       |
| 3     | Data III       | 5        | -5      | 5        | 1       |
| 4     | Data IV        | 5        | -5      | 5        | 1       |
| 5     | Data V         | 5        | -5      | 5        | 1       |
| 6     | Data VI        | 5        | -5      | 5        | 2       |

Table 4.32: The least variance estimates using the P II and BSP

| Units | Simulated Data | Variances |      |
|-------|----------------|-----------|------|
|       |                | P II      | BSP  |
| 1     | Data I         | 790       | 1224 |
| 2     | Data II        | 880       | 1018 |
| 3     | Data III       | 1117      | 1241 |
| 4     | Data IV        | 479       | 523  |
| 5     | Data V         | 1623      | 1770 |
| 6     | Data VI        | 844       | 865  |

Source: Researcher's computation

#### 4.1.4 Consistency of P II Using Simulated Data

Tables 4.33 – 4.38 shows the variance estimates for all possible samples that could be selected for the six simulated data sets. All these tables show that as the sample size increases the variances estimate decreases.

Table 4.33: Variance estimate for different sample sizes using P II (Data I:  $N = 4$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 5, \beta = -5$ ) |
|---------------------|---------------------------------------|
| 3                   | 9724                                  |
| 2                   | 790                                   |

Source: Researcher's computation

Table 4.34: Variance estimate for different sample sizes using P II (Data II:  $N = 5$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 5, \beta = -5$ ) |
|---------------------|---------------------------------------|
| 4                   | 880                                   |
| 3                   | 2944                                  |
| 2                   | 66543                                 |

Source: Researcher's computation

Table 4.35: Variance estimate for different sample sizes using P II (Data III:  $N = 6$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 5, \beta = -5$ ) |
|---------------------|---------------------------------------|
| 5                   | 1117                                  |
| 4                   | 2715                                  |
| 3                   | 5279                                  |
| 2                   | 119390                                |

Source: Researcher's computation

Table 4.36: Variance estimate for different sample sizes using P II (Data IV:  $N = 7$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 5, \beta = -5$ ) |
|---------------------|---------------------------------------|
| 6                   | 479                                   |
| 5                   | 1391                                  |
| 4                   | 3068                                  |
| 3                   | 5501                                  |
| 2                   | 6224                                  |

Source: Researcher's computation

Table 4.37: Variance estimate for different sample sizes using P II (Data IV:  $N = 8$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 5, \beta = -5$ ) |
|---------------------|---------------------------------------|
| 7                   | 1623                                  |
| 6                   | 4437                                  |
| 5                   | 9316                                  |
| 4                   | 17918                                 |
| 3                   | 38972                                 |
| 2                   | 519651                                |

Source: Researcher's computation

Table 4.38: Variance estimate for different sample sizes using P II (Data VI:  $N = 9$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 5, \beta = -5$ ) |
|---------------------|---------------------------------------|
| 8                   | 844                                   |
| 7                   | 2259                                  |
| 6                   | 4725                                  |
| 5                   | 9502                                  |
| 4                   | 21980                                 |
| 3                   | 88446                                 |
| 2                   | 1332290                               |

Source: Researcher's computation

## 4.2 Application to Real Life Data

Analysis was carried out on five life data for number of units,  $N = 4, 5, 6,$

8 and 9 using P I, P II, and BSP.

### 4.2.1 Presentation of Real Life Data

Table 4.39: Distribution of population of Swaziland by regions and the number of children with birth weight  $< 2.5 \text{ kg}$  ( $N = 4$ )

| Units | Regions    | Population ( $X_i$ ) | Number of births with reported weight | Percentage of number of children with birth weight $< 2.5 \text{ kg}$ | Number of children with birth weight $< 2.5 \text{ kg}$ ( $Y_i$ ) |
|-------|------------|----------------------|---------------------------------------|-----------------------------------------------------------------------|-------------------------------------------------------------------|
| 1     | Hhohho     | 282,734              | 672                                   | 9.5                                                                   | 64                                                                |
| 2     | Manzini    | 319,530              | 784                                   | 7.1                                                                   | 56                                                                |
| 3     | Shiselweni | 208,454              | 438                                   | 8.9                                                                   | 39                                                                |
| 4     | Lubombo    | 207,731              | 474                                   | 5.5                                                                   | 27                                                                |
|       | Total      | 1,018,449            | 2,368                                 |                                                                       |                                                                   |

Sources: Swaziland 2007 Census and Final Report of 2006 – 07 Swaziland Demographic and Health Survey

Table 4.40: Distribution of population of Liberia by regions and number of children with birth weight  $< 2.5 \text{ kg}$  ( $N = 5$ )

| Units | Regions         | Population ( $X_i$ ) | Number of births with reported weight | Percentage of number of children with birth weight $< 2.5 \text{ kg}$ | Number of children with birth weight $< 2.5 \text{ kg}$ ( $Y_i$ ) |
|-------|-----------------|----------------------|---------------------------------------|-----------------------------------------------------------------------|-------------------------------------------------------------------|
| 1     | North Western   | 294,849              | 118                                   | 8.9                                                                   | 11                                                                |
| 2     | South Central   | 1,569,334            | 783                                   | 10.1                                                                  | 80                                                                |
| 3     | South Eastern A | 296,940              | 88                                    | 7.2                                                                   | 7                                                                 |
| 4     | South Eastern B | 260,828              | 102                                   | 10.5                                                                  | 11                                                                |
| 5     | North Central   | 1,067,121            | 419                                   | 9.5                                                                   | 40                                                                |
|       | Total           | 3,489,072            |                                       |                                                                       |                                                                   |

Sources: Liberia 2008 Census Data and Final Report of 2013 Liberia Demographic and Health Survey

Table 4.41: Distribution of population of Nigeria by zones and number of children with birth weight  $< 2.5\text{ kg}$  ( $N = 6$ )

| Units | Zones         | Population ( $X_i$ ) | Number of births with reported weight | Percentage of number of children with birth weight $< 2.5\text{ kg}$ | Number of children with birth weight $< 2.5\text{ kg}$ ( $Y_i$ ) |
|-------|---------------|----------------------|---------------------------------------|----------------------------------------------------------------------|------------------------------------------------------------------|
| 1     | North Central | 20,369,956           | 534                                   | 7.5                                                                  | 41                                                               |
| 2     | North East    | 18,894,299           | 302                                   | 13.6                                                                 | 42                                                               |
| 3     | North West    | 35,915,467           | 489                                   | 27.2                                                                 | 134                                                              |
| 4     | South East    | 16,395,555           | 1317                                  | 4.3                                                                  | 57                                                               |
| 5     | South South   | 21,044,081           | 798                                   | 11.6                                                                 | 93                                                               |
| 6     | South West    | 27,722,432           | 1780                                  | 3.4                                                                  | 61                                                               |
|       | Total         | 140,341,790          |                                       |                                                                      |                                                                  |

Sources: Nigeria 2006 Population and Housing Data and Final Report of 2013 Nigeria National DHS

4.42: Distribution of population of The Gambia by Local Government Areas and number of children with birth weight  $< 2.5\text{ kg}$  ( $N = 8$ )

| Units | Local Government Areas | Population ( $X_i$ ) | Number of births with reported weight | Percentage of number of children with birth weight $< 2.5\text{ kg}$ | Number of children with birth weight $< 2.5\text{ kg}$ ( $Y_i$ ) |
|-------|------------------------|----------------------|---------------------------------------|----------------------------------------------------------------------|------------------------------------------------------------------|
| 1     | Banjul                 | 31,301               | 108                                   | 14.3                                                                 | 16                                                               |
| 2     | Basse                  | 239,916              | 359                                   | 10.1                                                                 | 37                                                               |
| 3     | Brikama                | 699,704              | 1920                                  | 11.8                                                                 | 227                                                              |
| 4     | Janjanbureh            | 126,910              | 386                                   | 9.3                                                                  | 36                                                               |
| 5     | Kanifing               | 382,096              | 1141                                  | 13.4                                                                 | 153                                                              |
| 6     | Kerewan                | 221,054              | 370                                   | 12.1                                                                 | 45                                                               |
| 7     | Kuntaur                | 99,108               | 167                                   | 9.9                                                                  | 17                                                               |
| 8     | Mansakonko             | 82,361               | 237                                   | 9.6                                                                  | 23                                                               |
|       | Total                  | 1,882,450            |                                       |                                                                      |                                                                  |

Sources: Preliminary Results of The Gambia 2013 Population and Housing Census and Final Report of 2013 Gambia DHS

Table 4.43: Distribution of population of South Africa by Provinces and number of children with birth weight  $< 2.5 \text{ kg}$  ( $N = 9$ )

| Units | Provinces       | Population ( $X_i$ ) | Number of births with reported weight | Percentage of number of children with birth weight $< 2.5 \text{ kg}$ | Number of children with birth weight $< 2.5 \text{ kg}$ ( $Y_i$ ) |
|-------|-----------------|----------------------|---------------------------------------|-----------------------------------------------------------------------|-------------------------------------------------------------------|
| 1     | Western Cape    | 5,822,734            | 323                                   | 11.2                                                                  | 37                                                                |
| 2     | Eastern Cape    | 6,562,053            | 266                                   | 5.3                                                                   | 15                                                                |
| 3     | Northern Cape   | 1,145,861            | 46                                    | 14.6                                                                  | 7                                                                 |
| 4     | Free State      | 2,745,590            | 130                                   | 6.8                                                                   | 9                                                                 |
| 5     | KwaZulu – Natal | 10,267,300           | 117                                   | 7.4                                                                   | 9                                                                 |
| 6     | North West      | 3,509,953            | 192                                   | 7.9                                                                   | 16                                                                |
| 7     | Gauteng         | 12,272,263           | 588                                   | 8.2                                                                   | 49                                                                |
| 8     | Mpumalanga      | 4,039,939            | 169                                   | 4.8                                                                   | 9                                                                 |
| 9     | Limpopo         | 5,404,868            | 288                                   | 8.4                                                                   | 25                                                                |
|       | Total           | 51,770,561           |                                       |                                                                       |                                                                   |

Sources: South Africa 2011 Census Data and Final Report of 2003 South Africa DHS

#### 4.2.2 Analysis of Real Life Data

Analysis was carried out on the five real life data for number of units,

$N = 4, 5, 6, 8$  and  $9$  using the P I and P II and BSP. Sample of sizes

$n = 3, 4, 5, 7$ , and  $8$  were selected for each of the data under consideration.

#### 4.2.2.1 Analysis of Data VII (Swaziland)

Variance using P I = 665 (Data VII:  $N = 4$ )

Table 4.44: Comparison of variances of P II and BSP (Data VII:  $N = 4$ )

|         |    | $\alpha$ |      |      |      |      |     |      |     |      |     |
|---------|----|----------|------|------|------|------|-----|------|-----|------|-----|
|         |    | -5       |      | -4   |      | -3   |     | -2   |     | -1   |     |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP | P II | BSP | P II | BSP |
|         | -5 | 740      | 1066 | 672  | 1019 | 620  | 961 | 587  | 892 | 574  | 817 |
|         | -4 | 814      | 1066 | 738  | 1019 | 671  | 961 | 619  | 893 | 587  | 817 |
|         | -3 | 885      | 1067 | 809  | 1020 | 734  | 962 | 668  | 894 | 617  | 818 |
|         | -2 | 943      | 1069 | 872  | 1022 | 796  | 965 | 722  | 897 | 658  | 821 |
|         | -1 | 963      | 1073 | 895  | 1028 | 820  | 972 | 745  | 905 | 679  | 830 |
|         | 1  | 1060     | 1037 | 1012 | 983  | 952  | 918 | 882  | 844 | 806  | 768 |
|         | 2  | 1063     | 1060 | 1016 | 1011 | 957  | 951 | 888  | 881 | 812  | 805 |
|         | 3  | 1065     | 1064 | 1018 | 1016 | 959  | 958 | 890  | 889 | 815  | 813 |
|         | 4  | 1066     | 1065 | 1019 | 1018 | 960  | 960 | 892  | 891 | 816  | 815 |
|         | 5  | 1066     | 1066 | 1019 | 1019 | 961  | 960 | 892  | 892 | 816  | 816 |

Source: Researcher's computation

Table 4.44 continues

|         |    | $\alpha$ |     |      |     |      |     |      |     |      |     |
|---------|----|----------|-----|------|-----|------|-----|------|-----|------|-----|
|         |    | 1        |     | 2    |     | 3    |     | 4    |     | 5    |     |
| $\beta$ |    | P II     | BSP | P II | BSP | P II | BSP | P II | BSP | P II | BSP |
|         | -5 | 597      | 673 | 626  | 620 | 663  | 587 | 705  | 574 | 751  | 578 |
|         | -4 | 580      | 673 | 598  | 620 | 627  | 587 | 664  | 574 | 707  | 578 |
|         | -3 | 576      | 674 | 582  | 621 | 601  | 587 | 631  | 574 | 669  | 577 |
|         | -2 | 585      | 676 | 578  | 623 | 587  | 588 | 609  | 573 | 641  | 576 |
|         | -1 | 598      | 684 | 587  | 628 | 594  | 591 | 614  | 574 | 644  | 574 |
|         | 1  | 665      | 641 | 615  | 604 | 585  | 586 | 575  | 587 | 581  | 603 |
|         | 2  | 669      | 665 | 618  | 615 | 587  | 586 | 575  | 576 | 580  | 583 |
|         | 3  | 671      | 670 | 619  | 619 | 587  | 587 | 574  | 575 | 579  | 580 |
|         | 4  | 672      | 672 | 620  | 620 | 587  | 587 | 574  | 574 | 579  | 579 |
|         | 5  | 672      | 672 | 620  | 620 | 587  | 587 | 574  | 574 | 578  | 579 |

Source: Researcher's computation

Table 4.45: Probabilities of inclusion,  $\pi_i$  (Data VII:  $N = 4$ )

| Units | Regions         | P II ( $\alpha = 4, \beta = 5$ ) | BSP ( $\alpha = 4, \beta = -2$ ) |
|-------|-----------------|----------------------------------|----------------------------------|
| 1     | Hhohho          | 0.7702446                        | 0.7705155                        |
| 2     | Manzini         | 0.8356341                        | 0.8340131                        |
| 3     | Shiselweni      | 0.6972719                        | 0.6979492                        |
| 4     | Lubombo         | 0.6968495                        | 0.6975222                        |
|       | Sample size (n) | 3                                | 3                                |

Source: Researcher's computation

Table 4.46: Joint Probabilities of inclusion,  $\pi_{ij}$  (Data VII:  $N = 4$ )

| Units Sampled | P II ( $\alpha = 4, \beta = 5$ ) | BSP ( $\alpha = 4, \beta = -2$ ) |
|---------------|----------------------------------|----------------------------------|
| 1, 2          | 0.6058786                        | 0.6045285                        |
| 1, 3          | 0.4675164                        | 0.4684647                        |
| 1, 4          | 0.467094                         | 0.4680377                        |
| 2, 3          | 0.532906                         | 0.5319623                        |
| 2, 4          | 0.5324836                        | 0.5315353                        |
| 3, 4          | 0.3941214                        | 0.3954715                        |
| Total         | 3                                | 3                                |

Source: Researcher's computation

Table 4.47: Least variance estimates (Data VII:  $N = 4$ )

| Units Sampled | P II ( $\alpha = 4, \beta = 5$ ) | BSP ( $\alpha = 4, \beta = -2$ ) |
|---------------|----------------------------------|----------------------------------|
| 1, 2          | 16.10726                         | 15.96177                         |
| 1, 3          | 109.7297                         | 109.3353                         |
| 1, 4          | 293.2262                         | 291.7486                         |
| 2, 3          | 11.46848                         | 11.96487                         |
| 2, 4          | 68.77869                         | 76.38292                         |
| 3, 4          | 68.77869                         | 68.10452                         |
| Total         | 574                              | 573                              |

Source: Researcher's computation

Result 1:  $\sum_{i=1}^4 \pi_i = n = 3 = \text{sample size selected}$

Result 2:  $\sum_{j=1, j \neq i}^4 \pi_{ij} = (n - 1)\pi_i = (3 - 1)\pi_i = 2\pi_i$

Result 3:  $\sum_{i=1}^4 \sum_{j=1, j \neq i}^4 \pi_{ij} = n(n - 1) = 3(3 - 1) = 6$

The double summation here means that  $6/2 = 3$

Result 4: The Sen – Yates – Grundy variance estimators are non – negative using P I and P II.

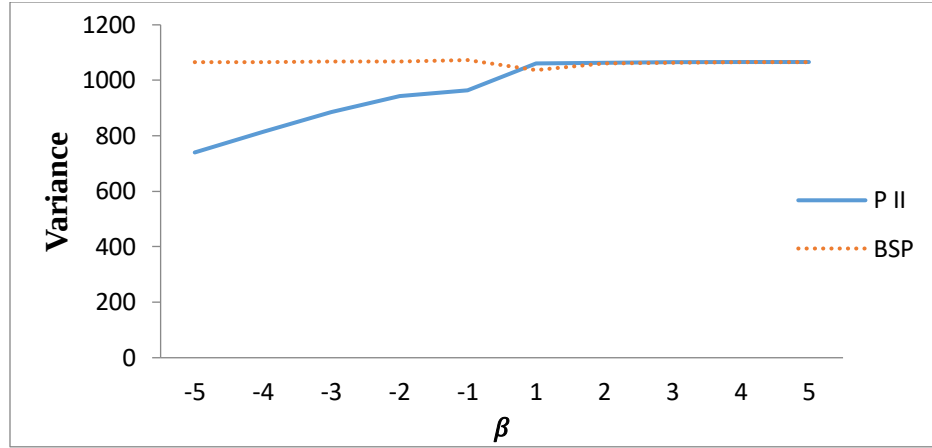


Figure 4.61: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data VII)

Interpretation of Figure 4.61:

Figure 4.61 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

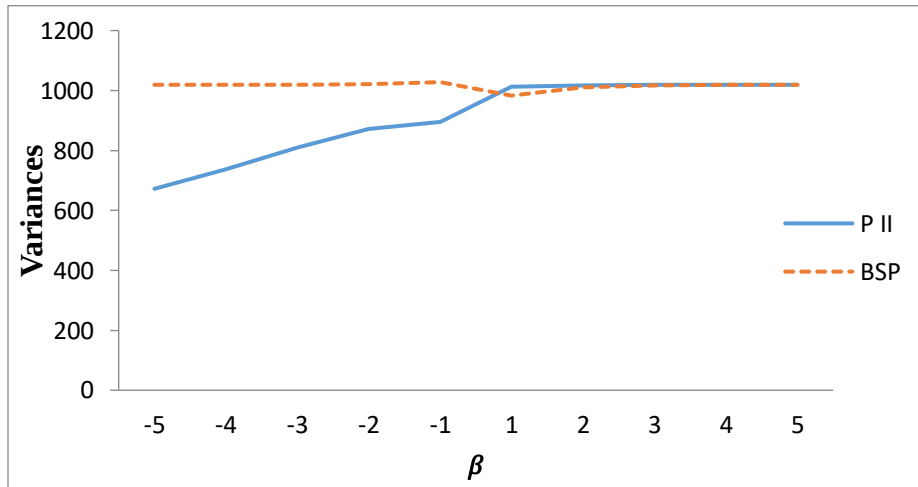


Figure 4.62: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data VII)

Interpretation of Figure 4.62:

