

A Quantitative Study of the Optimal Control Model for an Electric Power Generating System

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Abstract— The importance of electric power in today's world cannot be overemphasized for it is the key energy source for industrial, commercial and domestic activities. Its availability in the right quantity is essential to advancement of civilization. In fact, a direct relationship can be shown to exist between (electric) power availability and per capital of a nation. The economy of a nation is easily identified with the amount of electricity it supplies. The advanced economy of the world generates a lot electric power and hence they witness rapid economical and industrial development. Third-world countries often witness epileptic or unstable supply of electric power. It is common knowledge that to have constant and stable electricity supply, a country needs to invest in its power generation. In the light of this, an optimal control model is proposed herein for the electric power generating system. In the model, the power generated and the capital investment describe the state of the system while the generator capacity is chosen as the control. The goal is to minimize the cost of generation and maximize the generator output without violating operating limits on the equipment. In this study, we consider the minimization of the total operating cost incurred in the process of generating the required quantity of electric power as our specific objective. A quantitative treatment of the optimal control model of the electric power generating system is then undertaken using real-life data. The result of the analysis indicate that the model gave an actual representation of the system under consideration.

Keywords — *Electric power generating system, Mathematical model, Optimal control, Quantitative study*

I. INTRODUCTION

Energy is a basic necessity for the economic development of a nation. There are different forms of energy, but the most important form is the electrical energy [1]. A modern and civilized society is so much dependent on the use of electrical energy. As such activities relating to the generation of electrical energy has to be given the highest priority in the national planning process of any nation because of the importance of electrical energy to the economic and social development of the society. In fact, the greater the per capital consumption of electrical energy in a country, the higher the standard of living of its people [2]. Therefore, the

advancement of a country is measured in terms of its per capital consumption of electrical energy [3].

Furthermore, a cursory look at the data of electricity supply worldwide reveals that the amount of electric power generated clearly distinguishes between the developed, emerging and third-world nations. The advanced economy of the world generates a lot electric power and hence they witness rapid economical and industrial development. On the other hand, the weak economies often witness epileptic or unstable supply of electric power.

In the advanced society [4], the programmes for constant electricity supply are well-articulated, continuously evaluated and improved upon whereas in many of the third-world countries this is not so. It is common knowledge that to have constant and stable electricity supply, a country needs to invest heavily in its power generation [5].

Therefore, in providing a sound basis on which to operate an electric power generating system (EPGS), we propose an optimal control model for the EPGS. In this model, the power generated and the capital investment describe the state of the system and the generator capacity is chosen as the control, since we cannot neglect the operation limitation on the equipment because of its lifespan. Thus, the upper bound for the control was assigned the value 1 to represent the total capability of the machine and the lower bound as 0. The goal is to minimize the cost of generation but maximize the generator output. As such, we adopt the achievable objective of minimizing the total operating cost incurred in the process of generating the required quantity of electric power without violating operating limits on the equipment, [6]. In addition, we give the model developed a quantitative treatment using real-life data. In the ensuing analysis, attempts are made to validate the suitability of the model for the system being considered.

II. MODEL OF ELECTRIC POWER GENERATING SYSTEM

Let $G_i(t)$ represent the amount of power generated by the i^{th} generator at time t , $C_i(t)$, the capital investment on the i^{th} generator at time t . For the control classes, we choose measurable functions defined on a fixed interval,

$a_i \leq u_i \leq b_i$ ($i = 1, \dots, m$), since we cannot neglect the physical law governing power generating system and the operating limitation of the equipment.

Considering the i^{th} generator, the rate of change in generation at time t depends on the invested capital, $C_i(t)$ and the generator output, $G_i(t)$ which in turn depends on the power input, generator capacity, running cost and transmission losses. For m generators, (i.e., $i = 1, 2, \dots, m$), we have

$$\frac{dG_i(t)}{dt} = \alpha_i + q_i C_i(t) G_i(t) - k_i G_i(t), i = 1, 2, \dots, m$$

where α_i , q_i , and k_i are respectively the actual mechanical/electrical energy from the high pressure turbine and low pressure turbine (capacity of generator i), the corresponding running cost and the transmission loss rate, which depends on the distance from the grid centre. Also, the investment on the capital, $C_i(t)$ at time t is known to be dependent on labour cost, s_i , maintenance cost, y_i , fuel cost, $r_i C_i(t) G_i(t)$, capacity rate, x_i , and the cost of transmission to the grid centre, $\gamma_i C_i$. Because of the physical laws that govern power generation and the operating limitation on the equipment, we choose the generator capacity as our control, $u_i(t)$ since we cannot neglect the operation limitation on the equipment because of its lifespan, the upper bound for $u_i(t)$ is choosing to be 1 , to represent the total capability of the machine. Thus, we have

$$\frac{dC_i(t)}{dt} = (s_i + y_i) + r_i C_i(t) G_i(t) - x_i u_i C_i(t) + \gamma_i C_i, i = 1, 2, \dots, m.$$

