

Iterative Decomposition Method and Laplace Iterative Decomposition Method in Solving One-dimensional Time-Fractional Partial Differential Equations

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Abstract

In this paper, we consider the numerical treatment of one-dimensional time-fractional partial differential equations by Iterative Decomposition Method (IDM) and Laplace Iterative Decomposition Method (LIDM). Comparative study of the numerical results of illustrative examples of the two methods show that the two methods compared favourable in terms of efficiency, conveniency and accuracy. The major advantage demonstrated by the two methods over some other methods in the literature is their ability to handle non linear problems directly.

1 Introduction

Partial fractional order diffusion equations are generalizations of classical diffusion equation which are increasingly used in modeling practical supper diffusion problem in fluid flow, finance and other areas of application. This relevance arose the interest of many scholars who consequently pay a very much considerable attention to the solution of this class of problems (see Taiwo and Bello (2013)).

It is however discovered that most nonlinear fractional diffusion equations do not have exact analytical solutions, hence the need for numerical treatment. Recently, some scholars developed and used some methods to obtain the numerical approximation of (FPDEs). Examples of such methods are Adomian Decomposition Method (ADM) by Odibat and Momani, (2006), Homotopy Perturbation Method (HPM) by Odibat and Momani, (2008). The Variation Iteration Method (VIM) which was first proposed by He et al. (1999) is also used for numerical treatment of a wide class of fractional problems even (FDEs), and the modified VIM called Fractional Variation Iteration Method (FVIM) by Wu (2011). Here in this paper, just like Junqiang Song et. al (2012) made a comparative study about the Adomian Decomposition Method (ADM), Variation iteration method (VIM) and fractional variation al method (FVIM) in dealing with fractional partial differential equations (FPDEs) the IDM used by Taiwo and Odetunde, (2013) to obtain the approximate solution of linear fractional ordinary differential equations and integro-differential equations is

modified and extended to obtain the numerically solution of partial differential equations.

2 IDM and FPDE

Here, we illustrate the basic idea of the IDM by considering a general fractional non linear non homogeneous partial differential equations with initial condition of the form

$$D_t^\alpha u(x, t) + N[u(x, t)] + L[u(x, t)] = g(x, t) \quad (1)$$

where L is a linear operator, N is a nonlinear operator in x, t and D_t^α is the modified Riemann-Liouville derivative of order α , subject to the initial conditions

$$u^k(x, t) = c_k(x), k = 0, 1, 2, \dots, m-1 < \alpha \leq m \quad (2)$$

Applying the operator J_t^α , the inverse of the operator D_t^α to both sides of (1) yields

$$J_t^\alpha [D_t^\alpha u(x, t) + N(u(x, t)) + L[u(x, t)]] = J_t^\alpha g(x, t) \quad (3)$$

Using the useful property of Caputo's fractional derivative i.e.,

$$J_t^\alpha [D_t^\alpha u(x, t)] = u(x, t) - \sum_{k=0}^{m-1} \frac{t^k}{k!} u^{(k)}(x, 0), \quad m-1 < \alpha < m \quad (4)$$

Making use of (4) in (3) to obtain

$$u(x, t) = \sum_{k=0}^{m-1} \frac{\partial^k u(x, 0)}{\partial t^k} \frac{t^k}{k!} + J_t^\alpha [g(x, t)] - J_t^\alpha [Lu(x, t) + Nu(x, t)] \quad (5)$$

The IDM suggests that, the solution $u(x, t)$ could be decomposed into the infinite series of the convergent terms.

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t) \quad (6)$$

Substituting (6) in (5) to obtain

$$\sum_{n=0}^{\infty} u_n(x, t) = \frac{\partial^k u(x, 0)}{\partial t^k} \frac{t^k}{k!} + J_t^\alpha g(x, t) - J_t^\alpha \left[L \left(\sum_{n=0}^{\infty} u_n(x, t) \right) + N \sum_{n=0}^{\infty} u_n(x, t) \right] \quad (7)$$

From (7) the iteration are determined by the following recursive way

$$u_0(x, t) = \frac{\partial^k u(x, 0)}{\partial t^k} \frac{t^k}{k!}$$

$$u_1(x, t) = J_t^\alpha g(x, t) - J_t^\alpha [Lu_0 + Nu_0]$$

$$u_2(x, t) = J_t^\alpha g(x, t) - J_t^\alpha [Lu_1 + Nu_1]$$

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$$u_{n+1}(x, t) = J_t^\alpha g(x, t) - J_t^\alpha [Lu_n + Nu_n]$$

