

DTM-Pade Solution for Heat Transfer of Hydromagnetic Flow Over a Vertical Plate

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Abstract

This study deals with heat transfer of hydromagnetic flow over a vertical plate. The similarity solution is used to transform the system of partial differential equations to ordinary nonlinear differential equations. The non-linear equation of the flow field had been solved using Differential transformation method empowered by Pade approximants which compared well with the fourth order Runge-Kutta method. The results are presented graphically and discussed. It is found that power index, variable magnetic field, Prandtl and other quantities of physical interest significantly influenced the velocity and temperature profile.

Keywords: Heat transfer, DTM-Pade, MHD flow, Steady flow

1.0 Introduction

Flow through porous media plays important role in fields of industrial interest and engineering. In particular, flow on a stretching sheet finds wide application in polymer industries. Recently some Scholars [1,2] have studied the flow of a conducting viscous fluid over a stretching sheet with a constant rate of stretching when the flow is subject to variable magnetic field. The similarity solution for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media as explained in [3]. The work of [4] was based on numerical study of the influence of magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media by considering the Soret and Dufour effects. Numerical study on the effect of thermal radiation on boundary layer flow with heat and mass transfer past a moving vertical porous plate was carried out in [5]. The effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate with temperature dependent viscosity was investigated in [6]. Recently, Aziz [7] reported a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. The paper demonstrates that a similarity solution with the hot fluid on the lower surface of the plate is proportional to the inverse square root of the axial distance. Radiation heat transfer is important in many engineering processes involving high temperatures and good working knowledge of heat transfer plays an instrumental role in designing the pertinent equipment. In many practical applications depending on the surfaces properties and configuration, the radiative transport is often comparable with that of convective heat transfer. Here, it is worth to record that unlike convection/conduction, radiation heat transfer mechanism is rather more complex to account for due to various complexities involved. However, some reasonable approximations have been proposed to make radiation system solvable. The works of [8] and [9] described the essentials of radiative heat transfer. Many authors have reported pertinent radiative heat transfer studies in different configuration [10–17]. The study on the thermal radiation and buoyancy effects on MHD free convection heat generation flow over an accelerating permeable surface with temperature dependent viscosity was investigated in [18]. The Numerical Solution for Thermal Radiation effect on inclined Magnetic Field of MHD Free Convective Heat Transfer Dissipative Fluid Flow Past a Moving Vertical Porous Plate with Variable Suction was carried out in [19] and [22]. Also, the effect of thermal radiation on heat and mass transfer of a variable viscosity fluid past a vertical porous plate permeated by transverse magnetic field was reported in [20]. An investigation on MHD flow analysis using DTM-Pade' and numerical method was done in [21]. The study did not take into consideration the effects of buoyancy and heat transfer. Consequently, this work focused on heat transfer of hydromagnetic flow over a vertical plate using Differential Transform Method empowered by Pade approximant (DTM-Pade) to obtain the expressions for velocity and temperature profile.

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2.0 Problem Formulation

Let us consider a steady, magnetohydrodynamic boundary layer flow of a viscous incompressible electrically conducting fluid coupled heat by mixed convection flow over a vertical plate. The fluid is assumed to be Newtonian and its property variations due to temperature are limited to fluid density. The density variation and the effects of the buoyancy and porosity are taken into account in the momentum equation (Boussinesq approximation). In addition, a transverse magnetic field of strength B_0 has been applied to the plate as shown in the figure below:

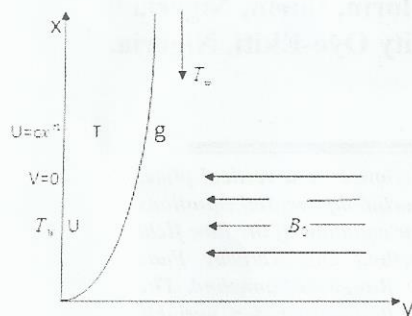


Figure 1

Flow configuration and coordinate system

The flow and heat transfer with the radiation effects are governed by the following equations; Continuity, Momentum and Energy equation can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \sigma \frac{B^2(x)u}{\rho} - \frac{\nu u}{k_p(x)} + gB_T(T - T_\infty) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (3)$$