Figure 4.62 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

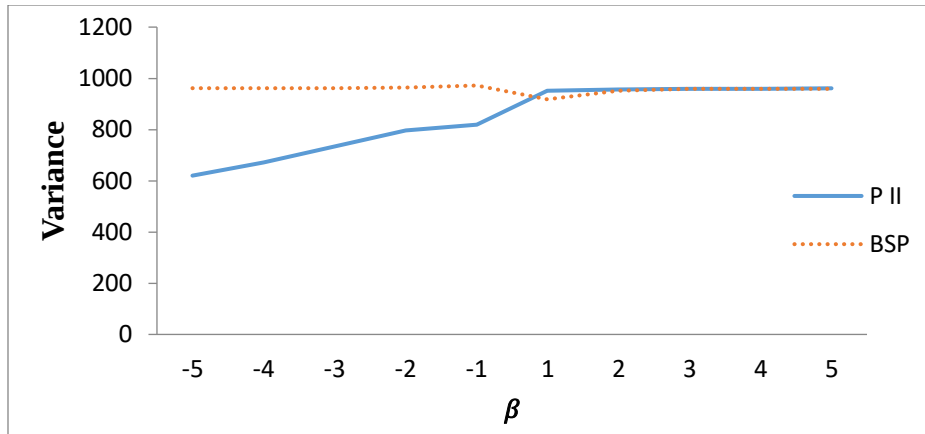


Figure 4.63: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data VII)

Interpretation of Figure 4.63:

Figure 4.63 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

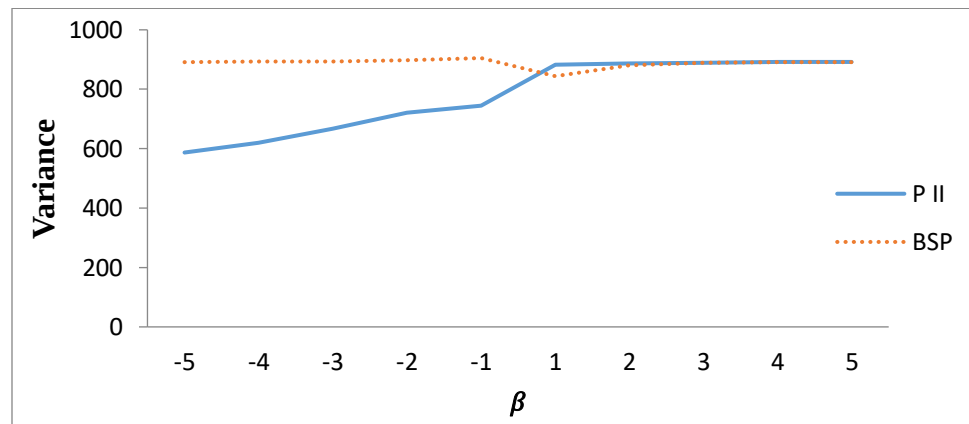


Figure 4.64: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data VII)

Interpretation of Figure 4.64:

Figure 4.64 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

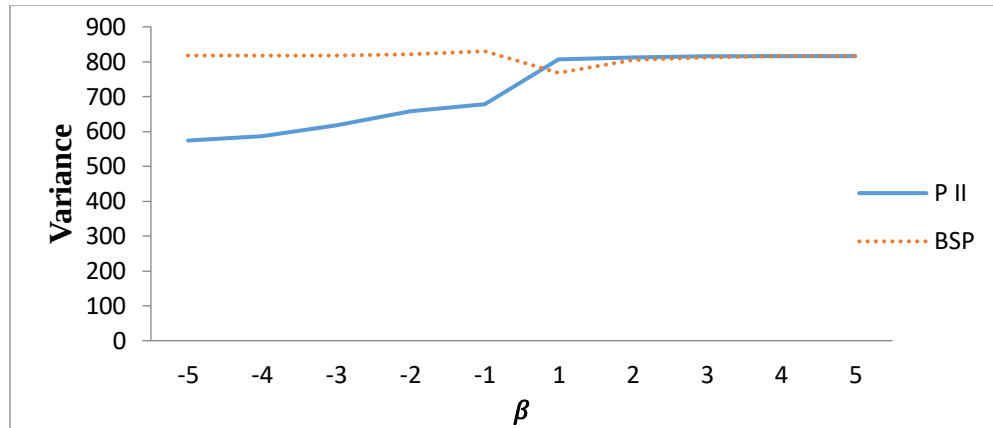


Figure 4.65: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data VII)

Interpretation of Figure 4.65:

Figure 4.65 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

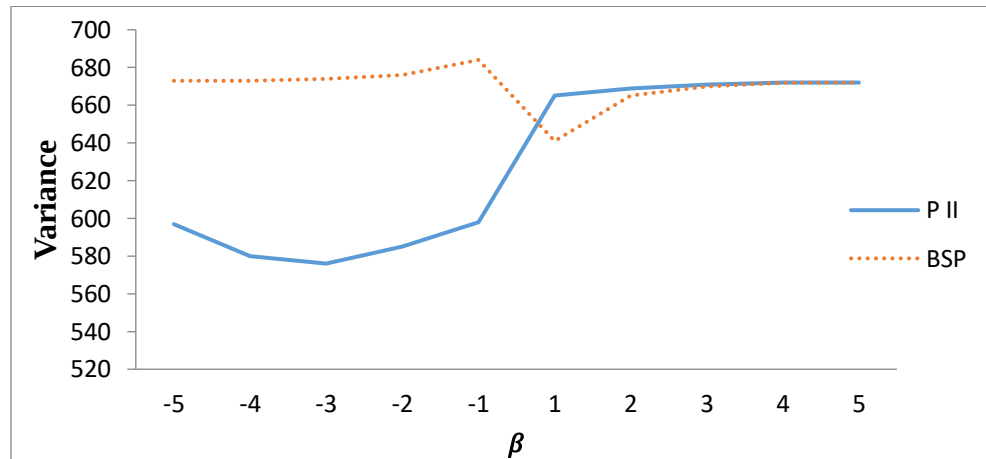


Figure 4.66: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data VII)

Interpretation of Figure 4.66:

Figure 4.66 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

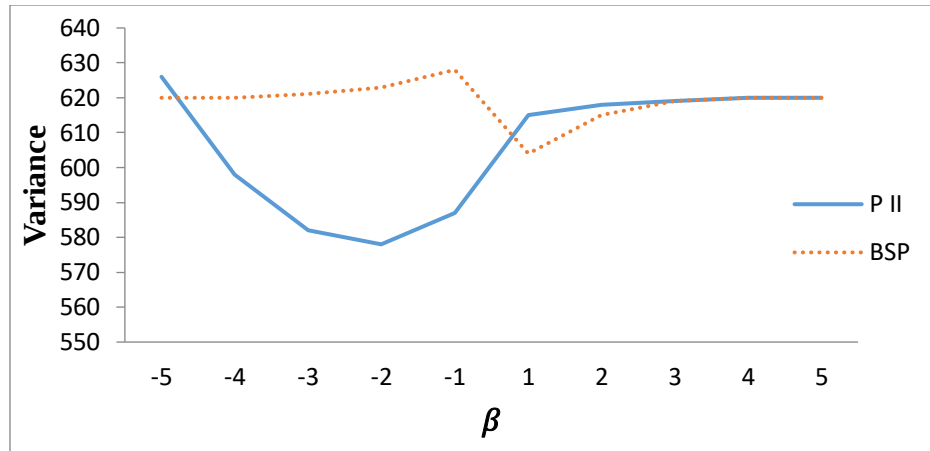


Figure 4.67: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data VII)

Interpretation of Figure 4.67:

Figure 4.67 shows that P II has lower variances than BSP when  $-4 \leq \beta \leq 1$ . It also shows that BSP has slightly lower variances than P II when  $\beta = -5$  and  $1 < \beta \leq 3$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

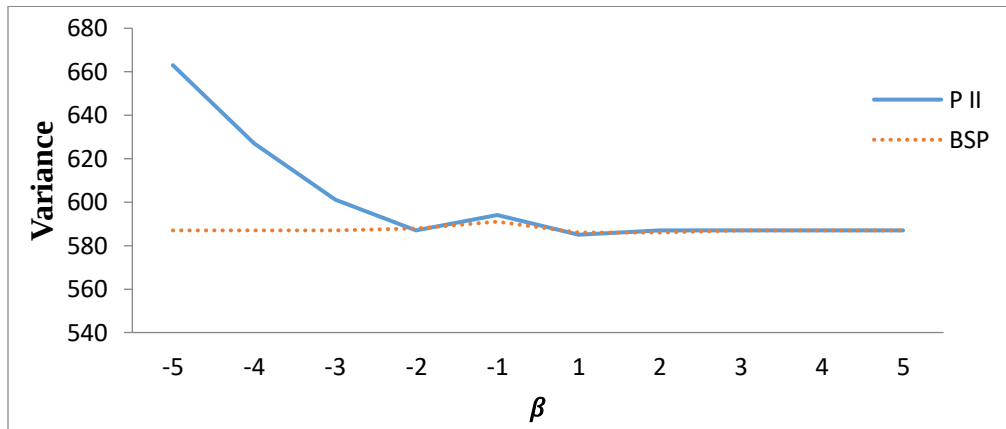


Figure 4.68: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data VII)

Interpretation of Figure 4.68:

Figure 4.68 shows that BSP has lower variances than P II when  $-5 \leq \beta \leq -2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

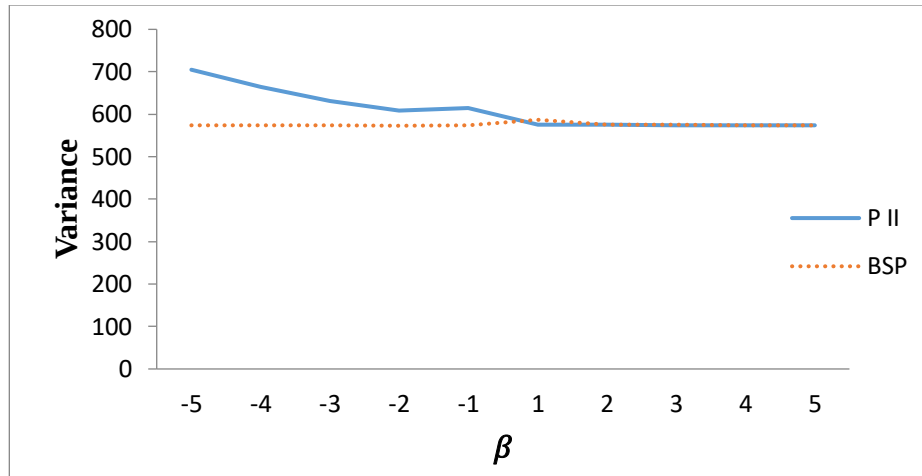


Figure 4.69: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data VII)

Interpretation of Figure 4.69:

Figure 4.69 shows that BSP has lower variances than P II when  $-5 \leq \beta \leq 1$  while the two procedures yield approximately same variances as the values of  $\beta$  increases, for  $\alpha = 4$ .

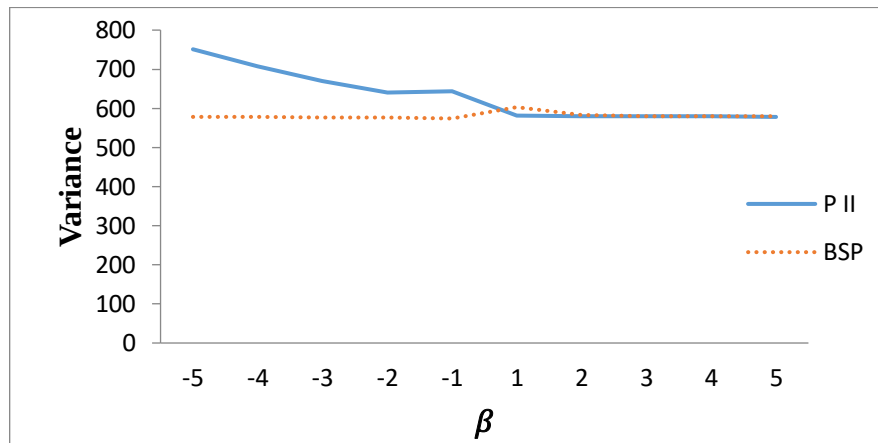


Figure 4.70: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data VII)

Interpretation of Figure 4.70:

Figure 4.70 shows that BSP has lower variances than P II when  $-5 \leq \beta \leq -1$ , while the two procedures yield approximately same variances as the values of  $\beta$  increases, for a fixed value of  $\alpha = 5$ .

#### 4.2.2.2 Analysis of Data VIII (Liberia)

Variance using P I = 1212 (Data VIII:  $N = 5$ )

Table 4.48: Comparison of variances of P II and BSP (Data VIII:  $N = 5$ )

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | -5       |      | -4   |      | -3   |      | -2   |      | -1   |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 1987     | 2702 | 1276 | 2692 | 874  | 2679 | 719  | 2645 | 657  | 2498 |
|         | -4 | 2481     | 2702 | 1964 | 2692 | 1253 | 2679 | 861  | 2645 | 712  | 2500 |
|         | -3 | 2636     | 2702 | 2458 | 2692 | 1908 | 2679 | 1201 | 2646 | 834  | 2505 |
|         | -2 | 2674     | 2702 | 2620 | 2692 | 2390 | 2680 | 1755 | 2648 | 1082 | 2515 |
|         | -1 | 2681     | 2702 | 2627 | 2692 | 2397 | 2681 | 1727 | 2654 | 995  | 2541 |
|         | 1  | 2701     | 2700 | 2691 | 2686 | 2677 | 2650 | 2636 | 2502 | 2458 | 1980 |
|         | 2  | 2702     | 2701 | 2691 | 2691 | 2678 | 2677 | 2640 | 2632 | 2478 | 2441 |
|         | 3  | 2702     | 2702 | 2692 | 2691 | 2679 | 2678 | 2642 | 2640 | 2488 | 2477 |
|         | 4  | 2702     | 2702 | 2692 | 2692 | 2679 | 2679 | 2643 | 2643 | 2493 | 2488 |
|         | 5  | 2702     | 2702 | 2692 | 2692 | 2679 | 2679 | 2644 | 2644 | 2495 | 2493 |

Source: Researcher's computation

Table 4.48 continues

|         |    | $\alpha$ |      |      |     |      |     |      |     |      |     |
|---------|----|----------|------|------|-----|------|-----|------|-----|------|-----|
|         |    | 1        |      | 2    |     | 3    |     | 4    |     | 5    |     |
| $\beta$ |    | P II     | BSP  | P II | BSP | P II | BSP | P II | BSP | P II | BSP |
|         | -5 | 619      | 1301 | 616  | 889 | 616  | 727 | 617  | 660 | 618  | 632 |
|         | -4 | 628      | 1307 | 619  | 892 | 616  | 728 | 616  | 661 | 617  | 632 |
|         | -3 | 647      | 1320 | 626  | 900 | 618  | 732 | 616  | 663 | 616  | 633 |
|         | -2 | 675      | 1350 | 636  | 916 | 621  | 740 | 617  | 667 | 616  | 635 |
|         | -1 | 636      | 1428 | 619  | 956 | 616  | 759 | 617  | 676 | 618  | 639 |
|         | 1  | 1212     | 768  | 846  | 653 | 707  | 624 | 651  | 617 | 628  | 616 |
|         | 2  | 1250     | 1168 | 862  | 819 | 713  | 693 | 654  | 644 | 629  | 625 |
|         | 3  | 1274     | 1245 | 873  | 858 | 719  | 711 | 656  | 653 | 630  | 628 |
|         | 4  | 1285     | 1274 | 880  | 873 | 722  | 718 | 658  | 656 | 631  | 630 |
|         | 5  | 1291     | 1286 | 883  | 880 | 723  | 722 | 659  | 658 | 631  | 631 |

Source: Researcher's computation

Table 4.49: Probabilities of inclusion,  $\pi_i$  (Data VIII:  $N = 5$ )

| Units | Regions         | P II ( $\alpha = 4, \beta = -3$ ) | BSP( $\alpha = 5, \beta = 1$ ) |
|-------|-----------------|-----------------------------------|--------------------------------|
| 1     | North Western   | 0.7500016                         | 0.7500112                      |
| 2     | South Central   | 0.9862192                         | 0.9886507                      |
| 3     | South Eastern A | 0.7500017                         | 0.7500116                      |
| 4     | South Eastern B | 0.7500007                         | 0.7500060                      |
| 5     | North Central   | 0.7637769                         | 0.7613204                      |
|       | Sample size (n) | 4                                 | 4                              |

Source: Researcher's computation

Table 4.50: Joint Probabilities of inclusion,  $\pi_{ij}$  (Data VIII: N=5)

| Units Sampled | P II ( $\alpha = 4, \beta = -3$ ) | BSP( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|--------------------------------|
| 1, 2          | 0.7362208                         | 0.7386619                      |
| 1, 3          | 0.5000033                         | 0.5000229                      |
| 1, 4          | 0.5000023                         | 0.5000172                      |
| 1, 5          | 0.5137785                         | 0.5113316                      |
| 2, 3          | 0.7362208                         | 0.7386624                      |
| 2, 4          | 0.7362198                         | 0.7386567                      |
| 2, 5          | 0.749996                          | 0.7499711                      |
| 3, 4          | 0.5000024                         | 0.5000177                      |
| 3, 5          | 0.5137786                         | 0.5113320                      |
| 4, 5          | 0.5137776                         | 0.5113264                      |
| Total         | 6                                 | 6                              |

Source: Researcher's computation

Table 4.51: Least variance estimates (Data VIII: N=5)

| Units Sampled | P II ( $\alpha = 4, \beta = -3$ ) | BSP( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|--------------------------------|
| 1, 2          | 20.66385                          | 1.685934e+01                   |
| 1, 3          | 3.555472                          | 3.554968e+00                   |
| 1, 4          | 4.057185e-11                      | 1.304849e-09                   |
| 1, 5          | 163.4083                          | 1.673833e+02                   |
| 2, 3          | 24.11387                          | 1.968289e+01                   |
| 2, 4          | 20.66394                          | 1.685976e+01                   |
| 2, 5          | 3.586832                          | 2.908742e+00                   |
| 3, 4          | 3.555515                          | 3.555215e+00                   |
| 3, 5          | 212.9059                          | 2.178424e+02                   |
| 4, 5          | 163.4091                          | 1.673876e+02                   |
| Total         | 616                               | 616                            |

Source: Researcher's computation

Result 1:  $\sum_{i=1}^5 \pi_i = n = 4 = \text{sample size selected}$

Result 2:  $\sum_{j=1, j \neq i}^5 \pi_{ij} = (n - 1)\pi_i = (4 - 1)\pi_i = 3\pi_i$

Result 3:  $\sum_{i=1}^5 \sum_{j=1, j \neq i}^5 \pi_{ij} = n(n - 1) = 4(4 - 1) = 12$

The double summation here means that  $12/2 = 6$

Result 4: The Sen – Yates – Grundy variance estimators are non – negative using P I and P II.

