

ENHANCED SEISMIC Q-COMPENSATION: ALGORITHM FORMULATION AND SYNTHETIC APPLICATION

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ABSTRACT

The process of compensating for the effects of attenuation on seismic records is often marred by numerical instability that causes the introduction of artifacts and the amplification of seismic noise at the expense of the desired seismic signals. This often leads to poor seismic image resolution, especially in seismic traces representing deep-seated high-attenuating rock layers that are typical for hydrocarbon reservoirs. In an effort to compensate for the effects of attenuation in seismic records, I propose a frame-work for correcting amplitude diminution and phase distortion in seismic waves, using a recursive Q inverse filtering scheme. The time varying inverse Q filter has a Fourier integral representation in which the direction of propagation for the up-going and down-going waves is reversed, and the wave number is defined as a complex conjugate of the wave number in the forward propagating waves. To overcome the issue of instability, the wave number is replaced with slowness to limit the frequency dependence of the computation, and the Q-compensation algorithm is implemented in a layer-by-layer step wise approach. The compensation algorithm is tested with synthetic but realistic seismic records. Results show that the algorithm is suitable for correcting energy dissipation and phase dispersion that usually degrade the resolution of subsurface seismic images. The compensated traces give higher resolved images of the reflectors at accurate depth location.

Keywords: Attenuation, Q-compensation, Inverse Filtering, Seismic Signal, Synthetic seismograms

INTRODUCTION

The amplitude and waveform of seismic waves are strongly impacted by the anelasticity and inhomogeneity in the media through which the waves travel. This usually results in loss, in parts, of the desired information in recorded seismic traces. The loss is often expressed in the form of amplitude diminution, wavelet distortion and delayed signal arrivals, which in turn degrade the quality of seismic data. Attenuation causes poor seismic interpretability and contributes to seismic-well data mis-tie. Correcting for the effects of amplitude reduction and waveform distortion in recorded seismic traces is beneficial to exploration seismology for many reasons. One reason is that signal amplitude contains information about the lithology and reservoir conditions (Winkler and Nur, 2009; Klimentos and McCan, 1990; Batzle *et al.*, 1996). Another reason is that the signal arrival time contains information on the location of the reflector. In order to enhance the interpretability of seismic records, seismic data processing usually involve a stage where the effects of attenuation on seismic records are corrected. This step is here known as Q-compensation. Q-compensation is the process by which the undesirable effects of Earth's anelasticity and inhomogeneity on seismic waves (i.e., amplitude reduction, waveform distortion and velocity dispersion) are corrected or compensated for.

A common approach to Q-compensation is inverse Q-filtering. Previous work on the application of inverse Q filtering to the improvement of subsurface seismic images includes Hale (1981 & 1982);

Bickel and Natarajan (1985); Hargreaves and Calvert (1991); Mittet *et al.* (1995); Wang (2002 & 2006); Knut Sorsdal (2009); Raji and Riethbrock (2011). In most of these studies, the procedure for implementing inverse Q filtering in a stack of layers is not emphasized. Inverse Q filtering is only discussed for a single layer model. The few algorithms that considered inverse Q filtering in stack of Earth layers are designed for constant Q layers (i.e. all the layers are having the same Q value or the same degree of absorption). In reality, different rock layers usually have different attenuation characteristics, and different Q values. Apart from this, Q-compensation algorithms at present have two main limitations: (i) accuracy of the compensated signal, (ii) and numerical instability. The accuracy of the compensated seismic signal is a measure of how close the Q-compensated seismic image compares to the real image of the reflector, if the Earth media were perfectly elastic. Instability is a common problem of the inverse Q filtering process. It is usually due to overflow or numerical instability in the estimation of the exponential functions, especially at high travel time components of the inverse Q-filtered traces. This problem often leads to the introduction of artifact and the amplification of ambient noise. in the later part of the seismic traces after Q-compensation (Figure 1).

In order to solve the issue of instability, either an approximation is defined for the amplitude compensation operator (Wang, 2002; Li *et al.*, 2009), or a stabilization factor is applied to the amplitude compensation operator (Wang, 2006). A Q-compensation algorithm that uses an amplitude approximation operator or an amplitude gain limit operator can solve the problem of instability but cannot recover an absorbed seismic signal whose amplitude is below a set limit. In other words, such an algorithm cannot recover the highly absorbed signals that correspond to the high travel time components of the seismic trace. Amplitude recovery for seismic signals absorbed in highly attenuating layers at great depth Typical for hydrocarbon reservoirs, cannot be achieved by attenuation compensation algorithms that use an amplitude gain limit operator or amplitude approximation. Against the back-drop of a constant Q algorithm, Q naturally changes with rock type, degree of saturation, and depth. Because of these limitations in the Q-compensation algorithms, an efficient Q-compensation algorithm is desirable. In this paper, I describe a Q-compensation algorithm that can simultaneously compensate for the loss in amplitude and correct phase dispersion in seismic traces recorded from a stack of constant or varying Earth Q layers.