In the above setting an important objective is to minimize the total operating cost incurred in the process of generating the required quantity of electric power, G , at any time t . The components of the total operating cost are $C(t)$ and $u(t)$. Thus, the expression for the objective function is of the form

$$J(u) = \int_{t_0}^{t_f} [\delta^T C(t) + \eta u^T u] dt,$$

where $\delta = (\delta_1, \delta_2, \dots, \delta_m)$ is the unit expenditure on the generators and η is a parameter to balance the size of the control.

The problem to study is to find the control u [7] that minimizes the cost function

$$J(u) = \int_{t_0}^{t_f} [\delta^T C(t) + \eta u^T u] dt, \quad (2.1)$$

subject to

$$\frac{dG_i(t)}{dt} = \alpha_i + q_i C_i(t) G_i(t) - k_i G_i(t), i = 1, \dots, m; \quad (2.2)$$

$$\frac{dC_i(t)}{dt} = (s_i + y_i) + r_i C_i(t) G_i(t) - x_i u_i C_i(t) + \gamma_i C_i \quad (2.3)$$

and

$$G_i(t_0) = G_0, C_i(t_0) = C_0, a_i \leq u_i \leq b_i.$$

III. QUANTITATIVE ANALYSIS

Electric power generation, as a real-life endeavor, needs to be studied as a multidisciplinary subject making use of contributions from the relevant fields. As shown in the proceeding developments in this work, mathematics has a lot to contribute in resolving practical problems in electric power generation as well as improving on its enhancement. In what follows, we fashioned solution to the electric power generating system model using real-life data presented in Table I.

In this work the desired solution is that power output from the generating station is maximized with minimum cost of production. There are two systems of differential equations in the optimality system with one involving the control. The systems is solved using an iterative fourth-order Runge-Kutta scheme defined [8] as

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf(t_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

$$k_3 = hf(t_n + \frac{h}{2}, y_n + \frac{k_2}{2})$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

and

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

The controls are updated at the end of each iteration using the formula for optimal controls.

TABLE I. GENERATOR PARAMETERS

Parameter	Meaning	Value for one-generator System	Value for three-generator System
α	actual mechanical/electrical energy available from the turbine	800MW	100MW, 100MW, 80MW $G_1 \leq 100\text{MW}$, $G_2 \leq 100\text{MW}$, $G_3 \leq 80\text{MW}$
q	total running cost	0.3217 per unit	0.3217, #0.3112, 0.312Per unit
r	fuel cost rate	0.347 per unit	0.3478 each per unit
x	actual capacity rate	0.606	0.606, 0.502, 0.402
k	rate of energy loss during transmission	0.002 MW per unit	0.002 MW each per unit
S	labour cost	200 per h	70 per h (assumed)

		(assumed)	
Y	maintenance cost	#100 per h (assumed)	50 per h (assumed)
γ	Cost of transmitting from generating station	# 0.3421 per unit	0.3421 per unit
u	generator actual capacity rate (control)	$a \leq u \leq b, 0 \leq u \leq 1$, a=0, b=1	$a \leq u \leq b,$ $0.3 \leq u \leq 0.9, a = 0.0,$ $b = 1.0$
δ	unit of power generating station	1, 2	1 each
η	Parameters to balance the size of the control, (Number of hours for which the machines can be on)	2, 6	3

SOURCE: National Grid Centre, Osogbo, Nigeria.

Using the data in Table I, the numerical solution obtained are as shown in the figures below.