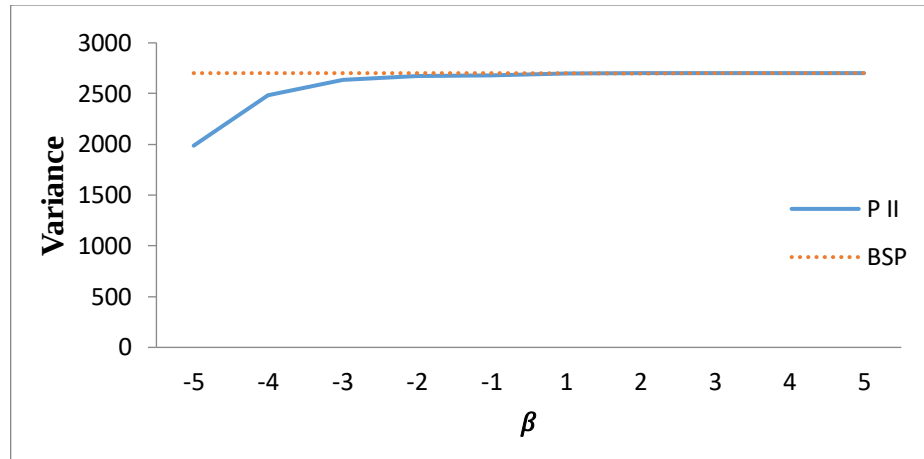


Figure 4.71: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data VIII)

Interpretation of Figure 4.71:

Figure 4.71 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -3$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

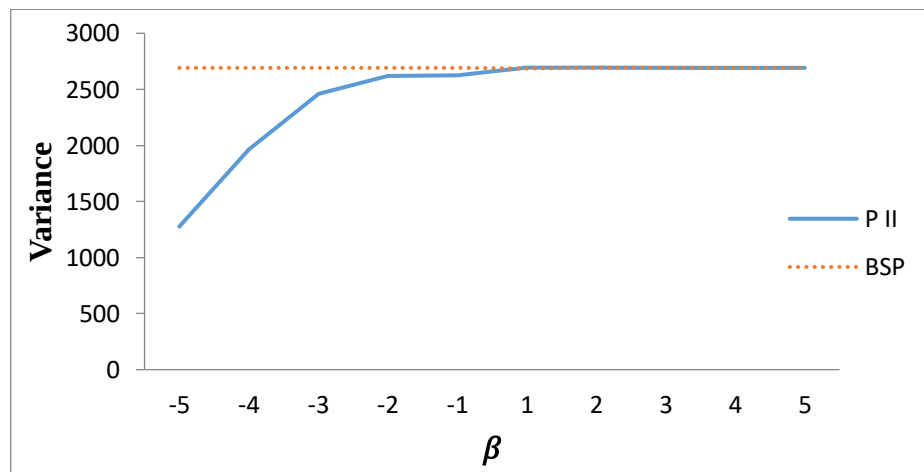


Figure 4.72: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data VIII)

Interpretation of Figure 4.72:

Figure 4.72 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq 1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

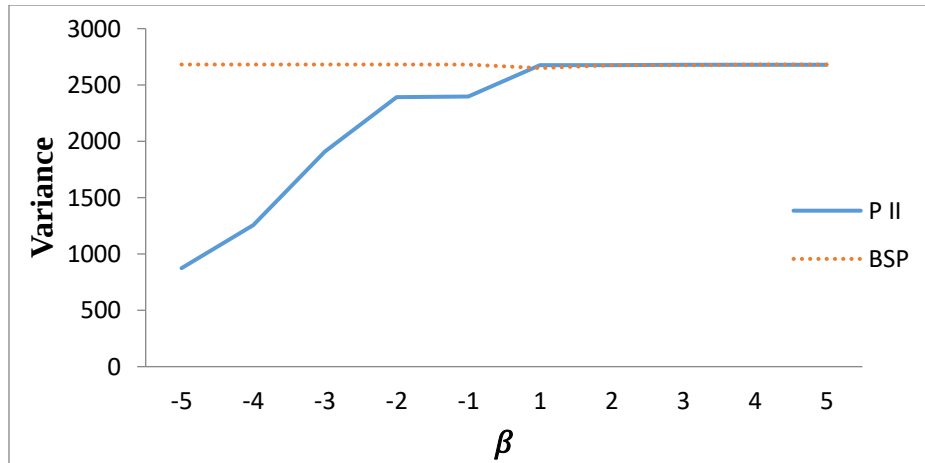


Figure 4.73: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data VIII)

Interpretation of Figure 4.73:

Figure 4.73 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq 1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

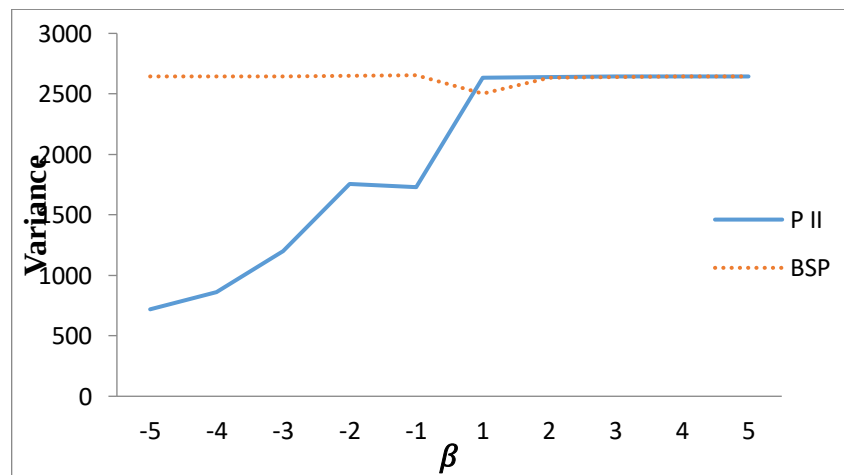


Figure 4.74: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data VIII)

Interpretation of Figure 4.74:

Figure 4.74 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq 1$ . It also shows that BSP has slightly lower variances than P II when  $1 < \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for a fixed value of  $\alpha = -2$ .

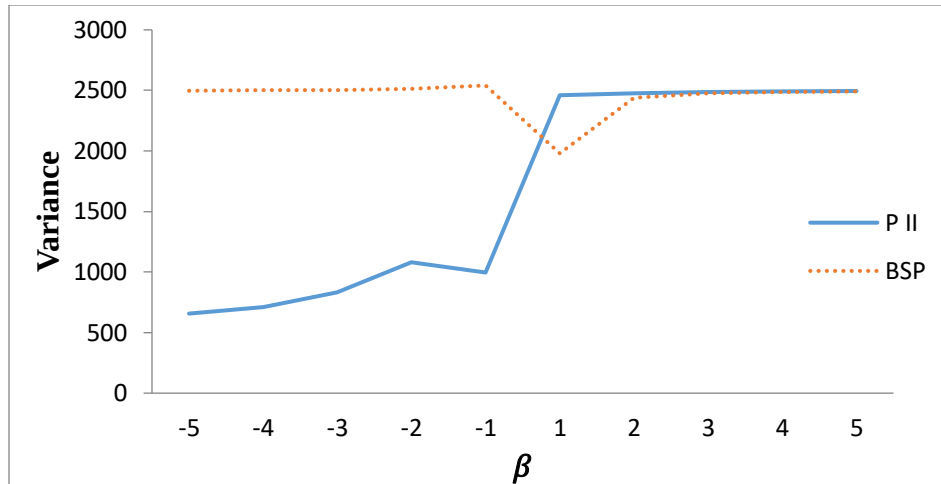


Figure 4.75: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data VIII)

Interpretation of Figure 4.75:

Figure 4.75 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq 1$ . It also shows that BSP has lower variances than P II when  $1 < \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for a fixed value of  $\alpha = -1$ .

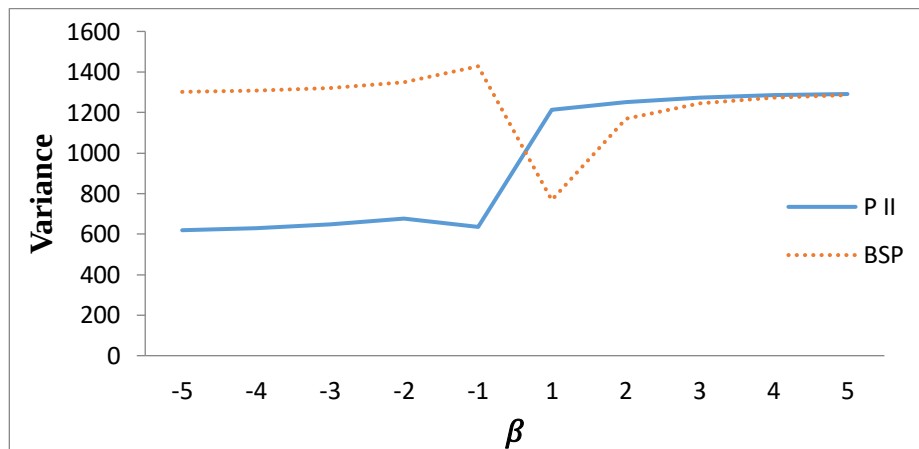


Figure 4.76: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data VIII)

Interpretation of Figure 4.76:

Figure 4.76 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 3$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

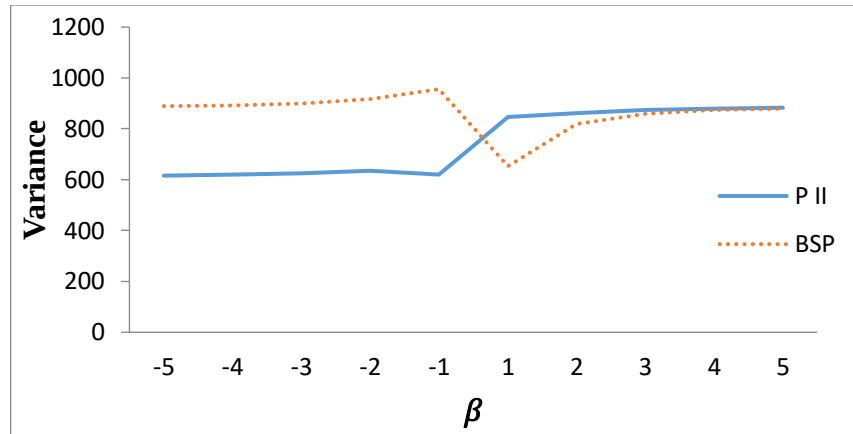


Figure 4.77: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data VIII)

Interpretation of Figure 4.77:

Figure 4.77 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 3$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

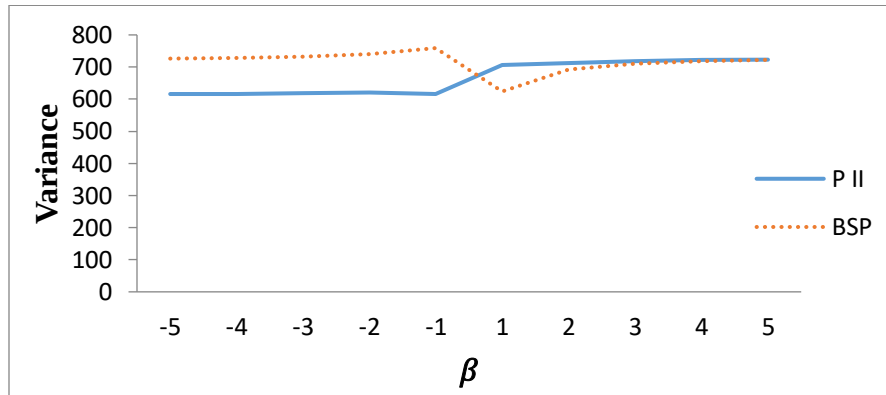


Figure 4.78: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data VIII)

Interpretation of Figure 4.78:

Figure 4.78 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

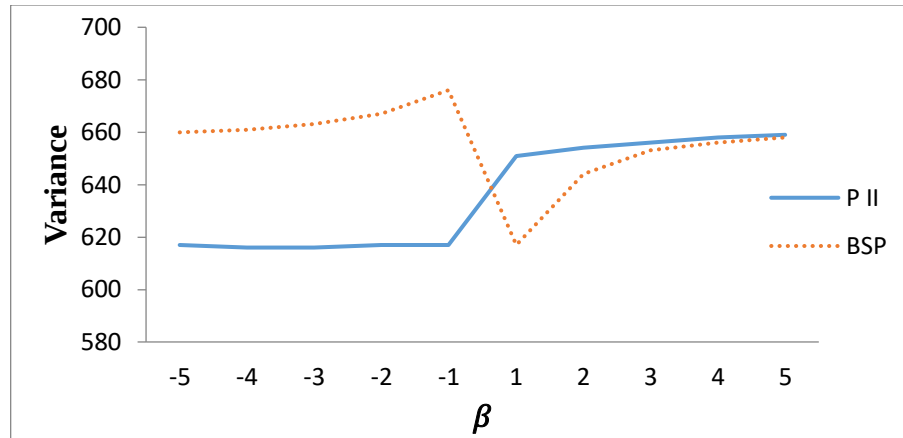


Figure 4.79: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data VIII)

Interpretation of Figure 4.79:

Figure 4.79 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 3$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 4$ .

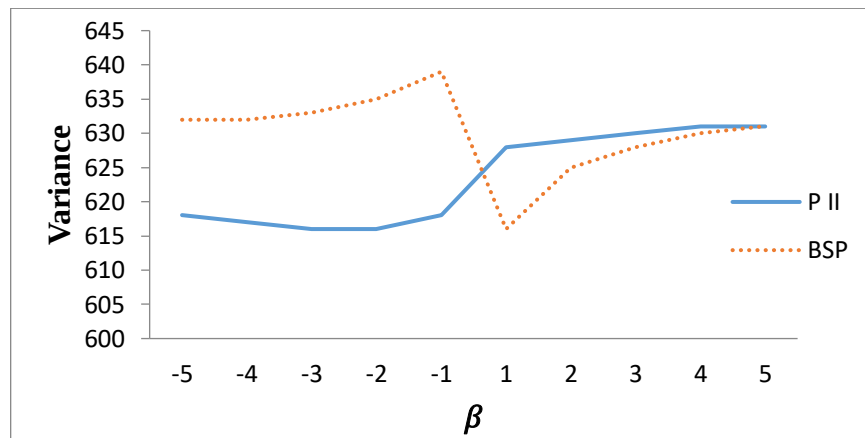


Figure 4.80: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data VIII)

Interpretation of Figure 4.80:

Figure 4.80 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 4$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

#### 4.2.2.3 Analysis of Data IX (Nigeria)

Variance using P I = 2030 (Data IX:  $N = 6$ )

Table 4.52: Comparison of variances of P II and BSP (Data IX:  $N = 6$ )

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | -5       |      | -4   |      | -3   |      | -2   |      | -1   |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 2335     | 2973 | 2060 | 2922 | 1779 | 2848 | 1534 | 2734 | 1347 | 2566 |
|         | -4 | 2563     | 2973 | 2333 | 2922 | 2057 | 2848 | 1776 | 2734 | 1531 | 2566 |
|         | -3 | 2728     | 2973 | 2556 | 2923 | 2324 | 2848 | 2047 | 2735 | 1766 | 2568 |
|         | -2 | 2834     | 2974 | 2712 | 2924 | 2533 | 2851 | 2293 | 2739 | 2012 | 2573 |
|         | -1 | 2884     | 2977 | 2785 | 2929 | 2637 | 2858 | 2426 | 2751 | 2160 | 2591 |
|         | 1  | 2969     | 2959 | 2916 | 2900 | 2838 | 2811 | 2719 | 2676 | 2719 | 2482 |
|         | 2  | 2972     | 2970 | 2920 | 2918 | 2844 | 2840 | 2729 | 2722 | 2558 | 2549 |
|         | 3  | 2972     | 2972 | 2922 | 2921 | 2847 | 2846 | 2732 | 2731 | 2563 | 2561 |
|         | 4  | 2973     | 2973 | 2922 | 2922 | 2847 | 2847 | 2733 | 2733 | 2565 | 2564 |
|         | 5  | 2973     | 2973 | 2922 | 2922 | 2848 | 2847 | 2734 | 2734 | 2565 | 2565 |

Source: Researcher's computation

Table 4.52 continues

|         |    | $\alpha$ |      |      |      |      |      |      |      |      |      |
|---------|----|----------|------|------|------|------|------|------|------|------|------|
|         |    | 1        |      | 2    |      | 3    |      | 4    |      | 5    |      |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II | BSP  | P II | BSP  | P II | BSP  |
|         | -5 | 1131     | 2061 | 1073 | 1780 | 1034 | 1535 | 1006 | 1348 | 986  | 1218 |
|         | -4 | 1216     | 2062 | 1130 | 1781 | 1072 | 1535 | 1033 | 1349 | 1006 | 1219 |
|         | -3 | 1339     | 2065 | 1212 | 1783 | 1127 | 1537 | 1070 | 1350 | 1032 | 1220 |
|         | -2 | 1496     | 2073 | 1319 | 1791 | 1198 | 1544 | 1117 | 1355 | 1064 | 1223 |
|         | -1 | 1607     | 2100 | 1397 | 1817 | 1249 | 1566 | 1150 | 1371 | 1085 | 1234 |
|         | 1  | 2030     | 1942 | 1750 | 1668 | 1511 | 1444 | 1330 | 1282 | 1206 | 1172 |
|         | 2  | 2050     | 2036 | 1769 | 1755 | 1525 | 1514 | 1341 | 1333 | 1213 | 1207 |
|         | 3  | 2058     | 2054 | 1776 | 1773 | 1532 | 1529 | 1346 | 1343 | 1217 | 1215 |
|         | 4  | 2060     | 2059 | 1779 | 1778 | 1534 | 1533 | 1347 | 1347 | 1218 | 1217 |
|         | 5  | 2061     | 2061 | 1780 | 1779 | 1534 | 1534 | 1348 | 1348 | 1218 | 1218 |

Source: Researcher's computation

Table 4.53: Probabilities of inclusion,  $\pi_i$  (Data IX:  $N = 6$ )

| Units | Zones           | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|-------|-----------------|-----------------------------------|---------------------------------|
| 1     | North Central   | 0.8006335                         | 0.8066915                       |
| 2     | North East      | 0.8002986                         | 0.8045171                       |
| 3     | North West      | 0.9843037                         | 0.9442845                       |
| 4     | South East      | 0.8000723                         | 0.8021628                       |
| 5     | South South     | 0.8008773                         | 0.8079377                       |
| 6     | South West      | 0.8138146                         | 0.8344064                       |
|       | Sample size (n) | 5                                 | 5                               |

Source: Researcher's computation

Table 4.54: Joint Probabilities of inclusion,  $\pi_{ij}$  (Data IX:  $N = 6$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 0.6009321                         | 0.6112086                       |
| 1, 3          | 0.7849372                         | 0.7509761                       |
| 1, 4          | 0.6007057                         | 0.6088543                       |
| 1, 5          | 0.6015107                         | 0.6146292                       |
| 1, 6          | 0.6144481                         | 0.6410979                       |
| 2, 3          | 0.7846024                         | 0.7488016                       |
| 2, 4          | 0.6003709                         | 0.6066798                       |
| 2, 5          | 0.6011759                         | 0.6124548                       |
| 2, 6          | 0.6141132                         | 0.6389235                       |
| 3, 4          | 0.784376                          | 0.7464473                       |
| 3, 5          | 0.785181                          | 0.7522223                       |
| 3, 6          | 0.7981184                         | 0.778691                        |
| 4, 5          | 0.6009495                         | 0.6101005                       |
| 4, 6          | 0.6138869                         | 0.6365692                       |
| 5, 6          | 0.6146919                         | 0.6423441                       |
| Total         | 10                                | 10                              |

Source: Researcher's computation

Table 4.55: Least variance estimates (Data IX:  $N = 6$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 0.1070216                         | 0.1178012                       |
| 1, 3          | 28.7546968                        | 118.9763997                     |
| 1, 4          | 26.6319932                        | 25.7138316                      |
| 1, 5          | 278.0967614                       | 249.6158563                     |
| 1, 6          | 34.0643929                        | 24.7878414                      |
| 2, 3          | 27.9592819                        | 117.0344590                     |
| 2, 4          | 23.4124058                        | 22.6570191                      |
| 2, 5          | 267.9115616                       | 242.5576020                     |
| 2, 6          | 30.5834609                        | 22.1320832                      |
| 3, 4          | 16.8478310                        | 74.1217354                      |
| 3, 5          | 1.5944922                         | 10.2162699                      |
| 3, 6          | 13.7059789                        | 56.0838005                      |
| 4, 5          | 133.4271202                       | 120.8482387                     |
| 4, 6          | 0.8355363                         | 0.2158496                       |
| 5, 6          | 102.2133779                       | 87.3490702                      |
| Total         | 986                               | 1172                            |

Source: Researcher's computation

Result 1:  $\sum_{i=1}^6 \pi_i = n = 5 = \text{sample size selected}$

Result 2:  $\sum_{\substack{j=1 \\ j \neq i}}^6 \pi_{ij} = (n - 1)\pi_i = (5 - 1)\pi_i = 4\pi_i$

Result 3:  $\sum_{\substack{i=1 \\ j \neq i}}^6 \sum_{j=1}^6 \pi_{ij} = n(n - 1) = 5(5 - 1) = 20$

The double summation here means that  $20/2 = 10$

Result 4: The Sen – Yates – Grundy variance estimators are non – negative using P I and P II.