Where u and v are the velocity components in x and y direction respectively, ρ is the fluid density, ν is the kinematic viscosity, σ is the electrical conductivity of the fluid, c_p is the specific heat at constant pressure, T_1 is the free stream temperature, B_T is the thermal expansion coefficient, g is the gravitational acceleration, q_r is the radiation heat flux. The boundary conditions are given by:

$$u = cx^n, v = 0, T = T_w \text{ at } y = 0$$

$$u = 0; T = T_\infty \text{ as } y \rightarrow \infty \quad (4)$$

Where c is the stretching rate, T and T_∞ are the temperatures. Radiation heat flux is given as

$$q_r = \frac{4\sigma \partial T^4}{3k \partial y}$$

where $T^4 = 4T_\infty^3 T - 3T_\infty^4$ is higher Taylor series, and σ and k are the Stefan-Boltzmann constant and the mean absorption coefficient respectively.

The variable magnetic field is given by $B(x) = B_0 x^{(n-1)/2}$
 $k_p(x)$ is the variable porosity is given by $k_p(x) = k_p' x^{1-n}$

2.1 Transformation Model

Introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}$$

And the similarity transformation [20]

$$\psi(x, y) = \sqrt{\frac{2\nu c}{n+1}} x^{\frac{n+1}{2}} f(\eta)$$

$$\eta(x, y) = \sqrt{\frac{c(n+1)}{2\nu}} x^{\frac{n+1}{2}} y$$

Equation (1) is identically satisfied, and equations (2) and (3) becomes

$$f'''(\eta) + ff''(\eta) - \beta f'^2(\eta) - \left(M + \frac{1}{K_p}\right) f'(\eta) + Gr\theta(\eta) = 0 \quad (5)$$

$$(1 + \frac{4}{3}R)\theta''(\eta) + Prf(\eta)\theta' = 0 \quad (6)$$

With the boundary conditions:

$$f(0) = 0, f'(0) = 1; \theta(0) = 1; f(\infty) = 0; \theta(\infty) = 0 \quad (7)$$

The Dimensionless parameters introduced are

$$R = \frac{4\sigma T_\infty^3}{k_r k}, Pr = \frac{\rho c_p v}{k}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \beta = \frac{2n}{n+1},$$

$$M = \frac{2\alpha B_0^2}{\rho c(1+n)}, \frac{1}{K_p} = \frac{2v}{c(1+n)k_p}, Gr = \frac{2g\beta(T_w - T_\infty)x^{2n-1}}{c^2(n+1)} \quad (8)$$

Where

β = Power index

Pr = Prandtl number

K_p = Permeability parameter

R = Radiation parameter

T_∞ = Free stream temperature concentration

f = Dimensionless stream function

θ = Dimensionless temperature

η = Dimensionless transverse coordinates

Gr = Grashoff number

T = Temperature

M = Magnetic parameter

3.0 Method of Solution

In order to obtain the approximate solution to the velocity profile, temperature profile (that is equation (5), (6) and the corresponding boundary condition equation (7). We employ the method of DTM-Pade simulation, which combines the differential transform method (DTM) and mathematical theory of Pade approximants to produce a very stable, convergent and appropriate methodology for nonlinear boundary value problems. Differential Transform method was originally proposed in electric circuit analysis problems, after words it was applied to several systems and differential equations. DTM offers analytical solution in the form of a polynomial and this can be used to solve nonlinear differential equations without linearization and discretization.

Differential transform method (DTM) is an alternative procedures for obtaining analytic Taylor series solution of the differential equations. With this method, we can obtain highly accurate results or exact solutions for differential equations. Here we provide a summary of the fundamentals of DTM analysis.