3 Modified IDM and FPDE

In this section, Laplace transform is applied to both sides of the equation (1) to obtain

$$\Lambda\{D_t^\alpha u(x, t)\} + \Lambda\{Ru(x, t)\} + \Lambda\{Nu(x, t)\} = \Lambda\{g(x, t)\} \quad (8)$$

Thus (8) can be expressed as

$$S^\alpha \{u(x, s)\} - \sum_{k=0}^{k-1} S^{\alpha-1-k} u^{(k)}(x, 0) + \Lambda\{Ru(x, t)\} + \Lambda\{Nu(x, t)\} = \Lambda\{g(x, t)\} \quad (9)$$

Thus, (9) further simplified to give

$$u(x, s) = \frac{1}{S^\alpha} \sum_{k=0}^{k-1} S^{\alpha-1-k} u^{(k)}(x, 0) - \frac{1}{S^\alpha} \Lambda\{Ru(x, t)\} - \frac{1}{S^\alpha} \Lambda\{Nu(x, t)\} + \frac{1}{S^\alpha} \Lambda\{g(x, t)\} \quad (10)$$

Hence (10) is then decomposed into

$$u_0(x, s) = \frac{1}{S^\alpha} \sum_{k=0}^{k-1} S^{\alpha-1-k} u^{(k)}(x, 0) \quad (11)$$

and

$$u_{n+1}(x, s) = -\frac{1}{S^\alpha} \Lambda\{Ru_n(x, t)\} - \frac{1}{S^\alpha} \Lambda\{Nu_n(x, t)\} + \frac{1}{S^\alpha} \Lambda\{g(x, t)\} \quad (12)$$

From (12) and (13), we subsequently obtained after simplification

$$u_0(x, t) = \Lambda^{-1} u_0(x, s)$$

and

$$u_1(x, s) = -\frac{1}{S^\alpha} \Lambda\{Ru_0(x, t)\} - \frac{1}{S^\alpha} \Lambda\{Nu_0(x, t)\} \quad (13)$$

Also,

$$u_1(x, t) = \Lambda^{-1} u_1(x, s) \quad (14)$$

Thus, for $n \geq 2$, after little modification, the IDM becomes

$$\begin{aligned}
 u_2(x,s) &= -\frac{1}{S^\alpha} \Lambda\{R(u_0+u_1)-R(u_0)\} - \frac{1}{S^\alpha} \Lambda\{N(u_0+u_1)-N(u_0)\} \\
 &\quad \text{M} \\
 u_{n+1}(x,s) &= -\frac{1}{S^\alpha} \Lambda\{R(u_0+u_1+L+u_n)-R(u_0+u_1+L+u_{n-1})\} \\
 &\quad -\frac{1}{S^\alpha} \Lambda\{N(u_0+u_1+L+u_n)-N(u_0+u_1+L+u_{n-1})\}
 \end{aligned}$$

To obtain $u_{n+1}(x,t)$, we take the inverse Laplace transform of $u_{n+1}(x,s)$. The solution is then obtained as

$$u(x,t) = \sum_0^\infty u_n(x,t) \quad (15)$$

4 Demonstration of IDM and LIDM on some Examples

Numerical Example 1

Consider the linear time-fractional diffusion equation

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \frac{\partial^2 u(x,t)}{\partial x^2}, \quad t > 0, x \in P, 0 < \alpha \leq 1 \quad (16)$$

subject to the initial conditions

$$u(x,0) = \sin x, \quad u_t(x,0) = 0 \quad (17)$$

IDM

Applying J_t^α to both sides of (16) to obtain

$$J_t^\alpha \left(\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} \right) = J_t^\alpha \left(\frac{\partial^2 u(x,t)}{\partial x^2} \right) \quad (18)$$

where the L.H.S of (18) is expressed as

$$\begin{aligned}
 J_t^\alpha \left(\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} \right) &= u(x,t) - \sum_{k=0}^{m-1} \frac{t^k}{k!} u^{(k)}(x,0) \\
 &= u(x,t) - u(x,0) - tu^{(1)}(x,0) - \frac{t^2}{2!} u^{(2)}(x,0)
 \end{aligned} \quad (19)$$

Substituting (17) into (19) to obtain

$$J_t^\alpha \left(\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} \right) = u(x,t) - \sin x \quad (20)$$