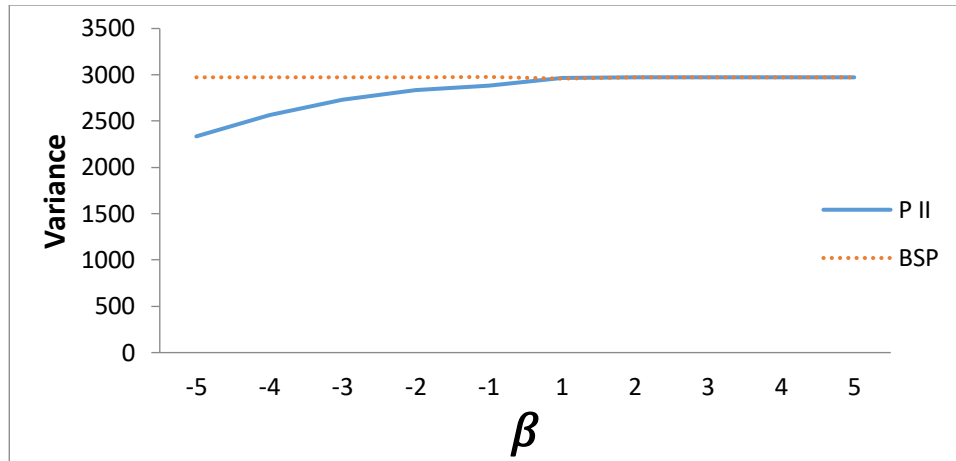


Figure 4.81: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data IX)

Interpretation of Figure 4.81:

Figure 4.81 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

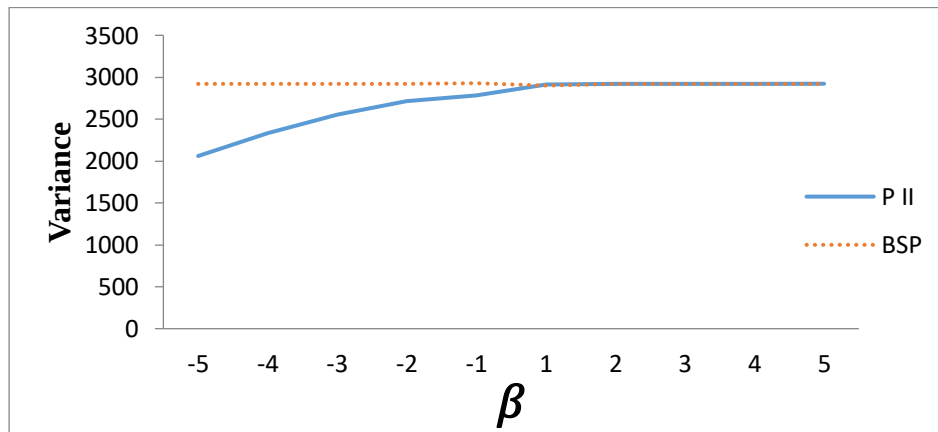


Figure 4.82: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data IX)

Interpretation of Figure 4.82:

Figure 4.82 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

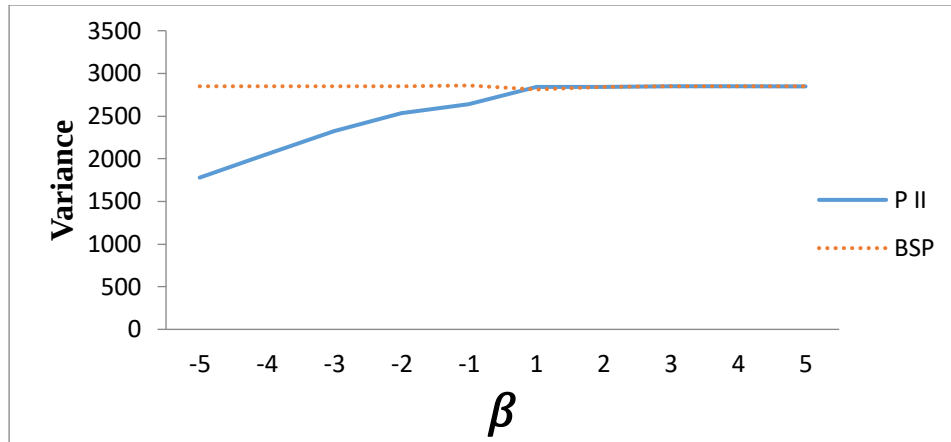


Figure 4.83: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data IX)

Interpretation of Figure 4.83:

Figure 4.83 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

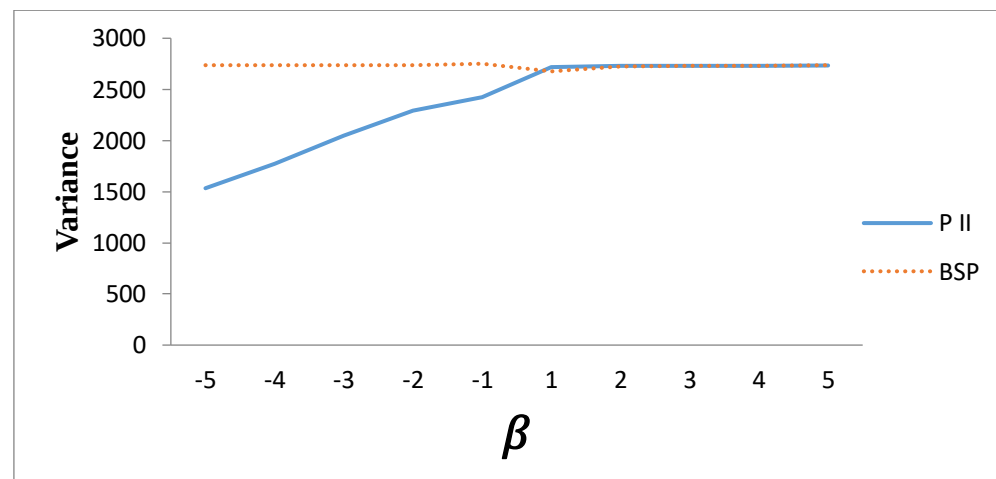


Figure 4.84: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data IX)

Interpretation of Figure 4.84

Figure 4.84 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

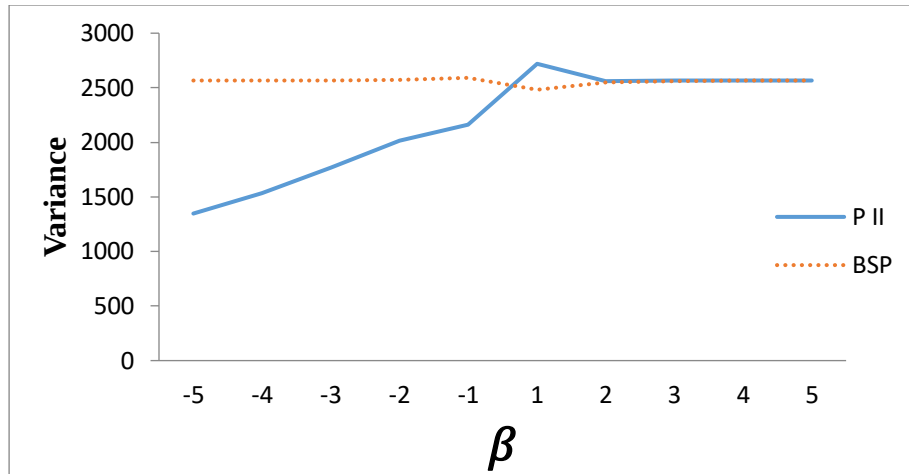


Figure 4.85: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data IX)

Interpretation of Figure 4.85:

Figure 4.85 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . BSP has lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

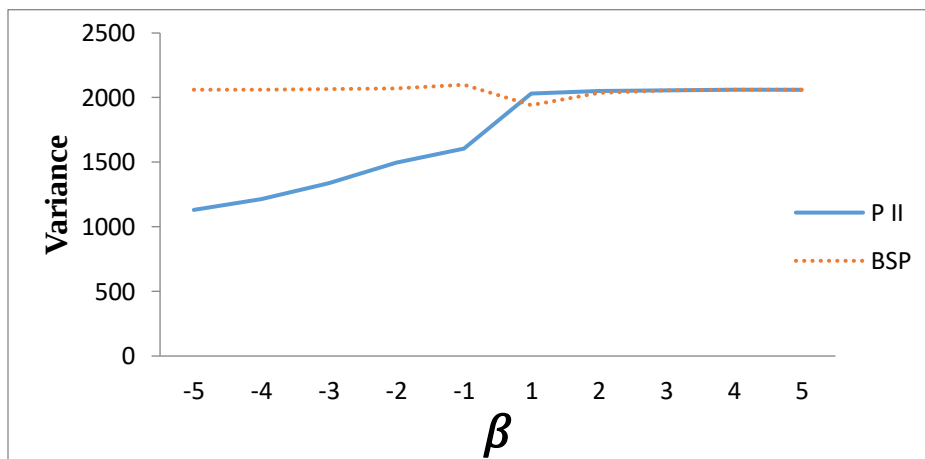


Figure 4.86: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data IX)

Interpretation of Figure 4.86:

Figure 4.86 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

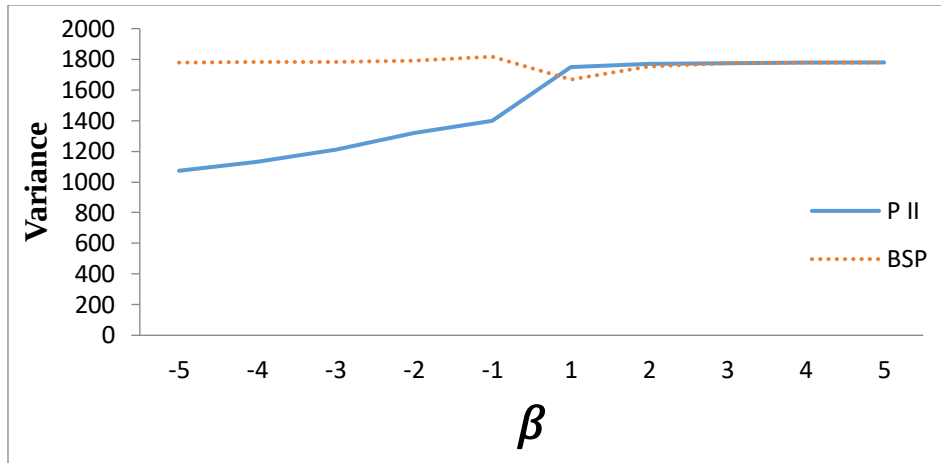


Figure 4.87: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data IX)

Interpretation of Figure 4.87:

Figure 4.87 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

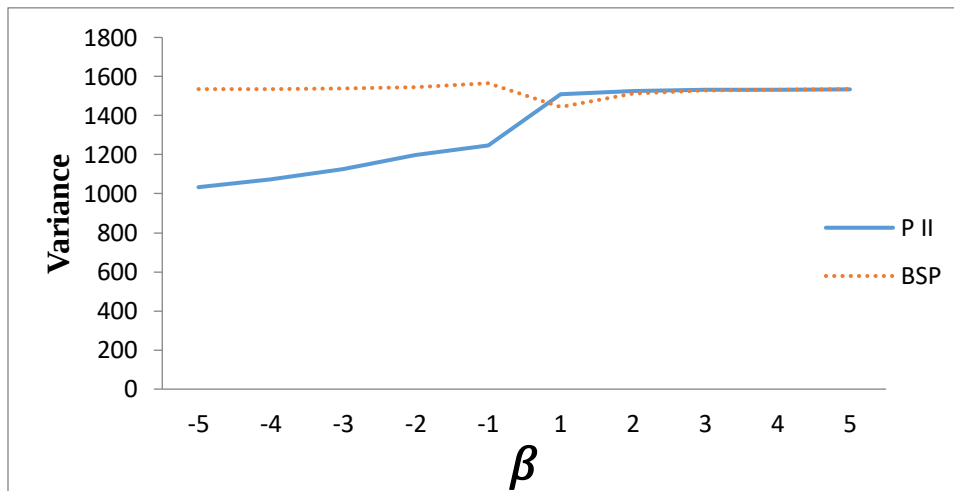


Figure 4.88: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data IX)

Interpretation of Figure 4.88:

Figure 4.88 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

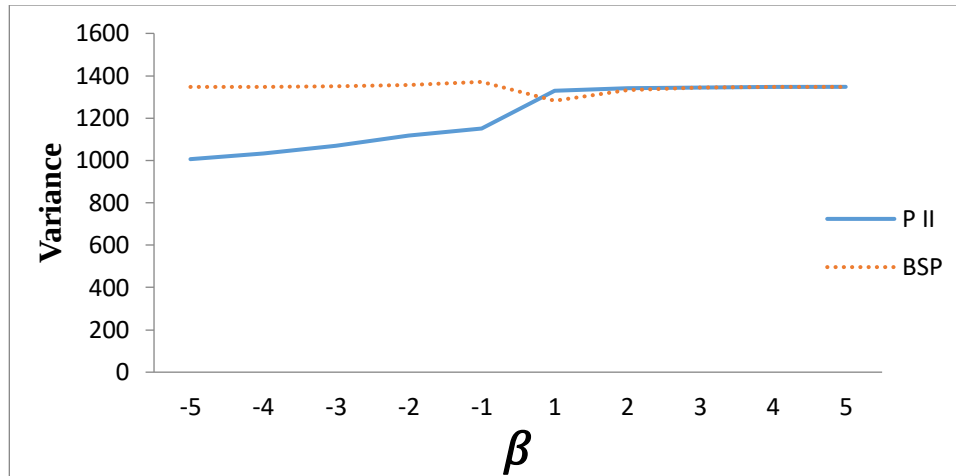


Figure 4.89: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data IX)

Interpretation of Figure 4.89:

Figure 4.89 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 4$ .

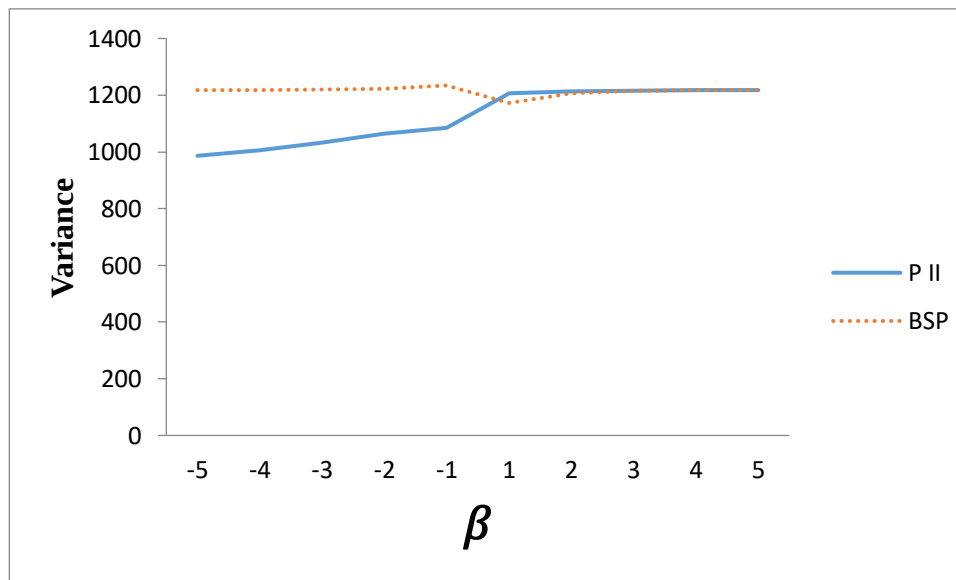


Figure 4.90: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data IX)

Interpretation of Figure 4.90:

Figure 4.90 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

#### 4.2.2.4 Analysis of Data X (The Gambia)

Variance using P I = 6660 (Data X:  $N = 8$ )

Table 4.56: Comparison of variances of P II and BSP (Data X:  $N = 8$ )

|         |    | $\alpha$ |       |       |       |       |       |       |       |       |       |
|---------|----|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|         |    | -5       |       | -4    |       | -3    |       | -2    |       | -1    |       |
| $\beta$ |    | P II     | BSP   | P II  | BSP   | P II  | BSP   | P II  | BSP   | P II  | BSP   |
|         | -5 | 9202     | 10636 | 6890  | 10625 | 5036  | 10596 | 4200  | 10530 | 3907  | 10306 |
|         | -4 | 10293    | 10637 | 9178  | 10625 | 6850  | 10597 | 5010  | 10532 | 4189  | 10310 |
|         | -3 | 10508    | 10637 | 10279 | 10626 | 9106  | 10598 | 6739  | 10534 | 4940  | 10316 |
|         | -2 | 10567    | 10637 | 10503 | 10626 | 10231 | 10599 | 8876  | 10536 | 6421  | 10327 |
|         | -1 | 10591    | 10637 | 10562 | 10626 | 10465 | 10600 | 9865  | 10540 | 7602  | 10356 |
|         | 1  | 10602    | 10637 | 10592 | 10626 | 10566 | 10597 | 10505 | 10513 | 10262 | 10076 |
|         | 2  | 10602    | 10637 | 10592 | 10626 | 10567 | 10599 | 10508 | 10534 | 10286 | 10281 |
|         | 3  | 10602    | 10637 | 10592 | 10626 | 10567 | 10600 | 10509 | 10536 | 10295 | 10305 |
|         | 4  | 10602    | 10637 | 10592 | 10626 | 10567 | 10600 | 10509 | 10537 | 10298 | 10312 |
|         | 5  | 10602    | 10637 | 10592 | 10626 | 10567 | 10600 | 10509 | 10537 | 10300 | 10315 |

Source: Researcher's computation

Table 4.56 continues

|         |    | $\alpha$ |      |      |      |             |      |      |      |      |             |
|---------|----|----------|------|------|------|-------------|------|------|------|------|-------------|
|         |    | 1        |      | 2    |      | 3           |      | 4    |      | 5    |             |
| $\beta$ |    | P II     | BSP  | P II | BSP  | P II        | BSP  | P II | BSP  | P II | BSP         |
|         | -5 | 3790     | 6910 | 3783 | 5050 | <b>3782</b> | 4205 | 3782 | 3909 | 3782 | 3816        |
|         | -4 | 3815     | 6921 | 3789 | 5057 | 3783        | 4208 | 3782 | 3910 | 3782 | 3816        |
|         | -3 | 3894     | 6949 | 3812 | 5075 | 3789        | 4216 | 3783 | 3912 | 3782 | 3817        |
|         | -2 | 4092     | 7025 | 3872 | 5124 | 3806        | 4236 | 3787 | 3919 | 3783 | 3819        |
|         | -1 | 4231     | 7246 | 3912 | 5270 | 3818        | 4296 | 3791 | 3939 | 3784 | 3825        |
|         | 1  | 6660     | 5695 | 4901 | 4410 | 4147        | 3969 | 3890 | 3834 | 3811 | <b>3795</b> |
|         | 2  | 6805     | 6638 | 4983 | 4881 | 4178        | 4138 | 3900 | 3886 | 3814 | 3810        |
|         | 3  | 6872     | 6817 | 5024 | 4989 | 4195        | 4180 | 3906 | 3900 | 3815 | 3813        |
|         | 4  | 6898     | 6875 | 5041 | 5026 | 4202        | 4195 | 3908 | 3905 | 3816 | 3815        |
|         | 5  | 6908     | 6896 | 5048 | 5039 | 4205        | 4200 | 3909 | 3907 | 3816 | 3815        |