Consider a function $u(x)$ which is analytic in a domain D and $x = x_0$ represent any point in the domain D . The function is represented by a power series whose center is located at x_0 . The differential transform of the function $u(x)$ is given as

$$u(k) = \frac{1}{k!} \left[\frac{d^k u(x)}{dx^k} \right]_{x=x_0} \quad (9)$$

Where $u(x)$ is the original function and $u(k)$ is the transformed function.

The inverse Transformation is defined as follows:

$$u(x) = \sum_{k=0}^{\infty} (x - x_0)^k u(k) \quad (10)$$

Combining equations (9) and (10), gives;

$$u(x) = \sum_{k=0}^{\infty} \frac{(x - x_0)^k}{k!} \left[\frac{d^k u(x)}{dx^k} \right] \quad (11)$$

The fundamental mathematical operation performed by differential transform method are given in the table (1.) below, especially the ones used in the course of this work.

3.1 Pade Approximant

Pade Approximant are used to approximate truncated power series. Pade approximants are applied to manipulate the obtained series problems which involve higher power series, since the radius of convergence of the power series may not be large enough to contain the two boundaries. It will appear difficult if we use power series to solve the problem, to overcome this difficulty, Pade approximant is the best approximation for a polynomial approximation of a function into rational functions of polynomials of given order. Suppose that a function $f(\eta)$ is represented by a power series,

$$f(\eta) = \sum_{i=0}^{\infty} c_i \eta^i \quad (12)$$

Equation (12) is the fundamental starting point of any analysis using Pade approximants c_i , $i = 0, 1, 2, \dots$ is reserved for the given set of coefficients and $f(\eta)$ is the associated function.

$[L/M]$ Pade approximant is a rational fraction

$$\frac{a_0 + a_1\eta + a_2\eta^2 + \dots + a_L\eta^L}{b_0 + b_1\eta + b_2\eta^2 + \dots + b_M\eta^M} \quad (13)$$

equation (13) has a Maclaurin expansion, this agrees with equation (10) also in equation (11). There are $L+1$ numerator and $M+1$ denominator coefficients. So there exist $L+1$ unknown coefficients in all.

This number suggests that normally $[L/M]$ ought to fit the power series in equation (10) through the orders $1, n, n^2, \dots, n^{L+M}$. In the notation of formal power series

$$\sum_{i=0}^{\infty} C_i \eta^i = \frac{a_0 + a_1\eta + a_2\eta^2 + \dots + a_L\eta^L}{b_0 + b_1\eta + b_2\eta^2 + \dots + b_M\eta^M} + O(\eta^{L+M+1}) \quad (14)$$

$$(b_0 + b_1\eta + b_2\eta^2 + \dots + b_M\eta^M)(c_0 + c_1\eta + \dots) = a_0 + a_1\eta + \dots + a_L\eta^L + O(\eta^{L+M+1}) \quad (15)$$

Equating the coefficients of $\eta^{L+1}, \eta^{L+2}, \dots, \eta^{L+M}$ we get.

$$b_M c_L - M + 1 + b_{M-1} c_L - M + 2 + \dots + b_0 c_L + 1 = 0,$$

$$b_M c_{L+1} - M + 2 + b_{M-1} c_{L+1} - M + 3 + \dots + b_0 c_{L+1} + 2 = 0,$$

$$\dots$$

$$b_M c_L + b_{M-1} c_{L+1} + \dots + b_0 c_{L+M} = 0 \quad (16)$$

If $j < 0$; we define $c_i = 0$ for consistency, for $b_0 = 1$; equation (13) therefore become a set of M linear equation for M unknown denominator coefficients

$$\begin{bmatrix} C_{L-M+1} & C_{L-M+2} & \dots & C_{L+1} \\ C_{L-M+1} & C_{L-M+3} & \dots & C_{L+2} \\ \vdots & \vdots & \dots & \vdots \\ C_L & C_{L+1} & \dots & C_{L+M} \end{bmatrix} \begin{bmatrix} b_M \\ b_{M-1} \\ \vdots \\ b_0 \end{bmatrix} = - \begin{bmatrix} C_{L+1} \\ C_{L+2} \\ \vdots \\ C_{L+M} \end{bmatrix} \quad (17)$$