Also, substituting (20) in (18) to obtain

$$u(x, t) - \sin x = J_t^\alpha \left(\frac{\partial^2 u(x, t)}{\partial x^2} \right) \quad (21)$$

From (21), the following are obtained recursively,

$$u_0 = \sin x$$

$$\begin{aligned} u_1(x, t) &= J_t^\alpha \left(\frac{\partial^2 u_0(x, t)}{\partial x^2} \right) = -\frac{t^\alpha}{\Gamma(\alpha + 1)} \sin x \\ u_2(x, t) &= J_t^\alpha \left(\frac{\partial^2 u_1(x, t)}{\partial x^2} \right) = \frac{t^{2\alpha}}{\Gamma(2\alpha + 1)} \sin x \\ u_3(x, t) &= J_t^\alpha \left(\frac{\partial^2 u_2(x, t)}{\partial x^2} \right) = -\frac{t^{3\alpha}}{\Gamma(3\alpha + 1)} \sin x \end{aligned}$$

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$u_n(x, t)$ for $n \geq 4$ are obtained following this procedure and the numerical solution is expressed in series as

$$u(x, t) = \sin x - \frac{t^\alpha}{\Gamma(\alpha + 1)} \sin x + \frac{t^{2\alpha}}{\Gamma(2\alpha + 1)} \sin x + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n t^{n\alpha}}{\Gamma(n\alpha + 1)} \sin x$$

LIDM

Here, taking the Laplace transform of both sides of (16) we obtain

$$\Lambda \left\{ \frac{\partial^\alpha u(x, t)}{\partial t^\alpha} \right\} = \Lambda \left\{ \frac{\partial^2 u(x, t)}{\partial x^2} \right\} \quad (22)$$

where,

$$\Lambda \left\{ \frac{\partial^\alpha u(x, t)}{\partial t^\alpha} \right\} = S^\alpha u(x, s) - S^{\alpha-1} u(x, 0) - S^{\alpha-2} u(x, 0) \quad (23)$$

Substituting the initial conditions given in (17) into (22) to obtain

$$\Lambda \left\{ \frac{\partial^\alpha u(x, t)}{\partial t^\alpha} \right\} = S^\alpha u(x, s) - S^{\alpha-1} \sin x \quad (24)$$

Also, substituting (24) into (22) to obtain

$$S^\alpha u(x, s) - S^{\alpha-1} \sin x = \Lambda \left\{ \frac{\partial^2 u(x, t)}{\partial x^2} \right\} \quad (25)$$

Simplification of (25) gives

$$u(x, s) = \frac{\sin x}{S} + \frac{\Lambda}{S^\alpha} \left\{ \frac{\partial^2 u(x, t)}{\partial x^2} \right\} \quad (26)$$

Thus, (26) is decomposed into

$$u_0(x, s) = \frac{\sin x}{S} \quad (27)$$

and

$$u_n(x, s) = \frac{L}{S^\alpha} \left\{ \frac{\partial^2 u_{n-1}(x, t)}{\partial x^2} \right\}. \quad (28)$$

Application of inverse Laplace transform on (27) and (28) and after simplification gives

$$u_0(x, t) = \sin x \quad (29)$$

$$u_1(x, t) = -\frac{t^\alpha \sin x}{\Gamma(\alpha + 1)} \quad (30)$$

$$u_2(x, t) = \frac{t^{2\alpha} \sin x}{\Gamma(2\alpha + 1)} \quad (31)$$

$$u_3(x, t) = -\frac{t^{3\alpha} \sin x}{\Gamma(3\alpha + 1)} \quad (32)$$

Thus,

$$u(x, t) = \lim_{n \rightarrow \infty} u_n(x, t) = \sum_{n=0}^{\infty} (-1)^n \frac{t^{n\alpha} \sin x}{\Gamma(n\alpha + 1)} \quad (33)$$

Example 2

We consider the following linear time-fractional wave equation

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{x^2}{2} \frac{\partial^2 u}{\partial x^2}, \quad t > 0, x \in \mathbb{P}, 1 < \alpha \leq 2 \quad (34)$$

subject to the initial conditions

$$u(x, 0) = x, \quad \frac{\partial u(x, 0)}{\partial t} = x^2 \quad (35)$$

IDM

Applying J'_α to both sides of (34) to obtain

$$(36) \quad J'_\alpha \left(\frac{\partial_\alpha n(x, t)}{\partial t_\alpha} \right) = J'_\alpha \left(\frac{1}{x^2} \frac{\partial x^2}{\partial t_\alpha} \right)$$

where the L.H.S of (36) is expressed as

$$(37) \quad J'_\alpha \left(\frac{\partial_\alpha n(x, t)}{\partial t_\alpha} \right) = n(x, t) - \sum_{k=0}^{K-1} \frac{t^k}{k!} n^{(k)}(x, 0) - \frac{t^2}{2!} n^{(2)}(x, 0) - L$$

