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NUMERICAL SOLUTION OF MULTI-ORDER FRACTIONAL DIFFERENTIAL EQUATIONS USING ORTHOGONAL POLYNOMIAL BASIS

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Abstract:

This paper presents the solution of multi-order fractional differential equations using a constructed Orthogonal polynomial as the basis function. An approximate solution was assumed and substituted into a general class of multi-order fractional differential equations of the form:

$$D^{\alpha_1}y(x) + D^{\alpha_2}y(x) + D^{\alpha_3}y(x) + D^{\alpha_4}y(x) + \dots = f(x) \quad (1)$$

with initial conditions $y(0) = \mu$.

where D^{α} are parameters denoting the fractional order derivative in the Caputo sense. The resulting equation was collocated at equally spaced interior points. The unknown constants in the assumed approximate solution were obtained using Gaussian elimination method. Some numerical examples are presented to illustrate the method.

Keywords: Multi-order fractional differential equations and Orthogonal polynomials.

1.0 Introduction

Fractional Calculus is a generalization of ordinary differentiation and integration to arbitrary orders. This class of calculus has been gaining some considerable attention in the recent years as researchers utilize it in solving problems in the field of Physics, Chemistry, Engineering and Technology, etc.

Most problems how ever of fractional differential equations are difficult to solve analytically. Hence, numerical methods are usually employed to solve such problems. To this end, many numerical techniques have been developed and these include Homotopy Analysis Method (HAM), Adomian Decomposition Method (ADM), Variational Iteration Method (VIM), etc. Khader (2011) solved some class of multi-order fractional differential equations and generalized the Legendre operational matrix of fractional calculus by combining truncated Legendre series with the Legendre operational matrix of fractional derivatives to solve initial value multi-order fractional differential equations. Mahnaz and Ali (2013) also solved multi-order fractional differential equations using operational matrix of Chebyshev polynomials. Omar (2014) solved some multi fractional order integro differential equations using VIM. Ibtisam (2011) solved some multi fractional order integro differential equations using Homotopy Analysis Method. In this work, some orthogonal polynomials were developed and applied to solve multi fractional order differential equations. Orthogonal polynomials have been seen as special functions of mathematical physics and veritable tools for solving differential equations.

Definition of relevant terms:

Here, we present some relevant definitions which are very useful in this work.

Orthogonality: Two functions, say $y_n(x)$ and $y_m(x)$ defined on the interval $a \leq x \leq b$ are said to be orthogonal if;

$$\langle y_n(x), y_m(x) \rangle = \int_a^b y_n(x)y_m(x)dx = 0 \quad (2)$$

If, on the other hand, a third function $w(x) > 0$ exists such that:

$$\langle y_n(x), y_m(x) \rangle = \int_a^b w(x)y_n(x)y_m(x)dx = 0 \quad (3)$$

then, we say that $y_n(x)$ and $y_m(x)$ are mutually orthogonal with respect to the weight function $w(x)$. Generally, we write:

the potential to capture the benefits of both artificial neural network learning rules to conclude and adjust the fuzzy inference systems, particularly Takagi Sugeno Kang fuzzy model (Jang [11], Sugeno and Kang [12], Takagi and Sugeno [13]).

In the literatures about the use of classification methods, the medical sector had a number of related work in which Takagi-Sugeno-Kang fuzzy inference system was applied. Ziasabounchi and Askerzade [14] developed ANFIS model based on hybrid learning algorithm, least squares estimate and gradient descent algorithm. To improve the cost-effectiveness of DR screening several image processing based methods have been developed in the last decade providing an alternative to first phase examinations performed by human graders Zsolt [15]. The key to the early recognition of DR is the reliable detection of micro aneurysms (MAs) on the retina, which serves as an essential part for most automatic DR screening systems (Antal and Hajdu [2], [16]). Asadi et al [5] developed a new supervised multi-layer feed forward neural network (SMFFNN) model to accelerate classification with high accuracy based on SPECT and SPECTF heart disease data sets. Others include (Sivasankar et al [17], Durairaj and Revathi [18], El-Rafaie et al [19], Giancardo [20]).