from these equations, b_i may be found. The numerator coefficients $a_0, a_1, \dots, a_L$ follow immediately from equation (12) by equating the coefficients of $1, n, n^2, \dots, n^{L+M}$ such as

$$a_0 = c_0,$$

$$a_1 = c_1 + b_1 c_0$$

$$a_2 = c_2 + b_1 c_1 + b_2 c_0$$

$$\dots$$

$$a_L = c_L + \sum_{i=1}^{\min[L/M]} b_i c_{L-i} \quad (18)$$

Therefore, Equation (17) and equation (18) normally determine the Pade Numerator and denominator. Equation (17) and (18) are called Pade equations. The $[L/M]$ Pade approximant is constructed which agrees with the equation (12). Thus, in order to solve equation (5) and (6). We take its differential transform by using the related definitions which yield:

$$F(k+3) = \left[\frac{1}{(k+1)(k+2)(k+3)} \left\{ \sum_{r=0}^k (r+1)(k-r+1)F(r+1)F(k-r+1) \right\} \right] -$$

$$\sum_{r=0}^k (k-r+2)(k-r+1)F(r)F(k-r+2) + \left(M + \frac{1}{K_p}\right)(k+1)F(k+1) - Gr\theta(k) \quad (19)$$

and

$$\theta(k+2) = \left[\frac{1}{(k+1)(k+2)} \left\{ -\frac{Pr}{(1 + \frac{1}{3})} \sum_{r=0}^k (k-r+1)F(r)\theta(k-r+1) \right\} \right] \quad (20)$$

Also, we take the differential transform of the boundary conditions in equation (7), which yield:

$$F(0) = 0, \quad F(1) = 1, \quad F(2) = A$$

$$\theta(0) = 1, \theta(1) = B \quad (21)$$

We calculate equation (19) and (20) Recursively,

$$F(3) = \frac{\beta}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6}$$

$$\theta(2) = 0$$

$$F(4) = \frac{\beta}{6} + \frac{A}{12} + \frac{(2M + \frac{2}{K_p})A}{24} - \frac{GrB}{24}$$

$$\theta(3) = \frac{PrB}{6(1 + \frac{1}{3})}$$

$$F(5) = \frac{\beta \left[\frac{A}{3} + M + \frac{1}{K_p} - Gr + 4A^2 \right]}{60} - \frac{\beta}{60} - \frac{M}{60} - \frac{1}{60K_p} + \frac{Gr}{60} - \frac{A^2}{30} + \frac{(3M + \frac{3}{K_p}) \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right)}{60}$$

$$\theta(1) = \frac{PrAB}{12 \left(1 + \frac{1R}{3} \right)}$$

$$F(6) = \frac{\beta \left[\frac{A}{3} - \frac{2A}{3} + \frac{(2M + \frac{2}{K_p})A}{3} - \frac{GrB}{3} + 12A \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right) \right]}{120}$$

$$\frac{\beta \left[\frac{A}{3} + M + \frac{1}{K_p} - Gr + 4A^2 \right]}{60} + \frac{A \left(-2M + \frac{2}{K_p} \right) A}{240} + \frac{GrB}{240} - \frac{A \left(\frac{A}{6} + M + 16K_p - \frac{Gr}{6} \right)}{15} +$$

$$\frac{(4M + \frac{4}{K_p}) \left(\frac{A}{6} - \frac{A}{12} + \frac{(2M + \frac{2}{K_p})A}{24} - \frac{GrB}{24} \right)}{120} + \frac{GrPrB}{720 \left(1 + \frac{1R}{3} \right)}$$

$$\theta(5) = \frac{Pr \left(-\frac{PrB}{2 \left(1 + \frac{1R}{3} \right)} + \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right) B \right)}{20 \left(1 + \frac{1R}{3} \right)}$$