Substituting (37) into (36) we obtain

$$(38) \quad J'_\alpha \left(\frac{\partial_\alpha n(x, t)}{\partial t_\alpha} \right) = n(x, t) - x - tx^2$$

Substituting (38) into (37) we obtain

$$(39) \quad n(x, t) = x + tx^2 + J'_\alpha \left(x^2 \frac{\partial x^2}{\partial t_\alpha} n(x, t) \right)$$

from (39) the following are obtained

$$n_0(x, t) = x + tx^2$$

$$\begin{aligned} n_1 &= J'_\alpha \left(x^2 \frac{\partial x^2}{\partial t_\alpha} n_0(x, t) \right) = x^2 \frac{\Gamma(\alpha+2)}{t^{\alpha+1}} \\ n_2 &= J'_\alpha \left(x^2 \frac{\partial x^2}{\partial t_\alpha} n_1(x, t) \right) = x^2 \frac{\Gamma(2\alpha+2)}{t^{2\alpha+1}} \\ n_3 &= J'_\alpha \left(x^2 \frac{\partial x^2}{\partial t_\alpha} n_2(x, t) \right) = x^2 \frac{\Gamma(2\alpha+2)}{t^{2\alpha+1}} \end{aligned}$$

following the same procedures for $n_n: n \geq 4$, the following approximation is obtained

$$n(x, t) = x + x^2 t + \frac{t^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{t^{1+2\alpha}}{\Gamma(2\alpha+2)} + \frac{t^{1+3\alpha}}{\Gamma(2\alpha+3)}$$

Taking $\alpha = 2$, the exact solution $n(x, t) = x + x^2 \sinh(t)$ is obtained.

LDM

Taking the Laplace transform of both sides of (33) we obtain

$$(40) \quad V \left\{ \frac{\partial^\alpha n}{\partial t^\alpha} \right\} = V \left\{ x^2 \frac{\partial^2 n}{\partial x^2} \right\}$$

where

$$(41) \quad V \left\{ \frac{\partial^\alpha n}{\partial t^\alpha} \right\} = S^\alpha n(x, s) - S^{\alpha-1} n(x, 0) - S^{\alpha-2} n'(x, 0)$$

substituting the initial conditions given in (35) into (41) we obtain

$$(42) \quad V \left\{ \frac{\partial^\alpha n}{\partial t^\alpha} \right\} = S^\alpha n(x, s) - S^{\alpha-1} x - S^{\alpha-2} x^2$$

substituting (42) into (40) we obtained after simplification

$$(43) \quad n(x, s) = \frac{s}{x} + \frac{s}{x^2} + \frac{s}{s^\alpha} V \left\{ \frac{x^2 \partial^2 n}{\partial x^2} \right\}$$

(43) is decomposed into

$$(44) \quad n_0(x, s) = \frac{s}{x} + \frac{s}{x^2}$$

and

$$(45) \quad n_n(x, s) = L \left\{ x^2 \frac{\partial^2 n_{n-1}}{\partial x^2} \right\} \frac{s^\alpha}{s}$$

Applying inverse Laplace transform on (44) and (45) and after simplification give

$$(46) \quad n_0(x, t) = x + tx^2$$

$$(47) \quad n_1(x, t) = \frac{\Gamma(\alpha + 2)}{t^{\alpha+1} x^2}$$

respectively and

$$(48) \quad n_2(x, t) = \frac{\Gamma(2\alpha + 2)}{t^{2\alpha+1} x^2}$$

Thus,

$$(49) \quad n = x + x^2 \left[t + \frac{t^{\alpha+1}}{\Gamma(\alpha + 2)} + \frac{t^{2\alpha+1}}{\Gamma(2\alpha + 2)} + \frac{t^{3\alpha+1}}{\Gamma(3\alpha + 2)} \right]$$

$$= x + x^2 \sinh(t) \quad \text{for } \alpha = 2$$

Example 3

Consider the following nonlinear time-fractional advection partial differential equation