Source: Researcher's computation

Table 4.57: Probability of inclusion,  $\pi_i$  (Data X:  $N = 8$ )

| Units | Local Government Areas | P II ( $\alpha = 3, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|-------|------------------------|-----------------------------------|---------------------------------|
| 1     | Banjul                 | 0.8571429                         | 0.8571429                       |
| 2     | Basse                  | 0.8571695                         | 0.8574571                       |
| 3     | Brikama                | 0.9988537                         | 0.9957746                       |
| 4     | Janjanbureh            | 0.857143                          | 0.8571548                       |
| 5     | Kanifing               | 0.8582484                         | 0.8608318                       |
| 6     | Kerewan                | 0.8571567                         | 0.8573484                       |
| 7     | Kuntaur                | 0.8571429                         | 0.8571463                       |
| 8     | Mansakonko             | 0.8571429                         | 0.8571442                       |
|       | Sample size (n)        | 7                                 | 7                               |

Source: Researcher's computation

Table 4.58: Joint Probability of inclusion,  $\pi_{ij}$  (Data X:  $N = 8$ )

| Units Sampled | P II ( $\alpha = 3, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 0.7143124                         | 0.7146                          |
| 1, 3          | 0.8559966                         | 0.8529174                       |
| 1, 4          | 0.7142859                         | 0.7142977                       |
| 1, 5          | 0.7153912                         | 0.7179746                       |
| 1, 6          | 0.7142996                         | 0.7144913                       |
| 1, 7          | 0.7142857                         | 0.7142891                       |
| 1, 8          | 0.7142857                         | 0.7142871                       |
| 2, 3          | 0.8560233                         | 0.8532317                       |
| 2, 4          | 0.7143126                         | 0.7146119                       |
| 2, 5          | 0.7154179                         | 0.7182889                       |
| 2, 6          | 0.7143263                         | 0.7148055                       |
| 2, 7          | 0.7143124                         | 0.7146034                       |
| 2, 8          | 0.7143124                         | 0.7146013                       |
| 3, 4          | 0.8559968                         | 0.8529294                       |
| 3, 5          | 0.8571021                         | 0.8566063                       |
| 3, 6          | 0.8560105                         | 0.853123                        |
| 3, 7          | 0.8559966                         | 0.8529208                       |
| 3, 8          | 0.8559966                         | 0.8529188                       |
| 4, 5          | 0.7153914                         | 0.7179866                       |
| 4, 6          | 0.7142997                         | 0.7145032                       |
| 4, 7          | 0.7142859                         | 0.7143011                       |
| 4, 8          | 0.7142859                         | 0.7142991                       |
| 5, 6          | 0.7154051                         | 0.7181802                       |
| 5, 7          | 0.7153912                         | 0.717978                        |
| 5, 8          | 0.7153912                         | 0.717976                        |
| 6, 7          | 0.7142996                         | 0.7144947                       |
| 6, 8          | 0.7142996                         | 0.7144926                       |
| 7, 8          | 0.7142857                         | 0.7142905                       |
| Total         | 21                                | 21                              |

Source: Researcher's computation

Table 4.59: Least variance estimates (Data X:  $N = 8$ )

| Units Sampled | P II ( $\alpha = 3, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 17.14427                          | 17.08267                        |
| 1, 3          | 8.323641                          | 31.00204                        |
| 1, 4          | 15.55552                          | 15.55321                        |
| 1, 5          | 721.0589                          | 700.6495                        |
| 1, 6          | 32.70011                          | 32.62482                        |
| 1, 7          | 0.03888885                        | 0.03888251                      |
| 1, 8          | 1.905555                          | 1.905514                        |
| 2, 3          | 6.481873                          | 24.11081                        |
| 2, 4          | 0.03879111                        | 0.03777628                      |
| 2, 5          | 516.5714                          | 500.236                         |
| 2, 6          | 2.488305                          | 2.479755                        |
| 2, 7          | 15.55027                          | 15.49314                        |
| 2, 8          | 7.619259                          | 7.587346                        |
| 3, 4          | 6.565616                          | 24.47262                        |
| 3, 5          | 0.5459373                         | 1.731907                        |
| 3, 6          | 5.841871                          | 21.75558                        |
| 3, 7          | 8.230791                          | 30.65654                        |
| 3, 8          | 7.68464                           | 28.62956                        |
| 4, 5          | 525.6382                          | 510.1257                        |
| 4, 6          | 3.149124                          | 3.137065                        |
| 4, 7          | 14.03886                          | 14.03643                        |
| 4, 8          | 6.572206                          | 6.571006                        |
| 5, 6          | 447.7096                          | 433.6333                        |
| 5, 7          | 710.5558                          | 690.3905                        |
| 5, 8          | 649.1558                          | 630.5461                        |
| 6, 7          | 30.48375                          | 30.41209                        |
| 6, 8          | 18.81878                          | 18.77116                        |
| 7, 8          | 1.4                               | 1.399959                        |
| Total         | 3782                              | 3795                            |

Source: Researcher's computation

Result 1:  $\sum_{i=1}^8 \pi_i = n = 7 = \text{sample size selected}$

Result 2:  $\sum_{\substack{j=1 \\ j \neq i}}^8 \pi_{ij} = (n - 1)\pi_i = (7 - 1)\pi_i = 6\pi_i$

Result 3:  $\sum_{\substack{i=1 \\ j \neq i}}^8 \sum_{j=1}^8 \pi_{ij} = n(n - 1) = 7(7 - 1) = 42$

The double summation here means that  $42/2 = 21$

Result 4: The Sen – Yates – Grundy variance estimators are non – negative using P I and P II.

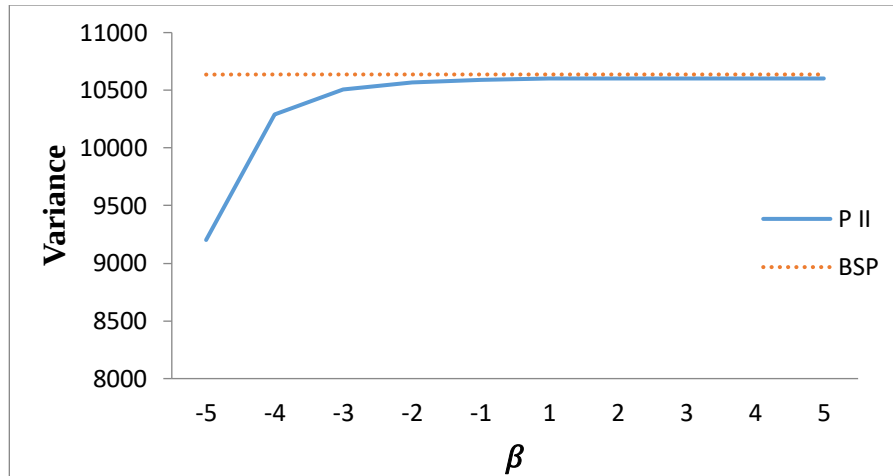


Figure 4.91: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data X)

Interpretation of Figure 4.91:

Figure 4.91 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

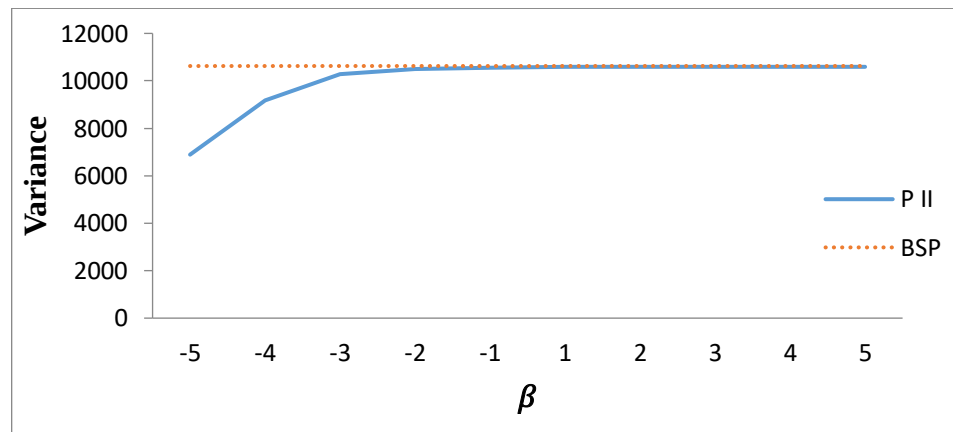


Figure 4.92: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data X)

Interpretation of Figure 4.92:

Figure 4.92 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -3$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

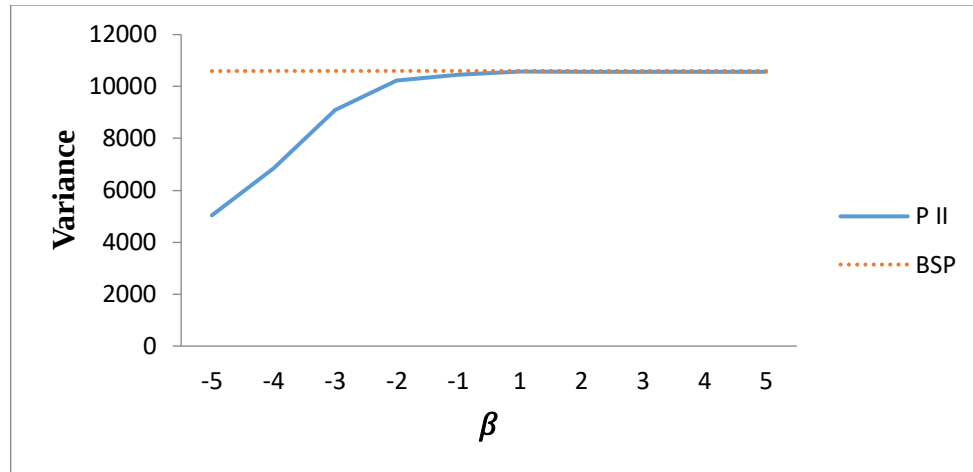


Figure 4.93: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data X)

Interpretation of Figure 4.93:

Figure 4.93 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

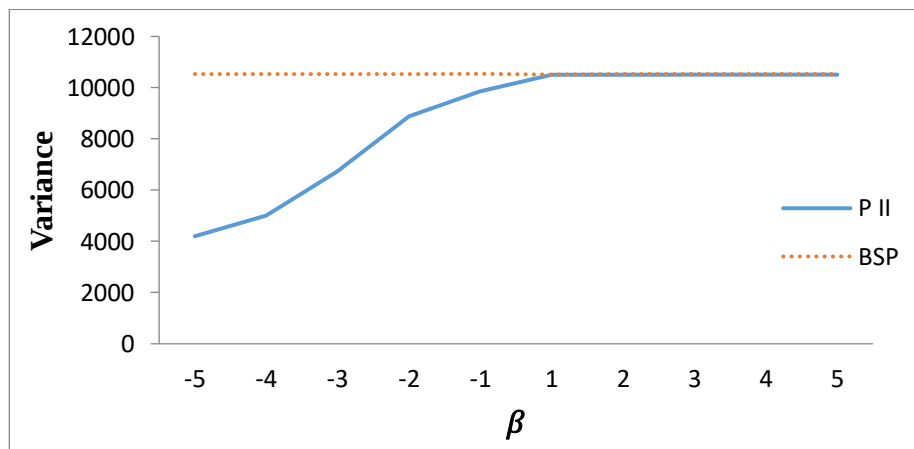


Figure 4.94: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data X)

Interpretation of Figure 4.94:

Figure 4.94 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq 1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

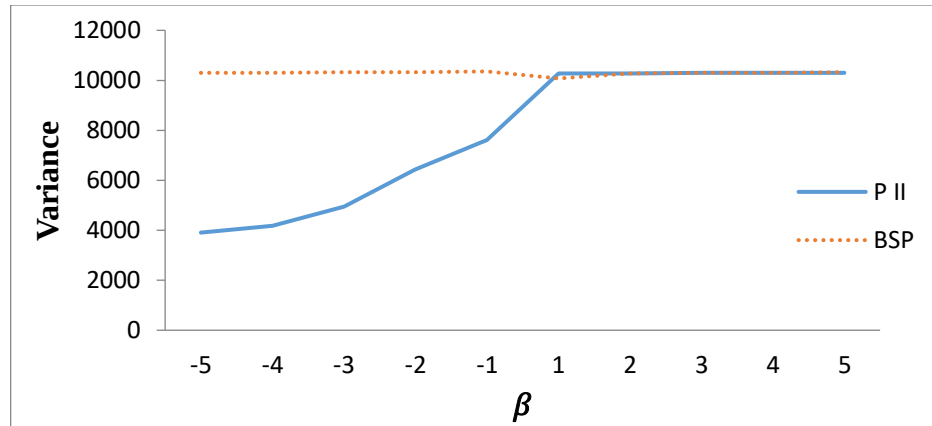


Figure 4.95: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data X)

Interpretation of Figure 4.95:

Figure 4.95 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

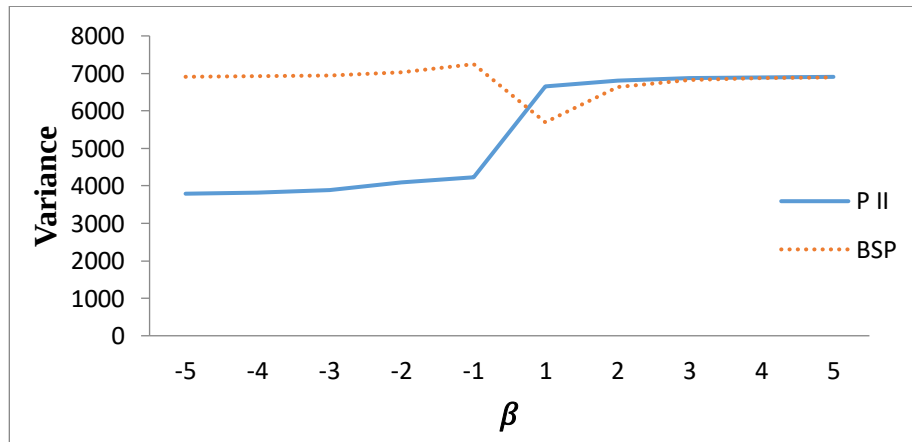


Figure 4.96: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data X)

Interpretation of Figure 4.96:

Figure 4.96 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

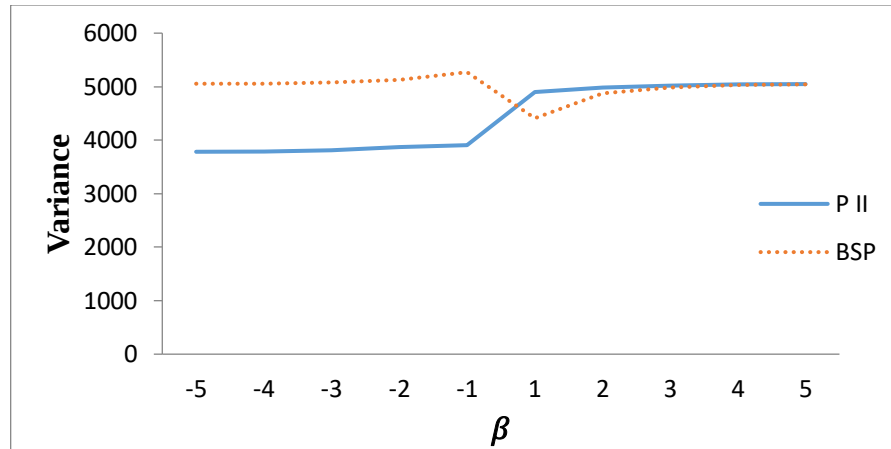


Figure 4.97: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data X)

Interpretation of Figure 4.97:

Figure 4.97 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

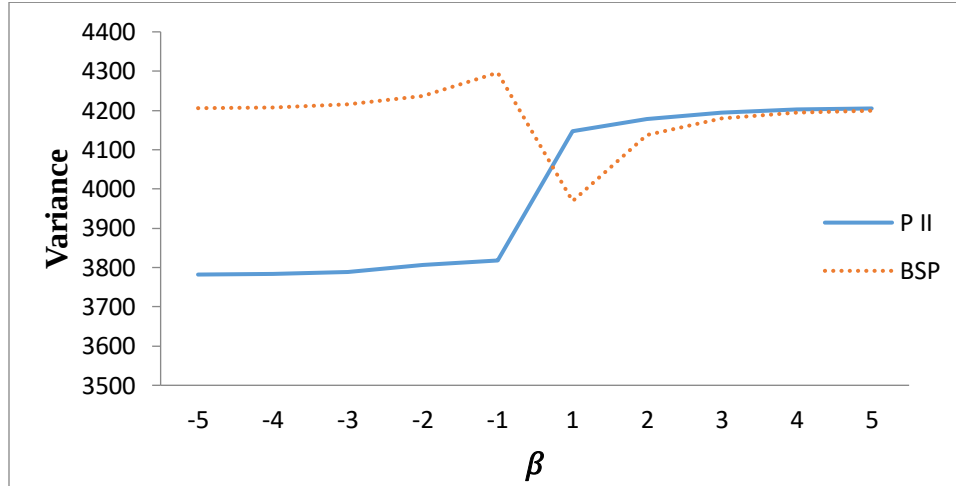


Figure 4.98: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data X)

Interpretation of Figure 4.98:

Figure 4.98 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 3$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

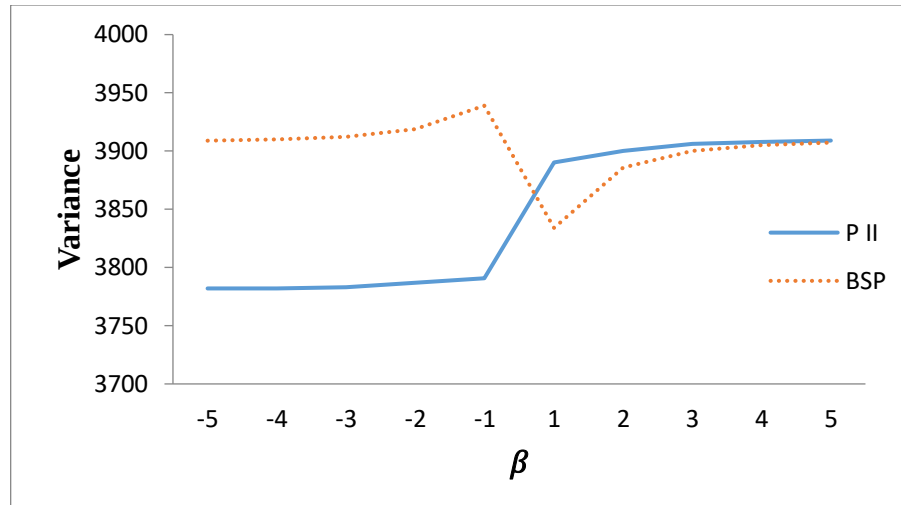


Figure 4.99: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data X)

Interpretation of Figure 4.99:

Figure 4.99 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 3$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 4$ .