$$F(7) = \frac{1}{240} \left[\frac{\beta \left[\frac{A}{3} + M + \frac{1}{K_p} - Gr + 4A^2 \right]}{6} - \frac{\beta}{6} - \frac{M}{6} - \frac{1}{6K_p} + \frac{Gr}{6} - \right.$$

$$\left. \frac{A^2}{3} + \frac{1}{6} \left(3M + \frac{3}{K_p} \right) \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right) + 16A \left[\frac{\beta}{6} A - \frac{A}{12} + \right. \right.$$

$$\left. \frac{A}{24} + \frac{(2M + \frac{2}{K_p})A}{24} - \frac{GrB}{24} \right] + 9 \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right)^2 \Big] -$$

$$\frac{1}{630} \left[\frac{\beta \left[\frac{A}{3} + M + \frac{1}{K_p} - Gr + 4A^2 \right]}{6} - \frac{\beta}{6} - \frac{M}{6} - \frac{1}{6K_p} + \frac{Gr}{6} + \frac{A^2}{315} - \right.$$

$$\left. \frac{1}{630} \left(3M + \frac{3}{K_p} \right) \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right) - \frac{1}{15} \left(A \left[\frac{\beta}{6} A - \frac{A}{12} + \frac{(2M + \frac{2}{K_p})A}{24} - \frac{GrB}{24} \right] - \right. \right.$$

$$\left. \frac{1}{35} \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right)^2 + \frac{1}{210} \left[\left(5M + \frac{5}{K_p} \right) \left(\frac{\beta \left[\frac{A}{3} + M + \frac{1}{K_p} - Gr + 4A^2 \right]}{60} - \right. \right. \right.$$

$$\left. \frac{1}{60} - \frac{M}{60} - \frac{1}{60K_p} + \frac{Gr}{60} - \frac{A^2}{30} + \frac{1}{60} \left(3M + \frac{3}{K_p} \right) \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right) \right] + \frac{GrPrAB}{2520 \left(1 + \frac{1R}{3} \right)}$$

and so on...

The closed form of the solution are equation (22) and (23) respectively

$$f(\eta) = \sum_{k=0}^{15} F(k) \eta^k \quad (22)$$

and

$$\theta(\eta) = \sum_{k=0}^{15} \theta(k) \eta^k \quad (23)$$

For Momentum equation we have:

$$f(\eta) = \eta + A\eta^2 + \left(\frac{\beta}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right) \eta^3 + \left(\frac{\beta}{6} + \frac{A}{12} + \frac{(2M + \frac{2}{K_p})A}{24} - \frac{GrB}{24} \right) \eta^4 +$$

$$\left(\frac{\beta \left[\frac{A}{3} + M + \frac{1}{K_p} - Gr + 4A^2 \right]}{60} - \frac{\beta}{60} - \frac{M}{60} - \frac{1}{60K_p} + \frac{Gr}{60} - \frac{A^2}{30} + \frac{(3M + \frac{3}{K_p}) \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right)}{60} \right) \eta^5 \dots \quad (24)$$

For thermal heat equation we have:

$$\theta(\eta) = 1 + B\eta + \left(\frac{PrB}{6 \left(1 + \frac{1R}{3} \right)} \right) \eta^3 + \left(\frac{PrAB}{12 \left(1 + \frac{1R}{3} \right)} \right) \eta^4 + \left(\frac{Pr \left(-\frac{PrB}{2 \left(1 + \frac{1R}{3} \right)} + \left(\frac{A}{6} + \frac{M}{6} + \frac{1}{6K_p} - \frac{Gr}{6} \right) B \right)}{20 \left(1 + \frac{1R}{3} \right)} \right) \eta^5 + \dots \quad (25)$$