$$D_t^\alpha u(x,t) + u(x,t)u_x(x,t) = x + xt^2, \quad t > 0, \quad x \in \mathbb{P}, \quad 0 < \alpha \leq 1 \quad (50)$$

subject to the initial conditions

$$u(x,0) = 0 \quad (51)$$

Solution

For both methods the following results were obtain

$$u_0(x,t) = 0 \quad (52)$$

$$u_1(x,t) = x \left(\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{2t^{2+\alpha}}{\Gamma(3+\alpha)} \right) \quad (53)$$

$$u_2(x,t) = x \left(\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{2t^{2+\alpha}}{\Gamma(3+\alpha)} - \frac{\Gamma(1+2\alpha)t^{3\alpha}}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} - \frac{4\Gamma(3+2\alpha)t^{3\alpha+2}}{\Gamma(1+\alpha)\Gamma(3+\alpha)\Gamma(3+3\alpha)} - \frac{4\Gamma(5+2\alpha)t^{3\alpha+4}}{\Gamma(3+\alpha)^2\Gamma(5+3\alpha)} \right) \quad (54)$$

$$u_3(x,t) = x \left(\frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{2t^{2+\alpha}}{\Gamma(3+\alpha)} - \frac{\Gamma(1+2\alpha)t^{3\alpha}}{\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} - \frac{4\Gamma(3+2\alpha)t^{3\alpha+2}}{\Gamma(1+\alpha)\Gamma(3+\alpha)\Gamma(3+3\alpha)} - \frac{4\Gamma(5+2\alpha)t^{3\alpha+4}}{\Gamma(3+\alpha)^2\Gamma(5+3\alpha)} + 2 \left(\frac{\Gamma(1+2\alpha)\Gamma(1+4\alpha)t^{5\alpha}}{\Gamma(1+\alpha)^3\Gamma(1+3\alpha)\Gamma(1+5\alpha)} + \frac{8\Gamma(5+2\alpha)^2\Gamma(8+6\alpha)t^{5\alpha+6}}{\Gamma(3+\alpha)^3\Gamma(5+3\alpha)\Gamma(7+5\alpha)} + \dots \right) \right) \quad (55)$$

5 Table of Results

Table 1: Numerical Values

$*t$	$2*x$	$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1.0$		
		IDM	LIDM	IDM	LIDM	IDM	LIDM	Exact
*0.25	0.25	0.14640842	0.14180807	0.17120511	0.17120511	0.19264006	0.19264006	0.19267840
	0.50	0.28371388	0.28371388	0.33176551	0.33176551	0.37330270	0.37330270	0.37337698
	0.75	0.40337938	0.40337938	0.47169834	0.47169834	0.53075518	0.53075518	0.53085580
	1.00	0.49796471	0.49796471	0.58230325	0.58230325	0.65520788	0.65520788	0.65533826
*0.50	0.25	0.10790621	0.10790621	0.13273945	0.13273945	0.14947323	0.14947323	0.15005809
	0.50	0.20910333	0.20910333	0.25722580	0.25722580	0.28965293	0.28965293	0.290786629
	0.75	0.29729942	0.29729942	0.36571910	0.36571910	0.41182342	0.41182342	0.41343481
	1.00	0.36701087	0.36701087	0.45147375	0.45147375	0.50838872	0.50838872	0.51037795
*0.75	0.25	0.07031034	0.07031034	0.10053521	0.10053521	0.11403776	0.11403776	0.11686536
	0.50	0.13624913	0.13624913	0.19481962	0.19481962	0.22098521	0.22098521	0.22646459
	0.75	0.19371660	0.19371660	0.27699110	0.27699110	0.31419287	0.31419287	0.32198335
	1.00	0.23913971	0.23913971	0.34194061	0.34194061	0.38786553	0.38786553	0.39748275