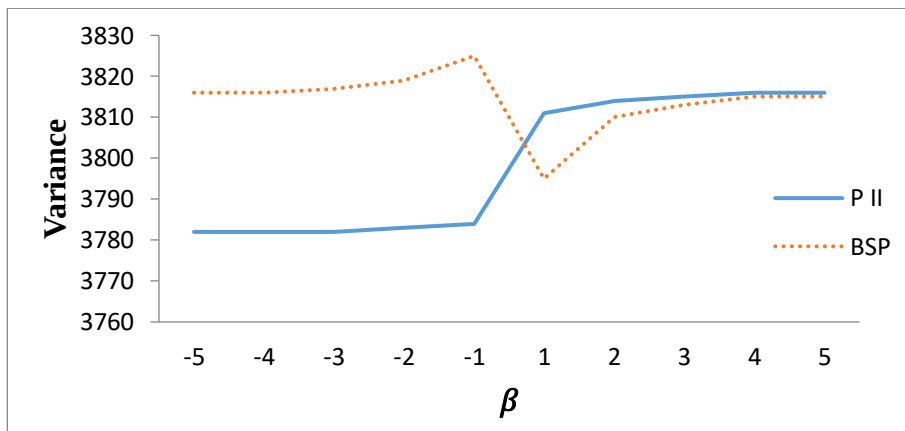


Figure 4.100: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data X)

Interpretation of Figure 4.100:

Figure 4.100 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has lower variances than P II when  $1 \leq \beta \leq 3$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

#### 4.2.2.5 Analysis of Data XI (South Africa)

Variance using P I = 279 (Data XI:  $N = 9$ )

Table 4.60: Comparison of variances of P II and BSP (Data XI:  $N = 9$ )

|         |    | $\alpha$ |     |      |     |      |     |      |     |      |     |
|---------|----|----------|-----|------|-----|------|-----|------|-----|------|-----|
|         |    | -5       |     | -4   |     | -3   |     | -2   |     | -1   |     |
| $\beta$ |    | P II     | BSP | P II | BSP | P II | BSP | P II | BSP | P II | BSP |
|         | -5 | 312      | 337 | 282  | 337 | 253  | 338 | 232  | 337 | 218  | 332 |
|         | -4 | 332      | 337 | 312  | 337 | 282  | 338 | 253  | 337 | 232  | 332 |
|         | -3 | 337      | 337 | 332  | 337 | 312  | 338 | 281  | 337 | 253  | 332 |
|         | -2 | 338      | 337 | 337  | 337 | 331  | 338 | 310  | 337 | 279  | 332 |
|         | -1 | 337      | 337 | 338  | 337 | 336  | 338 | 327  | 337 | 300  | 333 |
|         | 1  | 337      | 337 | 337  | 337 | 338  | 338 | 337  | 337 | 331  | 329 |
|         | 2  | 337      | 337 | 337  | 337 | 338  | 338 | 337  | 337 | 332  | 331 |
|         | 3  | 337      | 337 | 337  | 337 | 338  | 338 | 337  | 337 | 332  | 332 |
|         | 4  | 337      | 337 | 337  | 337 | 338  | 338 | 337  | 337 | 332  | 332 |
|         | 5  | 337      | 337 | 337  | 337 | 338  | 338 | 337  | 337 | 332  | 332 |

Source: Researcher's computation

Table 4.60 continues

|         |    | $\alpha$ |     |      |     |      |     |      |     |      |     |
|---------|----|----------|-----|------|-----|------|-----|------|-----|------|-----|
|         |    | 1        |     | 2    |     | 3    |     | 4    |     | 5    |     |
| $\beta$ |    | P II     | BSP | P II | BSP | P II | BSP | P II | BSP | P II | BSP |
|         | -5 | 200      | 282 | 194  | 253 | 190  | 232 | 186  | 218 | 182  | 208 |
|         | -4 | 208      | 282 | 200  | 253 | 194  | 232 | 190  | 218 | 186  | 208 |
|         | -3 | 217      | 282 | 207  | 254 | 200  | 233 | 194  | 218 | 189  | 208 |
|         | -2 | 230      | 283 | 216  | 254 | 206  | 233 | 199  | 218 | 193  | 208 |
|         | -1 | 241      | 285 | 223  | 256 | 211  | 234 | 202  | 220 | 196  | 209 |
|         | 1  | 279      | 273 | 251  | 246 | 231  | 226 | 217  | 214 | 200  | 204 |
|         | 2  | 281      | 280 | 253  | 252 | 232  | 231 | 218  | 217 | 208  | 207 |
|         | 3  | 282      | 281 | 253  | 253 | 232  | 232 | 218  | 218 | 208  | 208 |
|         | 4  | 282      | 282 | 253  | 253 | 232  | 232 | 218  | 218 | 208  | 208 |
|         | 5  | 282      | 282 | 253  | 253 | 232  | 232 | 218  | 218 | 208  | 208 |

Source: Researcher's computation

Table 4.61: Probabilities of inclusion,  $\pi_i$  (Data XI:  $N = 9$ )

| Units | Provinces       | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|-------|-----------------|-----------------------------------|---------------------------------|
| 1     | Western Cape    | 0.8750616                         | 0.8766392                       |
| 2     | Eastern Cape    | 0.8752037                         | 0.8780440                       |
| 3     | Northern Cape   | 0.875                             | 0.8750004                       |
| 4     | Free State      | 0.875                             | 0.8750353                       |
| 5     | KwaZulu – Natal | 0.8929234                         | 0.9074223                       |
| 6     | North West      | 0.8750004                         | 0.8751229                       |
| 7     | Gauteng         | 0.9817799                         | 0.9613679                       |
| 8     | Mpumalanga      | 0.8750016                         | 0.8752514                       |
| 9     | Limpopo         | 0.8750293                         | 0.8761166                       |
|       | Sample size (n) | 8                                 | 8                               |

Source: Researcher's computation

Table 4.62: Joint Probabilities of inclusion,  $\pi_{ij}$  (Data V:  $N = 9$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 0.7502654                         | 0.7546831                       |
| 1, 3          | 0.7500616                         | 0.7516396                       |
| 1, 4          | 0.7500617                         | 0.7516745                       |
| 1, 5          | 0.7679851                         | 0.7840614                       |
| 1, 6          | 0.750062                          | 0.7517620                       |
| 1, 7          | 0.8568416                         | 0.8380071                       |
| 1, 8          | 0.7500632                         | 0.7518906                       |
| 1, 9          | 0.7500909                         | 0.7527557                       |
| 2, 3          | 0.7502037                         | 0.7530444                       |
| 2, 4          | 0.7502037                         | 0.7530793                       |
| 2, 5          | 0.7681272                         | 0.7854663                       |
| 2, 6          | 0.7502041                         | 0.7531669                       |
| 2, 7          | 0.8569836                         | 0.8394119                       |
| 2, 8          | 0.7502053                         | 0.7532954                       |
| 2, 9          | 0.750233                          | 0.7541606                       |
| 3, 4          | 0.75                              | 0.7500358                       |
| 3, 5          | 0.7679234                         | 0.7824227                       |
| 3, 6          | 0.7500004                         | 0.7501233                       |
| 3, 7          | 0.8567799                         | 0.8363683                       |
| 3, 8          | 0.7500016                         | 0.7502519                       |
| 3, 9          | 0.7500293                         | 0.7511170                       |
| 4, 5          | 0.7679235                         | 0.7824576                       |
| 4, 6          | 0.7500004                         | 0.7501582                       |
| 4, 7          | 0.8567799                         | 0.8364033                       |
| 4, 8          | 0.7500016                         | 0.7502868                       |
| 4, 9          | 0.7500293                         | 0.7511519                       |
| 5, 6          | 0.7679238                         | 0.7825452                       |
| 5, 7          | 0.8747034                         | 0.8687902                       |
| 5, 8          | 0.767925                          | 0.7826737                       |
| 5, 9          | 0.7679527                         | 0.7835389                       |
| 6, 7          | 0.8567803                         | 0.8364908                       |
| 6, 8          | 0.750002                          | 0.7503743                       |
| 6, 9          | 0.7500297                         | 0.7512394                       |
| 7, 8          | 0.8567815                         | 0.8366193                       |
| 7, 9          | 0.8568092                         | 0.8374845                       |
| 8, 9          | 0.7500309                         | 0.7513680                       |
| Total         | 28                                | 28                              |

Source: Researcher's computation

Table 4.63: Least variance estimates (Data XI:  $N = 9$ )

| Units Sampled | P II ( $\alpha = 5, \beta = -5$ ) | BSP ( $\alpha = 5, \beta = 1$ ) |
|---------------|-----------------------------------|---------------------------------|
| 1, 2          | 13.13853                          | 1.258249e+01                    |
| 1, 3          | 24.47145                          | 2.400476e+01                    |
| 1, 4          | 21.31709                          | 2.089764e+01                    |
| 1, 5          | 18.06521                          | 1.518546e+01                    |
| 1, 6          | 11.99008                          | 1.172815e+01                    |
| 1, 7          | 0.1545289                         | 4.366399e-01                    |
| 1, 8          | 21.3168                           | 2.085883e+01                    |
| 1, 9          | 3.913888                          | 3.794698e+00                    |
| 2, 3          | 1.73667                           | 1.670286e+00                    |
| 2, 4          | 0.9765921                         | 9.352512e-01                    |
| 2, 5          | 0.8670118                         | 7.379744e-01                    |
| 2, 6          | 0.02734865                        | 2.910416e-02                    |
| 2, 7          | 2.849347                          | 6.444764e+00                    |
| 2, 8          | 0.9765831                         | 9.340635e-01                    |
| 2, 9          | 2.716616                          | 2.627145e+00                    |
| 3, 4          | 0.1088435                         | 1.087680e-01                    |
| 3, 5          | 0.07712649                        | 5.642115e-02                    |
| 3, 6          | 2.20407                           | 2.200448e+00                    |
| 3, 7          | 4.668884                          | 1.066033e+01                    |
| 3, 8          | 0.1088401                         | 1.083076e-01                    |
| 3, 9          | 8.813099                          | 8.693694e+00                    |
| 4, 5          | 0.0007429645                      | 1.992451e-03                    |
| 4, 6          | 1.333325                          | 1.330649e+00                    |
| 4, 7          | 4.173494                          | 9.553478e+00                    |
| 4, 8          | 7.005482e-12                      | 1.339703e-07                    |
| 4, 9          | 6.963353                          | 6.864120e+00                    |
| 5, 6          | 1.173805                          | 1.033727e+00                    |
| 5, 7          | 3.5384                            | 6.937193e+00                    |
| 5, 8          | 0.0007428218                      | 1.961043e-03                    |
| 5, 9          | 5.957985                          | 5.073056e+00                    |
| 6, 7          | 2.658359                          | 6.161561e+00                    |
| 6, 8          | 1.333312                          | 1.328809e+00                    |
| 6, 9          | 2.203065                          | 2.164331e+00                    |
| 7, 8          | 4.173438                          | 9.535686e+00                    |
| 7, 9          | 1.210091                          | 2.876068e+00                    |
| 8, 9          | 6.963266                          | 6.852186e+00                    |
| Total         | 182                               | 204                             |

Source: Researcher's computation

Result 1:  $\sum_{i=1}^9 \pi_i = n = 8 = \text{sample size selected}$

Result 2:  $\sum_{\substack{j=1 \\ j \neq i}}^9 \pi_{ij} = (n - 1)\pi_i = (8 - 1)\pi_i = 7\pi_i$

Result 3:  $\sum_{\substack{i=1 \\ j \neq i}}^9 \sum_{j=1}^9 \pi_{ij} = n(n - 1) = 8(8 - 1) = 56$

The double summation here means that  $56/2 = 28$

Result 4: The Sen – Yates – Grundy variance estimators are non – negative using P I and P II.

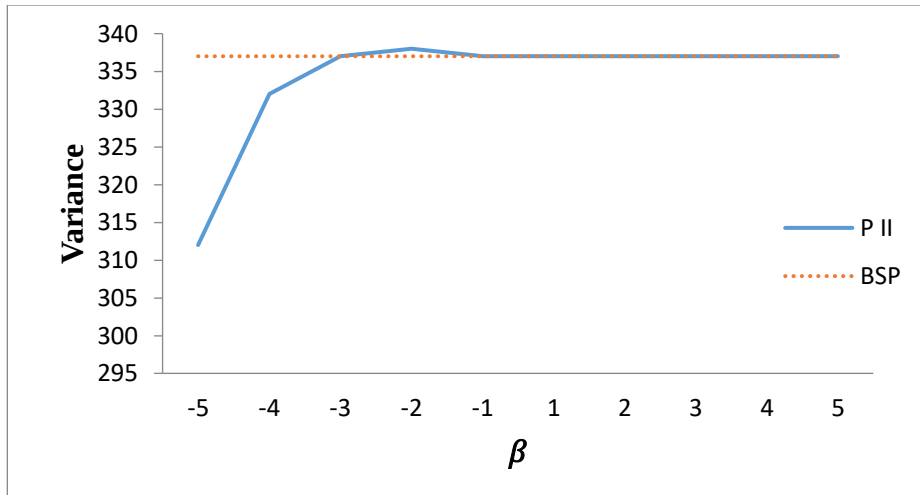


Figure 4.101: Comparison of variances using P II and BSP for  $\alpha = -5$  (Data XI)

Interpretation of Figure 4.101:

Figure 4.101 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -3$ . It also shows that BSP has slightly lower variances than P II when  $-3 < \beta \leq -2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -5$ .

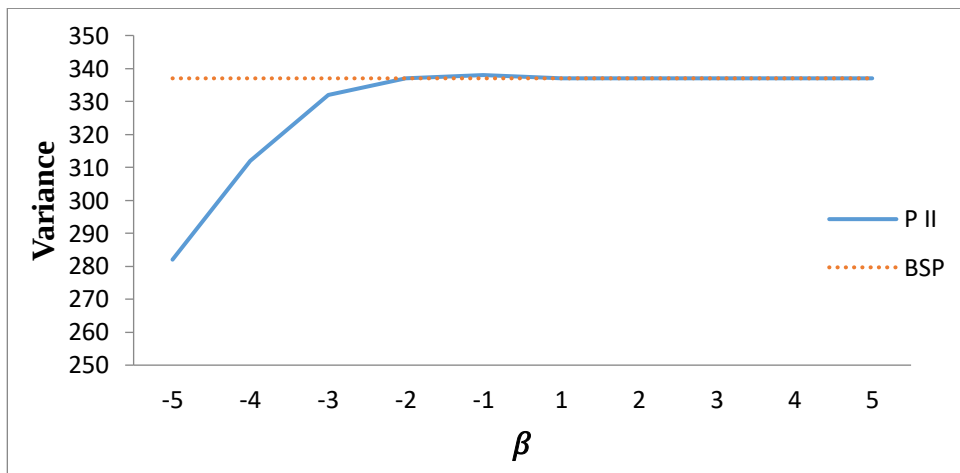


Figure 4.102: Comparison of variances using P II and BSP for  $\alpha = -4$  (Data XI)

Interpretation of Figure 4.102:

Figure 4.102 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -4$ .

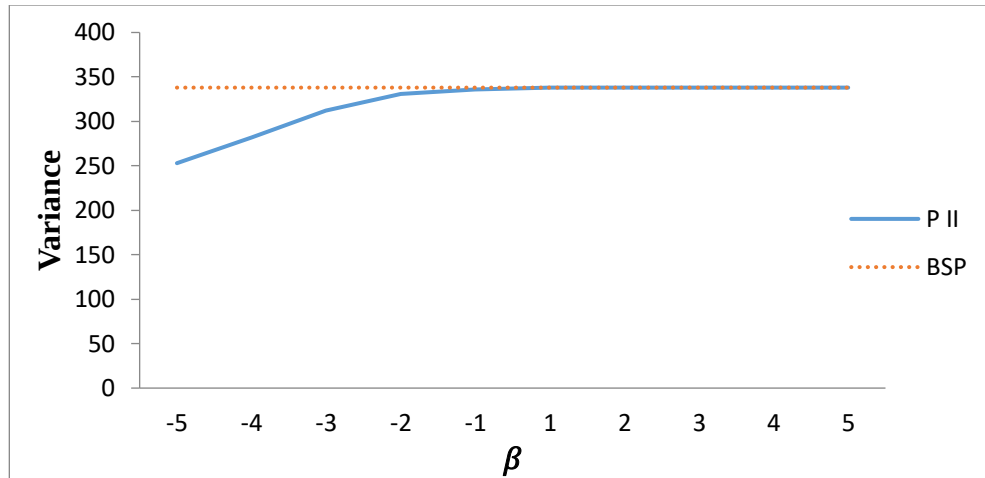


Figure 4.103: Comparison of variances using P II and BSP for  $\alpha = -3$  (Data XI)

Interpretation of Figure 4.103:

Figure 4.103 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -2$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -3$ .

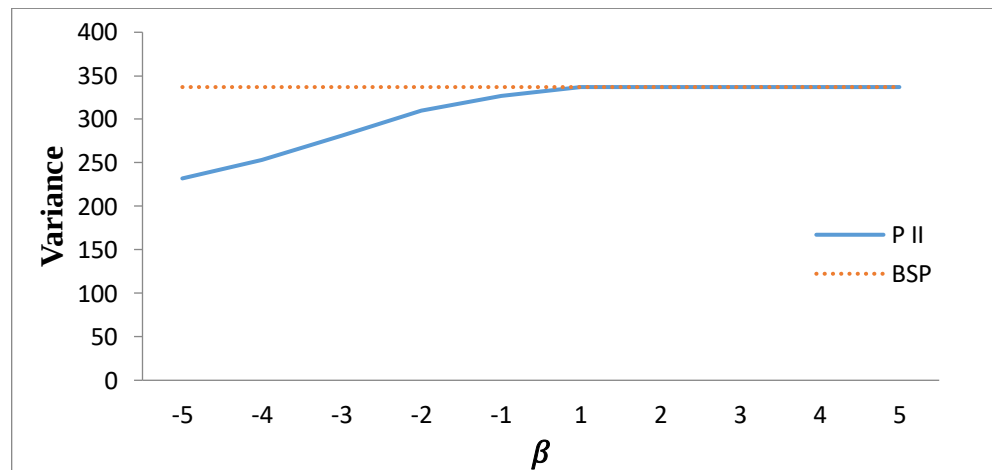


Figure 4.104: Comparison of variances using P II and BSP for  $\alpha = -2$  (Data XI)

Interpretation of Figure 4.104:

Figure 4.104 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -2$ .

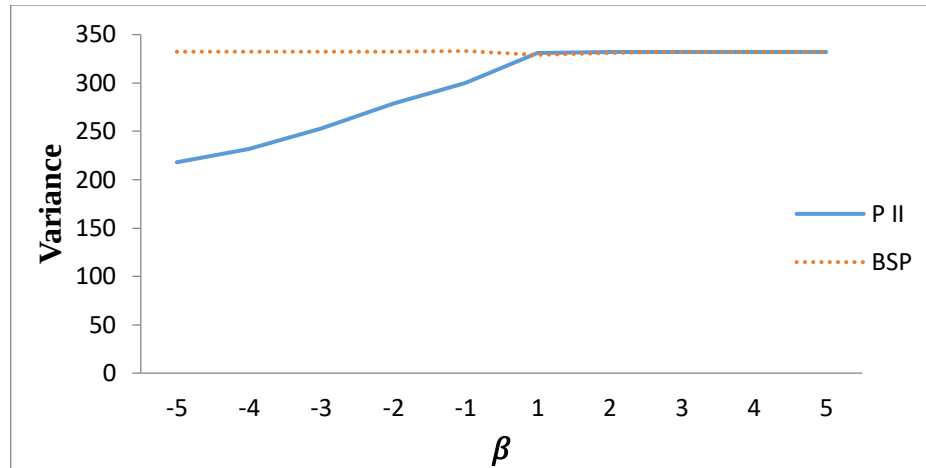


Figure 4.105: Comparison of variances using P II and BSP for  $\alpha = -1$  (Data XI)

Interpretation of Figure 4.105:

Figure 4.105 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . The two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = -1$ .