In the present boundary layer problem, owing to the free stream conditions, Differential transformation method diverges and cannot satisfy these boundary conditions. To handle boundary value problems on infinite domains, we apply pade

approximant on equation (24) and (25), in order to verify the accuracy of the DTM-Pade for $f''(\eta)$ and $\theta(\eta)$ for various pade [N,N], the accuracy of the result increases with increase in Pade [N,N]

$$\lim_{\eta \rightarrow \infty} f'(\eta) = 0 \quad (26)$$

and

$$\lim_{\eta \rightarrow \infty} \theta(\eta) = 0 \quad (27)$$

Solving the series solution equation (24) and (25) generated from those limit equation (26) and (27) simultaneously with the default parameters, $Pr = 0.72$, $M = 1$, $Gr = 1$, $\beta = 0.5$, $K_p = 100$, $R = 1$.

We obtain at these values our A and B to be -0.483805400 and -0.7204185277 respectively.

Substituting A and B and the default parameters into equation (24) and (25) on maple software, we obtain

$$\begin{aligned} f'(\eta) = & 1 - 0.96761080009\eta + 0.255000000\eta^2 - 0.04281139673\eta^3 + \\ & 0.000212500000\eta^4 + 0.004379005253\eta^5 - 0.001112214473\eta^6 - \\ & 0.0001575695039\eta^7 + 0.0001814142546\eta^8 - 0.00003420434684\eta^9 - \\ & 0.00001077532804\eta^{10} + 7.174660165 \times 10^{-6}\eta^{11} - \\ & 9.899049729 \times 10^{-7}\eta^{12} - 4.756091370 \times 10^{-7}\eta^{13} + 2.538353035 \times 10^{-7}\eta^{14} \\ \theta(\eta) = & 1 - 0.7204185277\eta + 0.03705009571\eta^3 - \\ & 0.008962518189\eta^4 - \\ & 0.0007701127037\eta^5 + 0.0008425506964\eta^6 - 0.0001683260955\eta^7 - \\ & 0.00001187757866\eta^8 + 0.00001477918507\eta^9 - 3.564819075 \times 10^{-6}\eta^{10} + \\ & 9.477710699 \times 10^{-8}\eta^{11} + 2.152766920 \times 10^{-7}\eta^{12} - 7.618540273 \times 10^{-8}\eta^{13} + \\ & 9.324455378 \times 10^{-9}\eta^{14} + 2.578994044 \times 10^{-9}\eta^{15} \end{aligned}$$

4.0 Discussion of Result

For purpose of results validation, physical meaningful values were chosen from the parameters; the effects of various parameters such as Magnetic Parameter M,

Porosity Parameter K_p , Power index β , Prandtl number Pr , Radiation R , Grashoff number Gr as well as the consistency of the methods are discussed. Thus to validate the accuracy of numerical result, we compared DTM-Pade method with fourth order RungeKutta method using shooting techniques and that DTM-Pade method compared well with RungeKutta method as seen in table 2 and it involved less computational effort than the fourth order RungeKutta method.

For $Gr=1$, $Pr=0.72$, $M=1$, $\beta=0.5$, $K_p=100$, $R=1$

Table 2: Comparison of DTM-Pade method with fourth order RungeKutta method using shooting techniques

η	DTM Pade Method		Runge- Kutta Method		Absolute Error	
	f'	θ	f'	θ	f'	θ
0	1.000000	1.000000	0.999999	0.999999	1×10^{-10}	1×10^{-10}
0.2	0.816337	0.856198	0.816339	0.856198	1.788×10^{-6}	4.93×10^{-8}
0.4	0.651061	0.713969	0.651065	0.713969	3.6447×10^{-6}	9.99×10^{-8}
0.6	0.502301	0.574565	0.502306	0.574565	5.6362×10^{-6}	1.559×10^{-7}
0.8	0.368414	0.438897	0.368422	0.438897	7.8228×10^{-6}	2.245×10^{-7}
1.0	0.248042	0.307572	0.248052	0.307573	1.02262×10^{-5}	3.141×10^{-7}