This paper involved diagnosis of two medical diseases, Diabetic Retinopathy and SPECT-Heart obtained from University of California at Irvine's (UCI) machine learning repository Lichman [21]. The remaining parts of this paper are organised as follows: in second part materials and methods of this work are presented. This led us to third part in which experiments and results were recorded and analyzed. Fourth part concludes the work.

Materials and Methods

This section deals with the materials and methods used in developing the proposed neuro-fuzzy classifier for the diagnosis of medical diseases.

Description of Input attributes

The dataset is available from UCI machine learning repository. The details information about the input variables is shown in Table 1.

Table 1: Information about input variables for Diabetic Retinopathy Data set

Description of Input variable	No. of MF	Min	Max	Std	Mean
Binary result of quality assessment: 0 = bad quality 1 = sufficient quality	2	0	1	0.2	1.0
Binary result of pre-screening: 1 indicates severe retinal abnormality and 0 its lack	2	0	1	0.3	0.9
The results of MA detection: MAs no., $\alpha = 0.5$	3	1	151	26	38.4
The results of MA detection: MAs no., $\alpha = 0.6$	3	1	120	23	35.1
The results of MA detection: MAs no., $\alpha = 0.7$	3	1	105	21	32.3
The results of MA detection: MAs no., $\alpha = 0.8$	3	1	97	20	28.9
The results of MA detection for exudates	3	1	89	15	21.2

Diabetic Retinopathy (DR) data set is obtained from UCI machine learning repository. This dataset contains features extracted from the Messidor image set to predict whether an image contains signs of DR or not. All features represent either a detected lesion, a descriptive feature of an anatomical part or an image-level descriptor. The underlying method image analysis and feature extraction as well as our classification technique is described in Antal and Hajdu [2].

This data set contains 20 attributes including classification attribute with 1,151 instances. It contains no missing values and the characteristics of attributes include integer and real values. The classification field refers to person contains sign of DR as 1 and no signs of DR as 0. In this research work, 7 input attributes are selected, they are quality assessment for bad and sufficient quality, pre-screening to indicates severe retinal abnormality and its lack, four number of MAs found at the

confidence levels $\alpha = 0.5, \dots, 0.8$, respectively, and one exudates is represented by a set of points. This type of data set was used in (Sivasankar et al [17], Giancardo [20]).

Table 2: Information about input variables for SPECT Heart Data set

Description of Input variable	No. of MF	Min	Max	Std	Mean
F1	2	0	1	0.498	0.446
F2	2	0	1	0.432	0.247
F3	2	0	1	0.489	0.393
F4	2	0	1	0.452	0.285
F5	2	0	1	0.492	0.404

The dataset describes diagnosing of cardiac Single Proton Emission Computed Tomography (SPECT) images. Each of the patients is classified into two categories: normal and abnormal. The database of 267 SPECT image sets (patients) was processed to extract features that summarize the original SPECT images. As a result, 44 continuous feature pattern was created for each patient. The pattern was further processed to obtain 22 binary feature patterns. SPECT data has 267 instances that are described by 23 binary attributes including the classification label. The dataset contains 267 samples (212 normal and 55 abnormal). 80 sample used in training the network while 187 samples used in testing the network. The CLIP3 algorithm was used to generate classification rules from these patterns. The CLIP3 algorithm generated rules that were 77.0% accurate (as compared with cardiologists' diagnoses).

In the proposed method, 5 input attributes with 267 instances are selected. Unlike in the existing method, two-thirds of examples were selected as training examples and the remaining one-third as test examples. Fig. 1, shows the diagram of human heart.