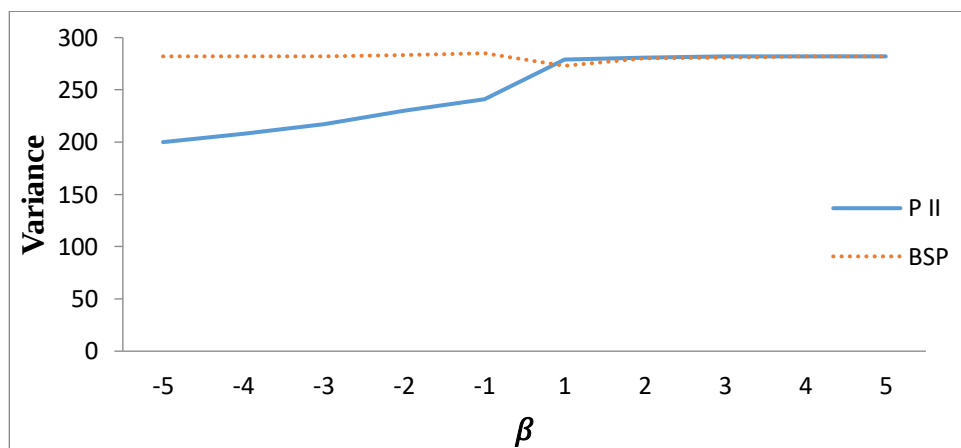


Figure 4.106: Comparison of variances using P II and BSP for  $\alpha = 1$  (Data XI)

Interpretation of Figure 4.106:

Figure 4.106 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 1$ .

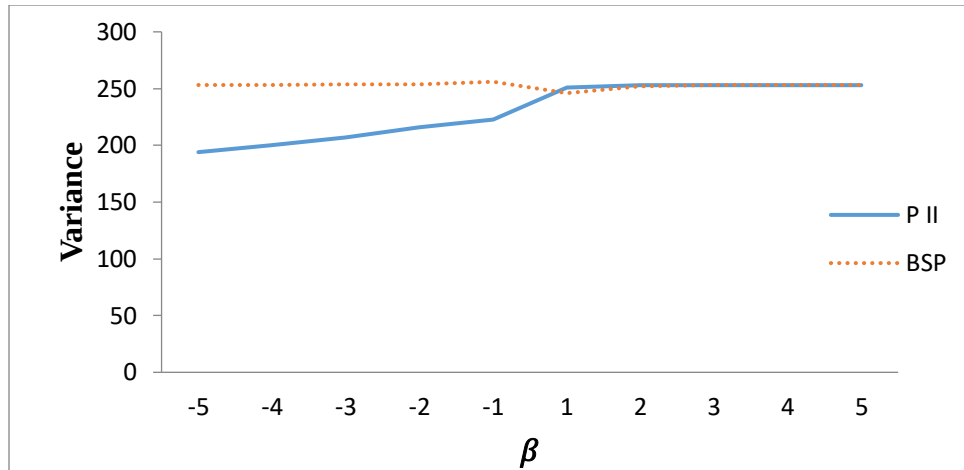


Figure 4.107: Comparison of variances using P II and BSP for  $\alpha = 2$  (Data XI)

Interpretation of Figure 4.107:

Figure 4.107 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 2$ .

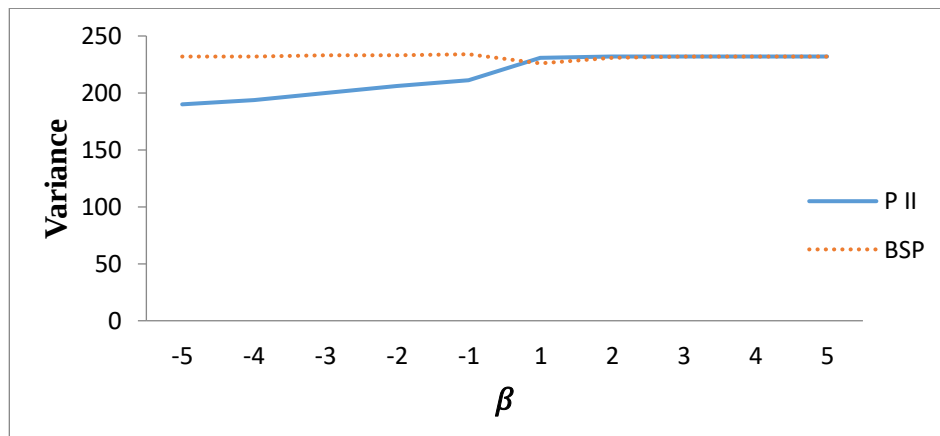


Figure 4.108: Comparison of variances using P II and BSP for  $\alpha = 3$  (Data XI)

Interpretation of Figure 4.108:

Figure 4.108 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 3$ .

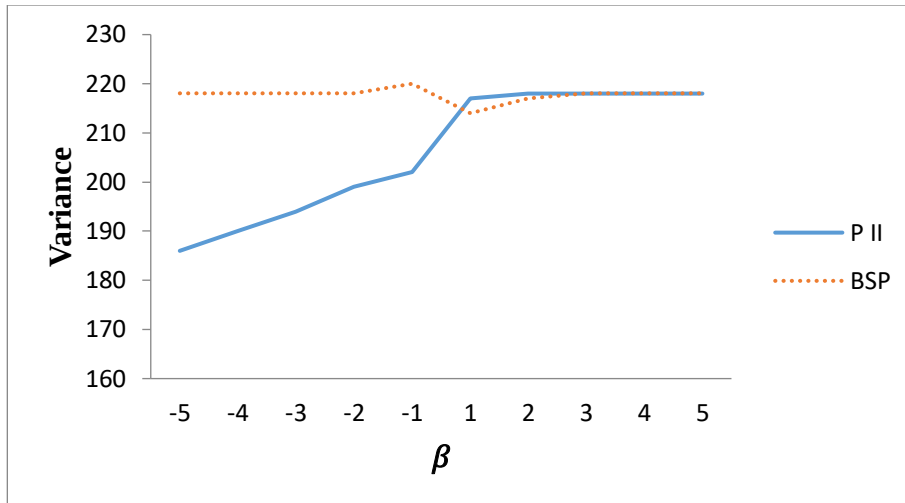


Figure 4.109: Comparison of variances using P II and BSP for  $\alpha = 4$  (Data XI)

Interpretation of Figure 4.109:

Figure 4.109 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq -1$ . It also shows that BSP has slightly lower variances than P II when  $1 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for a fixed value of  $\alpha$ .

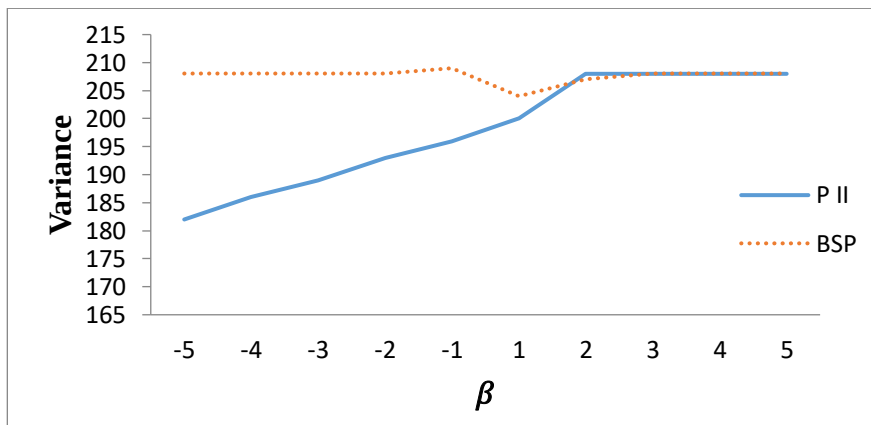


Figure 4.110: Comparison of variances using P II and BSP for  $\alpha = 5$  (Data XI)

Interpretation of Figure 4.110:

Figure 4.110 shows that P II has lower variances than BSP when  $-5 \leq \beta \leq 2$ , while the two procedures yield approximately same variances as the values of  $\beta$  increase, for  $\alpha = 5$ .

Table 4.64: The optimum values of  $\alpha$  and  $\beta$  at which the P II and BSP have least variance for all the five real life data using P II and BSP

| Serial Number | Data      | P II     |         | BSP      |         |
|---------------|-----------|----------|---------|----------|---------|
|               |           | $\alpha$ | $\beta$ | $\alpha$ | $\beta$ |
| 1             | Data VII  | 4        | 5       | 4        | -2      |
| 2             | Data VIII | 4        | -3      | 5        | 1       |
| 3             | Data IX   | 5        | -5      | 5        | 1       |
| 4             | Data X    | 3        | -5      | 5        | 1       |
| 5             | Data XI   | 5        | -5      | 5        | 1       |

Table 4.65: The least variance estimates using the P II and BSP

| Serial Number | Data      | Variances |      |
|---------------|-----------|-----------|------|
|               |           | P II      | BSP  |
| 1             | Data VII  | 574       | 573  |
| 2             | Data VIII | 616       | 616  |
| 3             | Data IX   | 969       | 1199 |
| 4             | Data X    | 3782      | 3795 |
| 5             | Data XI   | 182       | 204  |

### 4.2.3 Consistency of P II Using Life Data

Tables 4.66– 4.70 below show that as the sample size  $n$  increases, the variance estimate decreases. This shows that P II is consistent.

Table 4.66: Variance estimates for different sample sizes using P II (Data VII:  $N = 4$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 4, \beta = 5$ ) |
|---------------------|--------------------------------------|
| 3                   | 574                                  |
| 2                   | 3673                                 |

Table 4.67: Variance estimates for different sample sizes using P II (Data VIII:  $N=5$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 4, \beta = -3$ ) |
|---------------------|---------------------------------------|
| 4                   | 616                                   |
| 3                   | 4,271                                 |
| 2                   | 11,793,299                            |

Table 4.68: Variance estimates for different sample sizes using P II (Data IX:  $N = 6$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 5, \beta = -5$ ) |
|---------------------|---------------------------------------|
| 5                   | 986.1459                              |
| 4                   | 4829.713                              |
| 3                   | 30674.9                               |
| 2                   | 6172313                               |

Table 4.69: Variance estimates for different sample sizes using P II (Data X:  $N = 8$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 3, \beta = -5$ ) |
|---------------------|---------------------------------------|
| 7                   | 3782                                  |
| 6                   | 13453                                 |
| 5                   | 41488                                 |
| 4                   | 144895                                |
| 3                   | 786361                                |
| 2                   | 16643335192                           |

Table 4.70: Variance estimates for different sample sizes using P II (Data XI:  $N = 9$ )

| Sample Size ( $n$ ) | Variance ( $\alpha = 5, \beta = -5$ ) |
|---------------------|---------------------------------------|
| 8                   | 182                                   |
| 7                   | 571                                   |
| 6                   | 1486                                  |
| 5                   | 3994                                  |
| 4                   | 12905                                 |
| 3                   | 66694                                 |
| 2                   | 551183522                             |

## CHAPTER FIVE

### 5.0 Summary and Conclusion

#### 5.1 Summary

Variance estimates are obtained from all the eleven data under consideration using P I, P II, and the existing BSP. Using P I for each of the eleven data, one variance estimate is obtained in each case because P I does not contain the constants  $\alpha$  and  $\beta$  that vary in values. P I is a special case of P II when  $\alpha = \beta = 1$ . In this research work,  $\alpha$  and  $\beta$  only take values from  $-5$  to  $+5$ , excluding zero. Using P II and BSP for each data, there are one hundred pairs of variance estimates. This really gives room for comparison between P II and BSP as seen in Tables 4.7, 4.11, 4.15, 4.19, and 4.23.

For all the eleven data, the variance estimates obtained using P I shows the need for improvement. This shows why it is important to introduce the constant  $\alpha$  and  $\beta$ , and vary their values accordingly, thereby improving on the variance estimates. This leads to P II.

Firstly, the summary of the analysis obtained from the six simulated data are presented below.

For Data I, the variances obtained using P II and BSP are presented in Table 4.7. Figures 4.1 – 4.10 clearly show that P II has lower variances than BSP when both procedures do not yield approximately the same value. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$

increase. It is seen from Tables 4.31 and 4.32 that the least value of variance obtained when P II was used was 790 at  $\alpha = 5$  and  $\beta = -5$  while 1224 at  $\alpha = 5$  and  $\beta = 1$  was obtained when BSP was used. Considering Figures 4.1 – 4.10 for P II, it was also realized that the least values of the variance were gotten when  $\beta = -5$  while varying the values of  $\alpha$ .

For Data II, the variances obtained using P II and BSP are presented in Table 4.11. Figures 4.11 – 4.20 clearly show that P II has lower variances than BSP when both procedures do not yield approximately the same value. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase. It is seen from Tables 4.31 and 4.32 that the least value of variance obtained when P II was used was 880 at  $\alpha = 5$  and  $\beta = -5$  while 1018 at  $\alpha = 5$  and  $\beta = 1$  was obtained when BSP was used. Considering Figures 4.11 – 4.20 for P II, it was also realized that the least values of the variance were gotten when  $\beta = -5$  while varying the values of  $\alpha$ .

For Data III, the variances obtained using P II and BSP are presented in Table 4.15. Figures 4.21 – 4.30 clearly show that P II has lower variances than BSP when both procedures do not yield approximately the same value. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase except in Figures 4.26 – 4.29 where the variance estimates for P II reduced further. It is seen from Tables 4.31 and 4.32 that the least value of variance obtained when P II was used was 1117 at  $\alpha = 5$  and  $\beta = -5$  while 1241 at  $\alpha = 5$  and  $\beta = 1$  was obtained when BSP was used. Considering Figures 4.21

– 4.30 for P II, it was also realized that the least values of the variance are obtained when  $\beta = -5$  while varying the values of  $\alpha$ .

For Data IV, the variances obtained using P II and BSP are presented in Table 4.19. Figures 4.31 – 4.40 show that P II has lower variances than BSP when both procedures do not yield approximately the same value. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase. It is seen from Tables 4.31 and 4.32 that the least value of variance obtained when P II was used was 479 at  $\alpha = 5$  and  $\beta = -5$  while 523 at  $\alpha = 5$  and  $\beta = 1$  was obtained when BSP was used. Considering Figures 4.31 – 4.40 for P II, it was also realized that the least values of the variance were gotten when  $\beta = -5$  while varying the values of  $\alpha$ .

For Data V, the variances obtained using P II and BSP are presented in Table 4.23. Figures 4.41 – 4.50 clearly show that P II has lower variances than BSP when both procedures do not yield approximately the same value. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase. It is seen from Tables 4.31 and 4.32 that the least value of variance obtained when P II was used was 1623 at  $\alpha = 5$  and  $\beta = -5$  while 1770 at  $\alpha = 5$  and  $\beta = 1$  is obtained when BSP was used. Considering Figures 4.41 – 4.50 for P II, it was also realized that the least values of the variance are obtained when  $\beta = -5$  while varying the values of  $\alpha$ .

For Data VI, the variances obtained using P II and BSP are presented in Table 4.27. Figures 4.51 – 4.60 show that P II has lower variances than BSP when both

procedures do not yield approximately the same value. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase. It is seen from Tables 4.31 and 4.32 that the least value of variance obtained when P II was used was 844 at  $\alpha = 5$  and  $\beta = -5$  while 865 at  $\alpha = 5$  and  $\beta = 2$  was obtained when BSP was used. Considering Figures 4.51 – 4.60 for P II, it was also realized that the least values of the variance are obtained when  $\beta = -5$  while varying the values of  $\alpha$ .

For all the six simulated data under consideration, P II perform better than BSP in terms of efficiency, when  $-5 \leq \alpha \leq 5$  and  $-5 \leq \beta \leq -1$ . For  $-5 \leq \alpha \leq 5$  and  $1 \leq \beta \leq 5$ , the variance estimates for both procedures are getting very close to each other, and eventually approximately equal, as seen in the Figures 4.1 – 4.60.

Considering all the six simulated data, Tables 4.33 – 4.38 show that as the sample size increases, the variance estimate decreases. This shows that P II is consistent.

The summary of the analysis obtained from the five life data are presented below.

For Data VII, the variances obtained using P II and BSP are compared and presented in Table 4.44. Figures 4.61 – 4.66 clearly show that P II has lower variances than BSP, with difference when both procedures do not yield approximately the same value except when  $1 \leq \beta \leq 2$ . Figures 4.68 – 4.70 show that BSP has slightly lower variances than P II. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase. It is seen from Table 4.47 that the least value of variance obtained when P II was used was 574 at  $\alpha = 4$  and  $\beta = 5$  while 574 at  $\alpha = 4$  and  $\beta = -2$  was obtained when

BSP was used. Considering Figures 4.61 – 4.70 for P II, it was also realized that the least values of the variance were gotten when  $\beta = -5$  while varying the values of  $\alpha$ .

For Data VIII, the variances obtained using P II and BSP are compared and presented in Table 4.48. Figures 4.71 – 4.80 clearly show that P II has lower variances than BSP, with difference when both procedures do not yield approximately the same value except when  $1 \leq \beta \leq 2$  and in most of the figures. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase. It is seen from Table 4.51 that the least value of variance obtained when P II was used was 616 at  $\alpha = 4$  and  $\beta = -3$  while 616 at  $\alpha = 5$  and  $\beta = 1$  was obtained when BSP was used. Considering Figures 4.71 – 4.80 for P II, it was also realized that the least values of the variance were obtained when  $\beta = -5$  while varying the values of  $\alpha$ .

For Data IX, the variances obtained using P II and BSP are compared and presented in Table 4.52. Figures 4.81 – 4.90 clearly show that P II has lower variances than BSP with difference when both procedures do not yield approximately the same value except when  $1 \leq \beta \leq 2$  in most of the figures. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase. It is seen from Table 4.55 that the least value of variance obtained when P II was used was 986 at  $\alpha = 5$  and  $\beta = -5$  while 1172 at  $\alpha = 5$  and  $\beta = 1$  was obtained when BSP was used. Considering Figures 4.81

– 4.90 for P II, it was also realized that the least values of the variance were obtained when  $\beta = -5$  while varying the values of  $\alpha$ .

For Data X, the variances obtained using P II and BSP are compared and presented in Table 4.56. Figures 4.91 – 4.100 clearly show that P II has lower variances than BSP with difference when both procedures do not yield approximately the same value except when  $1 \leq \beta \leq 2$  in most of the figures. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase. It is seen from Table 4.59 that the least value of variance obtained when P II was used was 3782 at  $\alpha = 3$  and  $\beta = -5$  while 3795 at  $\alpha = 5$  and  $\beta = 1$  was obtained when BSP was used. Considering Figures 4.91 – 4.100 for P II, it was also realized that the least values of the variance were obtained when  $\beta = -5$  while varying the values of  $\alpha$ .

For Data XI, the variances obtained using P II and BSP are compared and presented in Table 4.60. Figures 4.101 – 4.110 show that P II has lower variances than BSP with difference when both procedures do not yield approximately the same value except when  $1 \leq \beta \leq 2$  in most of the figures. The variances obtained using P II and BSP become approximately the same as the values of  $\alpha$  and  $\beta$  increase. It is seen from Table 4.63 that the least value of variance obtained when P II was used was 182 at  $\alpha = 5$  and  $\beta = -5$  while 204 at  $\alpha = 5$  and  $\beta = 1$  was obtained when BSP was used. Considering Figures 4.101 – 4.110 for P II, it was also realized that the least values of the variance were obtained when  $\beta = -5$  while varying the values of  $\alpha$ .