Figure 2 shows the effect of magnetic field parameter M on the fluid flow velocity. It is noticed that as M increases the velocity distribution decreases, since the magnetic force exert a retarding force on the flow velocity

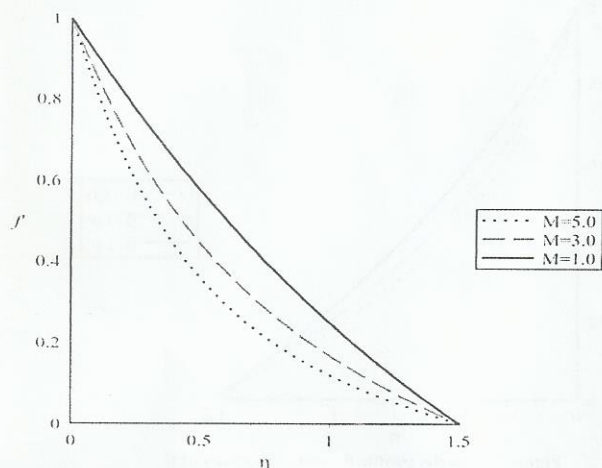
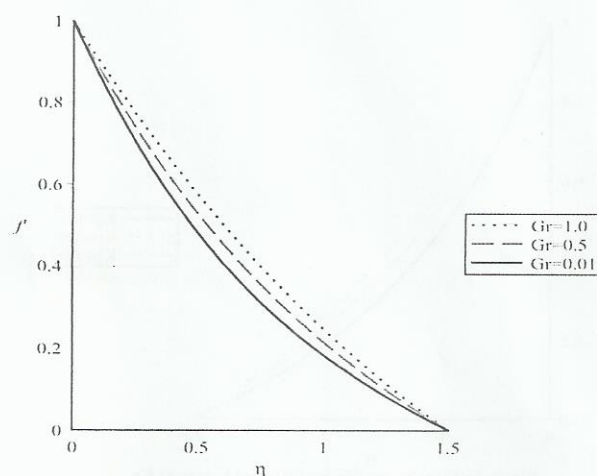
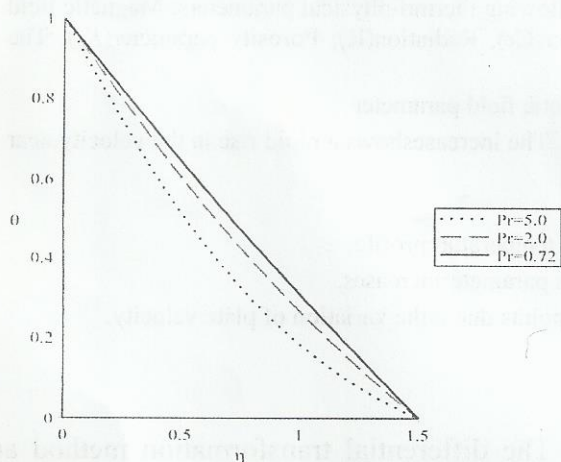
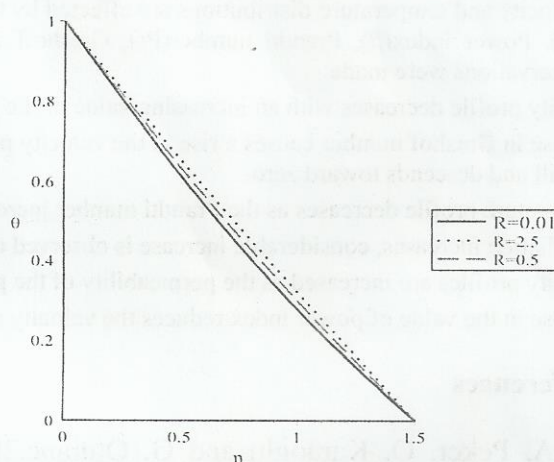
Figure 2: Velocity profile for different values of M Figure 3: Velocity profile for different values of Gr