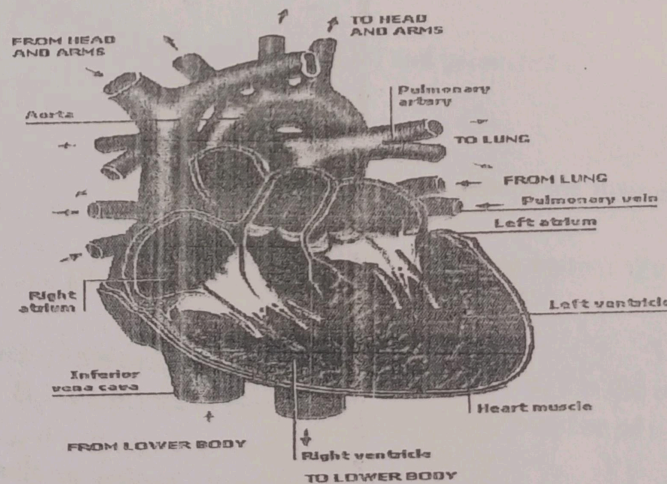


Fig. 1, Inside the human heart, Kindersley [22]

Design of a Proposed Neuro-Fuzzy Classifier

Adaptive neuro fuzzy inference system (ANFIS) was first introduced by Jang [11]. The ANFIS is a framework of adaptive techniques to assist learning and adaptation. To illustrate the ANFIS structure, two fuzzy IF-THEN rules, according to a first order Sugeno model are to be considered, for simplicity (see Sagir and Saratha [23], Jang et al [24]).

According to Jang et al [24], if $f(x, y)$ is a first order polynomial, then the Takagi Sugeno Fuzzy model is given as:

$$\text{IF } x = A_i \text{ and } y \text{ is } B_i \text{ THEN } z_i = f(x, y) \quad (1) \text{ where } A_i \text{ and } B_i \text{ are fuzzy}$$

sets in the rule antecedent part, while $z = px + qy + r = f(x, y)$ is a crisp function in the rule consequent part, and p, q & r are the optimal consequent parameters. Usually $f(x, y)$ is a polynomial in the input variables x and y .

Hybrid Learning Algorithm

In designing this classifier, a hybrid learning technique based on Least squares estimate and the Levenberg-Marquardt algorithm was used. The analytical derivation and chain rule method was applied for computation of the Jacobian Matrix.

Forward Pass

Least squares estimate (LSE) was used at the very beginning to get the initial values of the consequent parameters $S_2 = \{p_i, q_i, r_i\}$ details can be found in Sagir and Saratha [23]. Then at backward pass the Levenberg-Marquardt algorithm Marquardt [25], Yu and Wilamowski [26], is to update all parameters. After the consequent parameters S_2 are identified, the network output can be computed and the error measure E_k represents an objective function for k th of the training data can be obtained as:

$$E_k = (t_k - a_k)^2 \quad (2)$$

where t_k and a_k represent the target output vector and actual output vector, and N is the number of total points. The overall error measure E of the training data set can be computed using performance measure, mean square error (MSE) defined as:

$$MSE = \frac{1}{N} \sum_{k=1}^N E_k \quad (3)$$

Backward Pass

In the backward pass, error signals are propagated and antecedent parameters $S_1 = \{\sigma_i, c_i\}$ are to be updated by Levenberg-Marquardt algorithm.

The performance index to be optimised is defined by Madsen et al [27] and presented as:

$$F(w) = \frac{1}{2} e^T e \quad (4)$$

$F(w)$ is the total error function, $w = [w_1, w_2, \dots, w_k]$ comprising of all weights of the network, e is the error vector comprising the error of all the training samples.

The parameters of unique membership functions of current fuzzy inference system (FIS) is to be obtained, which is a novel approach that allows program to run faster defined as:

$$v = 1(R_{ij}) \quad (5)$$

where v is the index vector that keeps track of the unique membership functions MFs, I is the index table of the unique MF used in the rules, R_{ij} is a matrix of size number of rule by number of input, that identifies the membership functions for the i th rule and j th input.