For all the five real life data under consideration, P II perform better than BSP in terms of efficiency, when  $-5 \leq \alpha \leq 5$  and  $-5 \leq \beta \leq 1$  in most of the figures.

From  $-5 \leq \alpha \leq 5$  and  $1 \leq \beta \leq 5$ , the variance estimates for both procedures are getting very close to each other, and eventually approximately equal, as seen in the Figures 4.61 – 4.110.

Considering all the five real life data, Tables 4.66 – 4.70 show that as the sample size increases, the variance estimate decreases. This shows that P II is consistent.

## **5.2 Conclusion**

In our daily activities, we often have reasons to engage in sampling, which is defined as the process of selecting units of interest based on a well – defined probability law. There are a number of sample selection procedures in probability proportional to size sampling without replacement, with most focusing on selection of sample size two. This research work is about selection of sample of any size in PPSSWOR.

Two new procedures (P I and P II) for selection of any sample size in probability proportional to size sampling without replacement have been developed using the ideas of Alodat sample selection and Basit & Shahbaz sample selection procedures. Alodat procedure is restricted to the selection of sample of size two. BSP is for selection of sample of any size, but usually gives high variance values.

In order to know the most efficient sample selection procedure, the variances were calculated and compared using P I, P II, and BSP. Variance was not

computed using Alodat procedure because of its restriction to the selection of sample of size two.

P II and BSP contain two constant values;  $\alpha$  and  $\beta$  in their formula for calculating the probabilities of inclusion of single unit in the sample and also the joint probabilities of units in the sample. In this research work, these two constants takes integer values from -5 to +5, excluding zero.

P I results in only one value of variance. P II results in varying variances as the values of the constants  $\alpha$  and  $\beta$  change.

Eleven data are considered in all; six simulated data and five real life data. From the analysis, and in terms of efficiency, most of the figures show that P II dominates BSP with wide differences, when  $-5 \leq \beta \leq -1$  while varying the values of  $\alpha$ . BSP dominates P II, with slight differences, when  $1 \leq \beta \leq 2$  while varying the values of  $\alpha$ . The two procedures become approximately equal when  $2 < \beta \leq 5$ , while varying the values of  $\alpha$ . Also, it is realized that as the number of groups or units increase, better variance estimates are obtained in favour of P II.

The new procedures (P I and P II) have been used with real life data here, where Childs' weight at birth in six African countries namely; Swaziland, Liberia, Nigeria, The Gambia, and South Africa, are considered.

In all cases, P II performs better than P I. This shows the importance of the introduction of the constants  $\alpha$  and  $\beta$ .

In view of the analysis, it is therefore concluded that P II is better than the BSP in terms of efficiency, for specified values of constants  $\alpha$  and  $\beta$ .

Based on the analysis using both simulated and life data, P II is the most efficient sample selection procedure in probability proportional to size sampling without replacement in this research work. This study concluded that when  $-5 \leq \beta \leq -1$  and  $-5 \leq \alpha \leq 5$ , P II is better than the existing BSP. It is therefore recommended for use in probability proportional to size sampling without replacement.

The aim of this research work to modify two the existing Basit & Shahbaz (2007), and Alodat (2009) procedures in PPSSWOR to produce more efficient ones, has been achieved.

The stated objective to obtain two modified sample selection procedures (P I and P II) in PPSSWOR for any sample size has been achieved. The stated objective to derive the inclusion probability of single unit  $\pi_i$ , and joint probability of inclusion of units,  $\pi_{ij}$  in the sample for P I and P II has been achieved. Lastly, the objective to determine the most efficient sample selection procedure with minimum variance using both real life and simulated data has also been achieved.

### **5.3 Contribution to Knowledge**

This research work has been able to contribute to knowledge in the following ways;

- i. modification of existing sample selection procedure in probability proportional to size sampling without replacement to produce two new procedures that are more efficient than the existing procedures for specified values of constants  $\alpha$  and  $\beta$ ;

- ii. introduction of two modified formulae for calculating the inclusion probabilities of single unit in the sample,  $\pi_i$ , and also joint probabilities of inclusion of units in the sample,  $\pi_{ij}$ , and
- iii. derivation of two new procedures in PPSSWOR, that will always give positive variance estimates.

#### **5.4 Suggestion for Further Study**

The following are suggested for further study;

- i. derivation of sample selection procedure in PPSSWOR that will be better than the existing procedures, including P I and P II for all values of constants  $\alpha$  and  $\beta$ , and
- ii. formulation of a more efficient procedure for selecting samples of any size in PPSSWOR, that will lead to probabilities of inclusion of single unit in the sample, and also joint probabilities of inclusion  $\pi_i$  and  $\pi_{ij}$  respectively.

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## APPENDIX

USING LIFE DATA (DATA IX, when  $N = 6, n = 5$ )

A1: The R codes below are used to obtain the probabilities of selection, probabilities of inclusion, variance estimates and correlation coefficient between the variable of interest and the auxiliary variable for P I and P II.

```
x1=20369956
x2=18894299
x3=35915467
x4=16395555
x5=21044081
x6=27722432
x=c(x1,x2,x3,x4,x5,x6)
X=sum(x)
X
p=(x/X)
sum(p)
p1=(x1/X)
p1
p2=(x2/X)
p2
p3=(x3/X)
p3
p4=(x4/X)
p4
p5=(x5/X)
p5
p6=(x6/X)
p6
p=c(p1,p2,p3,p4,p5,p6)
p
sum(p)
y1=41
y2=42
y3=134
y4=57
y5=93
y6=61
y=c(y1,y2,y3,y4,y5,y6)
y
sum(y)
cor(x,y)
b=-5
n=5
N=6
```

```

a=5
A1=p1^a/(2-p1^b)
A1
A2=p2^a/(2-p2^b)
A2
A3=p3^a/(2-p3^b)
A3
A4=p4^a/(2-p4^b)
A4
A5=p5^a/(2-p5^b)
A5
A6=p6^a/(2-p6^b)
A6
A=c(A1,A2,A3,A4,A5,A6)
A
AA=sum(A)
h1=((1/AA)*(N-n)/(N-1)*A1)+((n-1)/(N-1))
h1
h2=((1/AA)*(N-n)/(N-1)*A2)+((n-1)/(N-1))
h2
h3=((1/AA)*(N-n)/(N-1)*A3)+((n-1)/(N-1))
h3
h4=((1/AA)*(N-n)/(N-1)*A4)+((n-1)/(N-1))
h4
h5=((1/AA)*(N-n)/(N-1)*A5)+((n-1)/(N-1))
h5
h6=((1/AA)*(N-n)/(N-1)*A6)+((n-1)/(N-1))
h6
h=c(h1,h2,h3,h4,h5,h6)
sum(h)
h12=((((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A1+A2)))+((n-1)*(n-2))/((N-1)*(N-2))
h12
h13=((((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A1+A3)))+((n-1)*(n-2))/((N-1)*(N-2))
h13
h14=((((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A1+A4)))+((n-1)*(n-2))/((N-1)*(N-2))
h14
h15=((((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A1+A5)))+((n-1)*(n-2))/((N-1)*(N-2))
h15
h16=((((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A1+A6)))+((n-1)*(n-2))/((N-1)*(N-2))
h16
h23=((((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A2+A3)))+((n-1)*(n-2))/((N-1)*(N-2))
h23
h24=((((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A2+A4)))+((n-1)*(n-2))/((N-1)*(N-2))
h24
h25=((((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A2+A5)))+((n-1)*(n-2))/((N-1)*(N-2))

```

$h_{25}$   
 $h_{26} = (((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A_2+A_6)) + ((n-1)*(n-2))/((N-1)*(N-2))$   
 $h_{26}$   
 $h_{34} = (((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A_3+A_4)) + ((n-1)*(n-2))/((N-1)*(N-2))$   
 $h_{34}$   
 $h_{35} = (((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A_3+A_5)) + ((n-1)*(n-2))/((N-1)*(N-2))$   
 $h_{35}$   
 $h_{36} = (((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A_3+A_6)) + ((n-1)*(n-2))/((N-1)*(N-2))$   
 $h_{36}$   
 $h_{45} = (((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A_4+A_5)) + ((n-1)*(n-2))/((N-1)*(N-2))$   
 $h_{45}$   
 $h_{46} = (((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A_4+A_6)) + ((n-1)*(n-2))/((N-1)*(N-2))$   
 $h_{46}$   
 $h_{56} = (((n-1)*(N-n))/(AA*(N-1)*(N-2))*(A_5+A_6)) + ((n-1)*(n-2))/((N-1)*(N-2))$   
 $h_{56}$   
 $H = c(h_{12}, h_{13}, h_{14}, h_{15}, h_{16}, h_{23}, h_{24}, h_{25}, h_{26}, h_{34}, h_{35}, h_{36}, h_{45}, h_{46}, h_{56})$   
 $H$   
 $HH = \text{sum}(H)$   
 $HH$   
 $v_{12} = (((h_1*h_2)-h_{12})/h_{12}) * ((y_1/h_1)-(y_2/h_2))^2$   
 $v_{12}$   
 $v_{13} = (((h_1*h_3)-h_{13})/h_{13}) * ((y_1/h_1)-(y_3/h_3))^2$   
 $v_{13}$   
 $v_{14} = (((h_1*h_4)-h_{14})/h_{14}) * ((y_1/h_1)-(y_4/h_4))^2$   
 $v_{14}$   
 $v_{15} = (((h_1*h_5)-h_{15})/h_{15}) * ((y_1/h_1)-(y_5/h_5))^2$   
 $v_{15}$   
 $v_{16} = (((h_1*h_6)-h_{16})/h_{16}) * ((y_1/h_1)-(y_6/h_6))^2$   
 $v_{16}$   
 $v_{23} = (((h_2*h_3)-h_{23})/h_{23}) * ((y_2/h_2)-(y_3/h_3))^2$   
 $v_{23}$   
 $v_{24} = (((h_2*h_4)-h_{24})/h_{24}) * ((y_2/h_2)-(y_4/h_4))^2$   
 $v_{24}$   
 $v_{25} = (((h_2*h_5)-h_{25})/h_{25}) * ((y_2/h_2)-(y_5/h_5))^2$   
 $v_{25}$   
 $v_{26} = (((h_2*h_6)-h_{26})/h_{26}) * ((y_2/h_2)-(y_6/h_6))^2$   
 $v_{26}$   
 $v_{34} = (((h_3*h_4)-h_{34})/h_{34}) * ((y_3/h_3)-(y_4/h_4))^2$   
 $v_{34}$   
 $v_{35} = (((h_3*h_5)-h_{35})/h_{35}) * ((y_3/h_3)-(y_5/h_5))^2$   
 $v_{35}$   
 $v_{36} = (((h_3*h_6)-h_{36})/h_{36}) * ((y_3/h_3)-(y_6/h_6))^2$   
 $v_{36}$   
 $v_{45} = (((h_4*h_5)-h_{45})/h_{45}) * ((y_4/h_4)-(y_5/h_5))^2$   
 $v_{45}$   
 $v_{46} = (((h_4*h_6)-h_{46})/h_{46}) * ((y_4/h_4)-(y_6/h_6))^2$

```

v46
v56=((h5*h6)-h56)/h56*((y5/h5)-(y6/h6))^2
v56
V=c(v12,v13,v14,v15,v16,v23,v24,v25,v26,v34,v35,v36,v45,v46,v56)
V
VAR=sum(V)
VAR
SE=sqrt(VAR)
SE

```

A2: The R codes below are used to obtain the probabilities of selection, probabilities of inclusion, variance estimates and correlation coefficient between the variable of interest and the auxiliary variable for BSP.

```

x1=20369956
x2=18894299
x3=35915467
x4=16395555
x5=21044081
x6=27722432
x=c(x1,x2,x3,x4,x5,x6)
X=sum(x)
X
p1=(x1/X)
p1
p2=(x2/X)
p2
p3=(x3/X)
p3
p4=(x4/X)
p4
p5=(x5/X)
p5
p6=(x6/X)
p6
p=c(p1,p2,p3,p4,p5,p6)
p
sum(p)
y1=41
y2=42
y3=134
y4=57
y5=93
y6=61
y=c(y1,y2,y3,y4,y5,y6)
y
sum(y)
cor(x,y)
b=1
n=5
N=6
a=5
M=(p^a*(1-p^b)/(1-2*p^b))
M
MM=sum(M)
h1=(1/MM)*((N-n)/(N-1))*(p1^a*(1-p1^b)/(1-2*p1^b))+((n-1)/(N-1))

```

$h_1$   
 $h_2 = (1/MM) * ((N-n)/(N-1)) * (p_2^a * (1-p_2^b) / (1-2*p_2^b)) + ((n-1)/(N-1))$   
 $h_2$   
 $h_3 = (1/MM) * ((N-n)/(N-1)) * (p_3^a * (1-p_3^b) / (1-2*p_3^b)) + ((n-1)/(N-1))$   
 $h_3$   
 $h_4 = (1/MM) * ((N-n)/(N-1)) * (p_4^a * (1-p_4^b) / (1-2*p_4^b)) + ((n-1)/(N-1))$   
 $h_4$   
 $h_5 = (1/MM) * ((N-n)/(N-1)) * (p_5^a * (1-p_5^b) / (1-2*p_5^b)) + ((n-1)/(N-1))$   
 $h_5$   
 $h_6 = (1/MM) * ((N-n)/(N-1)) * (p_6^a * (1-p_6^b) / (1-2*p_6^b)) + ((n-1)/(N-1))$   
 $h_6$   
 $h = c(h_1, h_2, h_3, h_4, h_5, h_6)$   
 $h$   
 $\text{sum}(h)$   
 $h_{12} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_1^a * (1-p_1^b) / (1-2*p_1^b)) + (p_2^a * (1-p_2^b) / (1-2*p_2^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{12}$   
 $h_{13} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_1^a * (1-p_1^b) / (1-2*p_1^b)) + (p_3^a * (1-p_3^b) / (1-2*p_3^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{13}$   
 $h_{14} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_1^a * (1-p_1^b) / (1-2*p_1^b)) + (p_4^a * (1-p_4^b) / (1-2*p_4^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{14}$   
 $h_{15} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_1^a * (1-p_1^b) / (1-2*p_1^b)) + (p_5^a * (1-p_5^b) / (1-2*p_5^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{15}$   
 $h_{16} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_1^a * (1-p_1^b) / (1-2*p_1^b)) + (p_6^a * (1-p_6^b) / (1-2*p_6^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{16}$   
 $h_{23} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_2^a * (1-p_2^b) / (1-2*p_2^b)) + (p_3^a * (1-p_3^b) / (1-2*p_3^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{23}$   
 $h_{24} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_2^a * (1-p_2^b) / (1-2*p_2^b)) + (p_4^a * (1-p_4^b) / (1-2*p_4^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{24}$   
 $h_{25} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_2^a * (1-p_2^b) / (1-2*p_2^b)) + (p_5^a * (1-p_5^b) / (1-2*p_5^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{25}$   
 $h_{26} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_2^a * (1-p_2^b) / (1-2*p_2^b)) + (p_6^a * (1-p_6^b) / (1-2*p_6^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{26}$   
 $h_{34} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_3^a * (1-p_3^b) / (1-2*p_3^b)) + (p_4^a * (1-p_4^b) / (1-2*p_4^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$   
 $h_{34}$   
 $h_{35} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_3^a * (1-p_3^b) / (1-2*p_3^b)) + (p_5^a * (1-p_5^b) / (1-2*p_5^b))) + ((n-1)/(N-1)) * ((n-2)/(N-2))$

$$h_{35}$$

$$h_{36} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_3^a * (1-p_3^b)) / (1-2*p_3^b) + (p_6^a * (1-p_6^b)) / (1-2*p_6^b)) + ((n-1)/(N-1)) * ((n-2)/(N-2))$$

$$h_{36}$$

$$h_{45} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_4^a * (1-p_4^b)) / (1-2*p_4^b) + (p_5^a * (1-p_5^b)) / (1-2*p_5^b)) + ((n-1)/(N-1)) * ((n-2)/(N-2))$$

$$h_{45}$$

$$h_{46} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_4^a * (1-p_4^b)) / (1-2*p_4^b) + (p_6^a * (1-p_6^b)) / (1-2*p_6^b)) + ((n-1)/(N-1)) * ((n-2)/(N-2))$$

$$h_{46}$$

$$h_{56} = (((n-1)/(N-1)) * ((N-n)/(N-2)) * (1/MM)) * ((p_5^a * (1-p_5^b)) / (1-2*p_5^b) + (p_6^a * (1-p_6^b)) / (1-2*p_6^b)) + ((n-1)/(N-1)) * ((n-2)/(N-2))$$

$$h_{56}$$

$$H = c(h_{12}, h_{13}, h_{14}, h_{15}, h_{16}, h_{23}, h_{24}, h_{25}, h_{26}, h_{34}, h_{35}, h_{36}, h_{45}, h_{46}, h_{56})$$

$$H$$

$$\text{sum}(H)$$

$$v_{12} = (((h_1 * h_2) - h_{12}) / h_{12}) * ((y_1/h_1) - (y_2/h_2))^2$$

$$v_{12}$$

$$v_{13} = (((h_1 * h_3) - h_{13}) / h_{13}) * ((y_1/h_1) - (y_3/h_3))^2$$

$$v_{13}$$

$$v_{14} = (((h_1 * h_4) - h_{14}) / h_{14}) * ((y_1/h_1) - (y_4/h_4))^2$$

$$v_{14}$$

$$v_{15} = (((h_1 * h_5) - h_{15}) / h_{15}) * ((y_1/h_1) - (y_5/h_5))^2$$

$$v_{15}$$

$$v_{16} = (((h_1 * h_6) - h_{16}) / h_{16}) * ((y_1/h_1) - (y_6/h_6))^2$$

$$v_{16}$$

$$v_{23} = (((h_2 * h_3) - h_{23}) / h_{23}) * ((y_2/h_2) - (y_3/h_3))^2$$

$$v_{23}$$

$$v_{24} = (((h_2 * h_4) - h_{24}) / h_{24}) * ((y_2/h_2) - (y_4/h_4))^2$$

$$v_{24}$$

$$v_{25} = (((h_2 * h_5) - h_{25}) / h_{25}) * ((y_2/h_2) - (y_5/h_5))^2$$

$$v_{25}$$

$$v_{26} = (((h_2 * h_6) - h_{26}) / h_{26}) * ((y_2/h_2) - (y_6/h_6))^2$$

$$v_{26}$$

$$v_{34} = (((h_3 * h_4) - h_{34}) / h_{34}) * ((y_3/h_3) - (y_4/h_4))^2$$

$$v_{34}$$

$$v_{35} = (((h_3 * h_5) - h_{35}) / h_{35}) * ((y_3/h_3) - (y_5/h_5))^2$$

$$v_{35}$$

$$v_{36} = (((h_3 * h_6) - h_{36}) / h_{36}) * ((y_3/h_3) - (y_6/h_6))^2$$

$$v_{36}$$

$$v_{45} = (((h_4 * h_5) - h_{45}) / h_{45}) * ((y_4/h_4) - (y_5/h_5))^2$$

$$v_{45}$$

$$v_{46} = (((h_4 * h_6) - h_{46}) / h_{46}) * ((y_4/h_4) - (y_6/h_6))^2$$

$$v_{46}$$

$$v_{56} = (((h_5 * h_6) - h_{56}) / h_{56}) * ((y_5/h_5) - (y_6/h_6))^2$$

$$v_{56}$$

```
V=c(v12,v13,v14,v15,v16,v23,v24,v25,v26,v34,v35,v36,v45,v46,v56)
V
VAR=sum(V)
VAR
SE=sqrt(VAR)
SE
```