Figure 3 depicts the effects of thermal Grashoff number on velocity profiles. It is shown that an increases in Gr causes a rise in the velocity profiles, a rise in Grashoff number boost the buoyancy force. The increase shows a rise in the velocity near the wall and descends towards zero

Figure 4 depict the effect of Prandtl number on the temperature profile, it has been observed that the temperature profile decreases as the Prandtl increases. The reason is that smaller value of Pr are equivalent to increasing thermal conductivities and therefore heat is able to diffuse away from the heated surface more rapidly than higher values of Pr

Figure 4: Temperature profile for different values of Pr Figure 5: Temperature profile for different values of R

The influence of Radiation Parameter R on the temperature profile is shown on Figure 5. As R increases, Considerable increase is observed in the temperature profile, from the peak value at the wall across the boundary layer regime to the free stream, at which the temperature are negligible for any value of R . The increase in temperature is accompanied by increase in the temperature layer

The effects of the porosity parameter K_p on the velocity profiles is presented in Figure 6, from these Figure, it is observed that the velocity profiles are increased as the permeability of the Porous Parameter increases.

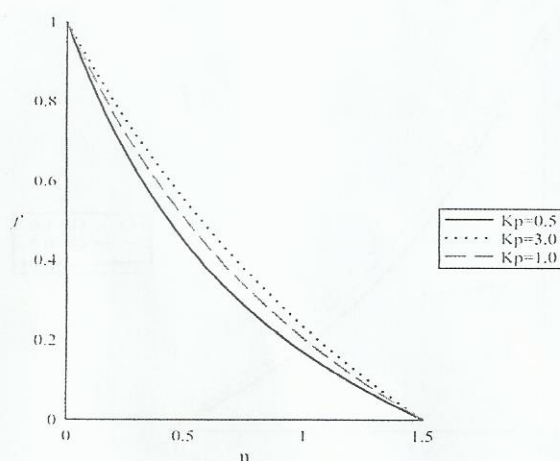
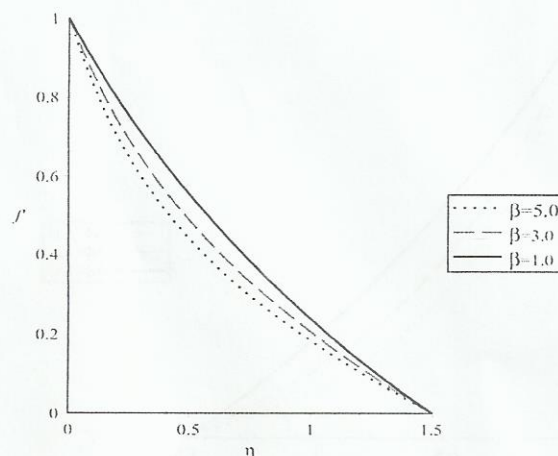
Figure 6: Velocity profile for different values of K_p Figure 7: Velocity profile for different values of β

Figure 7 shows the velocity profiles for different values of β . The values represent the integer and fractional values of n . This contributes to non linear variation of the plate velocity as well as magnetic field strength. Increase in the value of power index β reduces the velocity at all points due to the variation of plate velocity

5.0 Conclusions

A numerical study is performed for the problem of hydromagnetic boundary layer flow with heat transfer over a vertical plate. The velocity and temperature distributions are affected by the following thermo-physical parameters: Magnetic field parameter(M), Power index(β), Prandtl number(Pr), Grashoff number(Gr), Radiation(R), Porosity parameter(K_p). The following observations were made:

1. Velocity profile decreases with an increasing value of the magnetic field parameter
2. Increase in Grashof number causes a rise in the velocity profile. The increase shows a rapid rise in the velocity near the wall and descends toward zero.
3. Temperature profile decreases as the Prandtl number increases.
4. As radiation increases, considerable increase is observed on the temperature profile.
5. Velocity profiles are increased as the permeability of the porous parameter increases.
6. Increase in the value of power index reduces the velocity at all points due to the variation of plate velocity.

6.0 References

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