The Jacobian matrix is to be built-up column-wise, which contains first order partial derivatives of network error using analytical derivation and chain rule method

$$J_k = \frac{\partial f_i}{\partial p_j} \quad (6)$$

Approximate Hessian matrix, which contains second order partial derivative of network error using the cross product of Jacobian.

$$H_{i,j} = \frac{\partial^2 f_i}{\partial x_i \partial x_j} \quad (7)$$

To ensure that equation is invertible, another approximation to Hessian matrix is introduced, that's

$$\mathbf{H} \approx \mathbf{J}_k^T \mathbf{J}_k + \Psi \mathbf{I} \quad (8)$$

where Ψ is called combination coefficient or learning parameter, $\mathbf{I}$ is the identity matrix. Therefore, from equation (7) and equation (8), we can update Hessian matrix as:

$$\mathbf{H}^* = [\mathbf{H} + \Psi \mathbf{I}] \quad (9)$$

With Levenberg-Marquardt method, the increment of the parameter in training will be obtained as:

$$\Delta \mathbf{X}_k = (\mathbf{J}_k^T \mathbf{J}_k + \Psi \mathbf{I})^{-1} \mathbf{J}_k^T \mathbf{e} \quad (10)$$

The network parameter needs to be updated as:

$$\mathbf{X}_{k+1} = \mathbf{X}_k - (\mathbf{J}_k^T \mathbf{J}_k + \Psi \mathbf{I})^{-1} \mathbf{J}_k^T \mathbf{e}_k \quad (11)$$

where $\mathbf{X}$ is parameter vector, $\mathbf{J}_k$ is the Jacobian matrix, and k is the index of iterations.

Results and Discussion

Based on the two data sets obtained from UCI machine learning repository, throughout the experiments, two-thirds of examples were selected as training examples and the remaining one-third as test examples. The obtained results with two data sets for robustness were compared based on machine learning process in terms of accuracy, sensitivity and specificity Kahramani and Allahverdi [28], Saed [29].

Table 3. Comparison of performance error and convergence time between proposed model and conventional model for DR data set

Technique	No. of Epoch	MSE	Time (sec)
ANFIS-LM	500	0.16268	15.11
ANFIS-GD	500	0.18967	36.02

Table 4. Comparison of test results between proposed model and conventional model for DR data set

	Proposed model		Conventional model	
	Train (%)	Test (%)	Train (%)	Test (%)
Accuracy	73.31	76.76	73.44	75.98
Sensitivity	82.73	85.08	83.57	84.53
Specificity	65.04	69.31	64.55	68.32
F-measure	72.18	75.88	72.13	75.00
Precision	81.10	83.83	81.73	83.13

Table 5. Comparison of accuracy results with some related existing models for DR data set

Methodology adopt	Accuracy (%)
Proposed method	76.76
Conventional method	75.98
PCA+MGA, Sivasankar et al [17]	66.00
Nearest Neighbour, Giancardo [20]	66.80
Random Forests, Giancardo [20]	75.80

The performance of the classifier was determined by the computation of performance error and classification accuracy. Comparison of accuracy with some existing classifiers, as shown in Table 3, Table 4 and Table 5, respectively, for Diabetic Retinopathy data set are presented. Based on comparison of the results as shown in Table 5, the proposed classifier produced reasonable results in diagnosing the possible presence of medical disease. This assertion is based on observation: in Table 5, it is clearly confirmed that none of the research studies achieved higher than 75.00% accuracy.

Table 6. Comparison of performance error and convergence time between proposed model and conventional model for SPECT Heart data set

Technique	No. of Epoch	MSE	Time (Sec)
ANFIS-LM	750	0.02919	2.99
ANFIS-GD	750	0.02928	18.47

Table 7. Comparison of results between proposed model and conventional model for SPECT Heart data set

	Proposed model		Conventional model	
	Train (%)	Test (%)	Train (%)	Test (%)
Accuracy	94.21	96.43	94.21	94.68
Sensitivity	90.44	98.29	90.44	92.71
Specificity	88.03	89.61	88.03	88.83
F-measure	95.08	97.04	95.08	95.15
Precision	92.00	96.34	92.00	93.46

Table 8. Comparison of test accuracy results with some related existing models for SPECT Heart data set

Methodology adopt	Accuracy (%)	Sensitivity (%)	Specificity (%)
Proposed method	96.34	98.29	89.61
Conventional method	93.46	90.44	88.03
RS, El-Rafaie et al [19]	93 $\pm$ 3.8	95.00	85.00
SMFFNN, Asadi et al [5]	92.00	*	*
SBPN, Asadi et al [5]	87.00	*	*
BPN+PCA, Asadi et al [5]	73.30	*	*

The proposed classifier yields better results than other classifiers but with faster convergence speed, as presented in Table 6. The accuracy of the proposed classifier was obtained as 96.34 %, MSE as 0.02919 and elapsed time as 2.99 seconds, for SPECT Heart data set. The Levenberg-Marquardt algorithm is more effective and it achieved a lower MSE and thus higher mapping precision, as presented in Table 7 – Table 8. The asterisk (*) indicates that there is no such type of result in the respective existing classifiers.