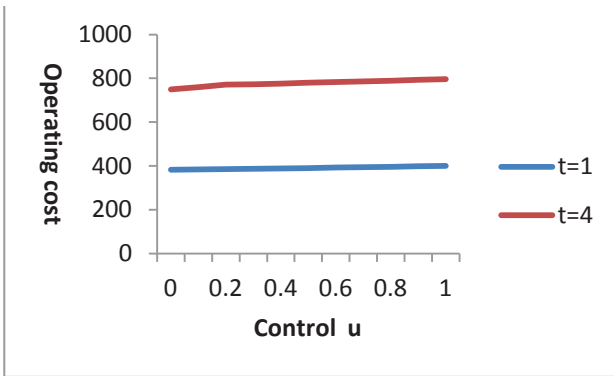


Figure 1: Graph of operating cost against control for one-generator system

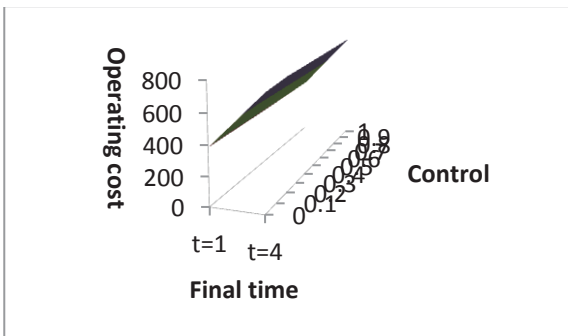


Figure 2: Two-dimensional diagram of operating cost against final time and control for one-generator system

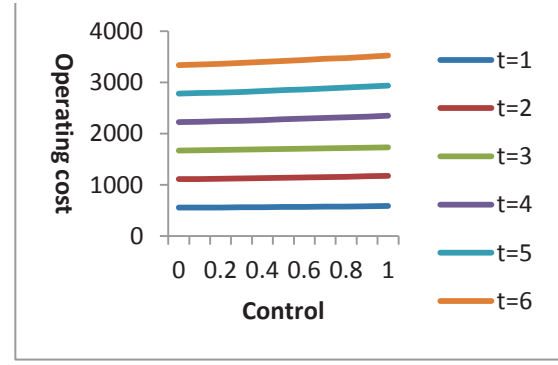


Figure 3: Graph of operating cost against control for three-generator system

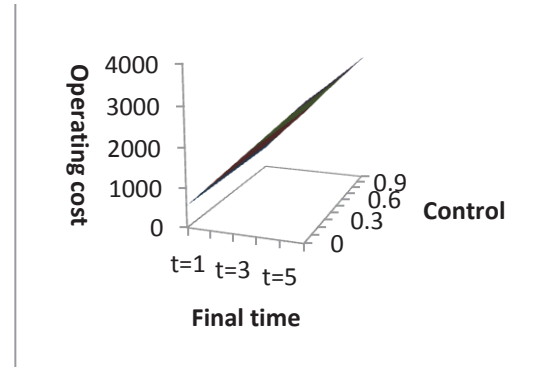


Figure 4: Two-dimensional diagram of operating cost against final time and control for three-generator system

IV. DISCUSSION ON RESULTS

From the results obtained, it was observed that the model gave an actual representation of the system under consideration, and the generator output is maximized with minimum cost of production under a safe and secure atmosphere. The numerical results obtained show that generator output can be improved upon under a secured condition of the machine with minimum cost.

Also, it can be established that the model gave the maximum generator output so far, and that the more we generate the more we spend on it. Therefore, the choice of u_i is greatly dependent on the number of generating machines that are available and the number of hours for which the generation is to be carried out. Thus for this exercise, $u_1 = 0.8$, $u_2 = 0.7$, and $u_3 = 0.5$ are recommended for the three generators system above with $J = 574.84430$ at $t_f = 1$ and $J = 3449.06581$ at $t_f = 6$.

However, the physical capability of the machines or simply the physical characteristics of the generating machines is very important and so the control has to be put into consideration while trying to minimize the cost. As such, we can monitor the control i.e., we can generate more with minimum cost and still

maintain the good condition of the generating machine. It was also observed that the more time we spend, the more power and the more cost we have. Thus, from the results obtained, it is observed that for efficiency and effective functioning of the generating machines in each station, monitoring of the control is very essential.

V. CONCLUSION

The importance of electric power in today's world cannot be overemphasized for it is the key energy source for industrial, commercial and domestic activities. Its availability in the right quantity is essential to advancement of civilization.

The art of electric power generation as complex as it may seem can be aided with the use of mathematical modelling. The real-life data from power generating stations have been subjected to mathematical analysis for the purpose of improving the power system for the good of all.

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