Table 2: Numerical Values

$*t$	$2*x$	$\alpha = 1.5$		$\alpha = 1.75$		$\alpha = 2.0$		
		IDM	LIDM	IDM	LIDM	IDM	LIDM	Exact
*0.25	0.25	0.26622298	0.26622298	0.26593959	0.26593959	0.26578827	0.26578827	0.26578827
	0.50	0.56489190	0.56489190	0.56375836	0.56375836	0.56315308	0.56315308	0.56315308
	0.75	0.89600678	0.89600678	0.893456630	0.893456630	0.53075518	0.53075518	0.53085580
	1.00	1.25956762	1.25956762	1.25503343	1.25503343	1.25261232	1.25261232	1.25261232
*0.50	0.25	0.10790621	0.10790621	0.13273945	0.13273945	0.14947323	0.14947323	0.15005809
	0.50	0.20910333	0.20910333	0.25722580	0.25722580	0.28965293	0.28965293	0.290786629
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*0.75	0.25	0.07031034	0.07031034	0.10053521	0.10053521	0.11403776	0.11403776	0.11686536
	0.50	0.72761955	0.72761955	0.71446834	0.71446834	0.70557913	0.70557913	0.70557918
	0.75	1.26214400	1.26214400	1.23255378	1.23255378	1.21255304	1.21255304	1.21255316
	1.00	1.91047821	1.91047821	1.91047821	1.91047821	1.82231652	1.82231652	1.82231673

5 Table of Results

Table 1: Numerical Values

$*t$	$2*x$	$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1.0$		
		IDM	LIDM	IDM	LIDM	IDM	LIDM	Exact
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Table 2: Numerical Values

$*t$	$2*x$	$\alpha = 1.5$		$\alpha = 1.75$		$\alpha = 2.0$		
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	0.75	0.89600678	0.89600678	0.893456630	0.893456630	0.53075518	0.53075518	0.53085580
	1.00	1.25956762	1.25956762	1.25503343	1.25503343	1.25261232	1.25261232	1.25261232
*0.50	0.25	0.10790621	0.10790621	0.13273945	0.13273945	0.14947323	0.14947323	0.15005809
	0.50	0.20910333	0.20910333	0.25722580	0.25722580	0.28965293	0.28965293	0.290786629
	0.75	0.29729942	0.29729942	0.36571910	0.36571910	0.41182342	0.41182342	0.41343481
	1.00	0.36701087	0.36701087	0.45147375	0.45147375	0.50838872	0.50838872	0.51037795
*0.75	0.25	0.07031034	0.07031034	0.10053521	0.10053521	0.11403776	0.11403776	0.11686536
	0.50	0.72761955	0.72761955	0.71446834	0.71446834	0.70557913	0.70557913	0.70557918
	0.75	1.26214400	1.26214400	1.23255378	1.23255378	1.21255304	1.21255304	1.21255316
	1.00	1.91047821	1.91047821	1.91047821	1.91047821	1.82231652	1.82231652	1.82231673

Table 3: Numerical Values

$*t$	$2*x$	$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1.0$		
		IDM	LIDM	IDM	LIDM	IDM	LIDM	Exact
*0.25	0.25	0.12422501	0.26622298	0.09230374	0.09230374	0.06250058	0.06250058	0.0625000
	0.50	0.24845002	0.24845002	0.18460748	0.18460748	0.12500117	0.12500117	0.125000
	0.75	0.37267504	0.37267504	0.27691122	0.27691122	0.18750175	0.18750175	0.187500
	1.00	0.49690005	0.49690005	0.36921496	0.36921496	0.2526000234	0.2526000234	0.250000
*0.50	0.25	0.18377520	0.18377520	0.15148283	0.15148283	0.12507592	0.12507592	0.125000
	0.50	0.20910333	0.20910333	0.25722580	0.25722580	0.28965293	0.28965293	0.250000
	0.75	0.29729942	0.29729942	0.36571910	0.36571910	0.41182342	0.41182342	0.375000
	1.00	0.36701087	0.36701087	0.45147375	0.45147375	0.50838872	0.50838872	0.500000
*0.75	0.25	0.27227270	0.27227270	0.21407798	0.21407798	0.18881843	0.18881843	0.187500
	0.50	0.54454540	0.54454540	0.42815596	0.42815596	0.37763687	0.37763687	0.375000
	0.75	0.81681810	0.81681810	0.64223394	0.64223394	0.56645530	0.56645530	0.562500
	1.00	1.08909080	1.08909080	0.85631192	0.85631192	0.75527373	0.75527373	0.750000

6 Discussion of Results

In table 1 above, the results obtained for different values of α using IDM and LDM were displaced. The exact solution is only known for $\alpha = 1$ for the different values of α considered the two methods gives same results. In table 2, the results obtained for different values of α using IDM and LDM were presented. The values of $\alpha = 2$ are the only case for which we know the exact solution $u(x, t) = x + x^2 \sinh(t)$. In table 3, the results obtained for different values of α using IDM and LDM were presented. The values of $\alpha = 1$ is the only case for which we know the exact solution $u(x, t) = xt$.

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