Graphs of MSE Vs No. of Iterations

In each of the figure there are two curves, which show the proposed method in blue colour and the conventional method in red colour, both are trained simultaneously with the same number of epoch. The proposed method was able to produced greater improvement in error rate. The high number of epoch signifies that the network proceeds to approximate convergence. This indicates that in some sense of training set is adequately attained small training error quickly and efficiently enough to generalise to unseen data (test data) or instances. The aim is to minimise both the false negative (which may let a disease go untreated and get worse) without inflating false positives too much (which may be very costly and emotionally for patients) and being able to attainable lowest measure. This takes the algorithm to reach a stable solution on each of the error (MSE) in the least amount of time. Below are the figures for each of the data set.

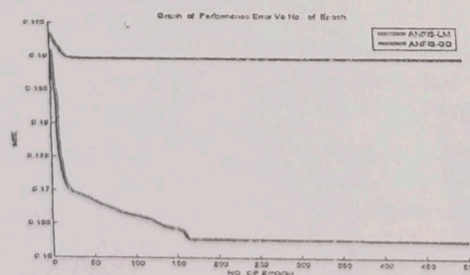


Fig. 2, Graph of MSE Vs No. of Epoch for Diabetic Retinopathy Data set

In Fig.2, it appears that the proposed method outperformed conventional method as error decrease per training examples of the network and continuous to drop. The error starts stabilizing after 160 epochs for the proposed method and after 50 epochs for conventional method. This is because, conventional method has weaker convergence rate as number of iteration is very high. The training process does not over fit the training data and the proposed method has gained and produced the estimation of generalisation in final error achieved of 0.16268 at 15.11 seconds as against 0.18967 at 36.02 seconds of conventional method, both after 500 epochs.

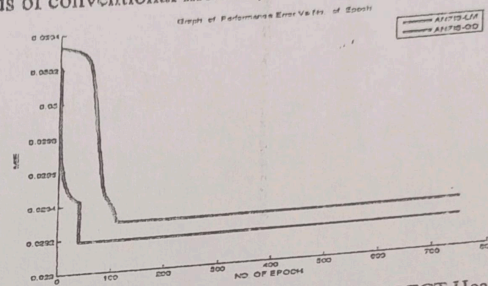


Fig.3, Graph of MSE Vs No. of Epoch for SPECT Heart data set

In Fig. 3, it appears that the proposed method outperformed conventional method as error decrease per training examples of the network and continuous to drop. The error starts stabilizing after 50 epochs for the proposed method and after 125 epochs for conventional method. The training process does not over fit the training data and the proposed method has gained and produced the estimation of generalisation in final error achieved of 0.02919 at 2.99 seconds as against 0.02928 at 18.47 seconds of conventional method, both after 750 epochs.

Conclusion and Future Research Work

The objective of this study is to develop a neuro-fuzzy classifier for the diagnosis of medical diseases. This study proposed two major novelty techniques: firstly, indexing unique membership functions in a row-wise vector using a vectorisation technique. Secondly, the Levenberg-Marquardt algorithm was used instead of conventional approach, gradient descent algorithm. For robustness, the applicability of the proposed technique in data classification using two benchmark data sets in the area of medical diagnosis was demonstrated. There are some improvements of the accuracy results of the proposed method than the results of existing methods.

The proposed method could be enhanced in the future towards the improvement by applying adaptive neuro fuzzy inference system (ANFIS) based metaheuristic algorithms like the genetic algorithm, firefly algorithm or cat swarm optimisation instead of filter approach for feature selection.

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