In the subsequent sections, we describe the mathematical algorithm and the procedure for implementing attenuation compensation in seismic traces recorded from various Earth Q models. To give a clear description of the procedure of Q-compensation, a brief description of the process of seismic attenuation is given. The Q-compensation algorithm is tested with synthetic seismograms. In order to demonstrate the improvements in the compensated traces, the difference plots between the compensated and elastic traces are shown. The term Q-compensation is used interchangeably with attenuation compensation throughout this paper.

THE Q-COMPENSATION PROCEDURE

Q-compensation is the inverse procedure for seismic attenuation in which wave propagation is reversed in time along the same path of the forward propagation. This can be achieved by means of an inverse Q-filter. An important step before constructing an inverse Q filter is to determine the Q-factor that caused the attenuation. For the purpose of the Q-compensation algorithm, I briefly review plane wave propagation in an absorptive Earth medium, and then describe the compensation (inversion) procedure. The plane wave two-way forward propagation $A(k_h, z, \omega)$ from a source at depth z , to a reflector at further depth step Δz , and back to a recorder at z (Figure 2) can be defined as:

$$(1) \quad A(k_h, z - \Delta z, \omega) = A(k_h, z, \omega) \exp(-i(k - k_h) \Delta z)$$

Where i is the imaginary number and k is the wave-number. k_h and ω are the horizontal wave-number and the angular frequency, respectively.

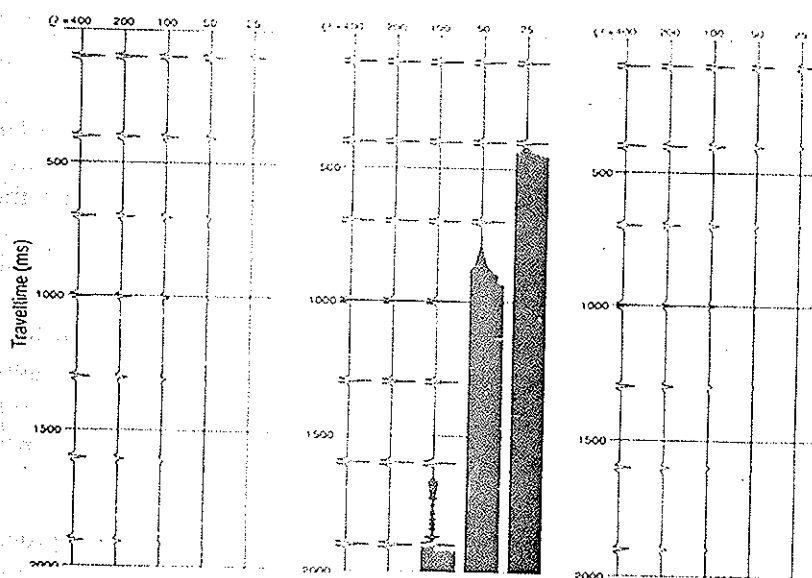


Figure 1. Attenuation and Q-Compensation – showing the problem of instability in full amplitude and phase compensation. (a) Synthetic traces showing the effect of attenuation- $Q=400, 200, 100, 50, 25$ (b) Q-compensation for full amplitude and phase compensation, result which clearly indicates numerical instability. (c) Q-compensation for both amplitude and phase compensation- with full band phase compensation but band limited amplitude compensation. Adapted from Wang (2002).

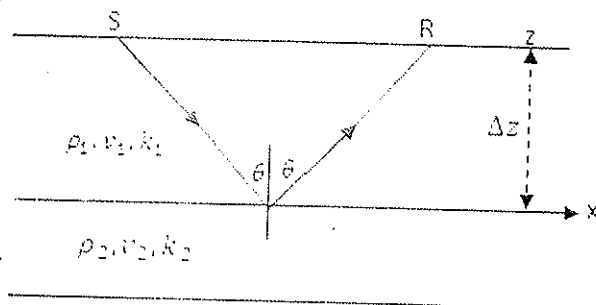


Figure 2. Schematic diagram of wave propagation in Earth media. The wave will be compensated for earth Q-effects on its ways from the source (S) down to the reflector and to the receiver (R)

$$\frac{w}{v(w)}; \quad k_x = \frac{w \sin(\theta)}{v(w)}. \quad (2)$$

Where θ is the angle of incidence. Absorption or attenuation is introduced into wave propagation by replacing the wave velocity with a complex frequency-dependent velocity. Following Futterman, (1962), I define the velocity v of an Earth layer as a frequency dependent complex velocity:

$$v = \beta(w) - \frac{j}{2\beta(w)Q}. \quad (3)$$

Where $\beta(w) = v(1 - (\pi Q)^{-1} \ln(\frac{w}{w_c}))$.

β is the real valued part of the velocity, and $\frac{j}{2\beta(w)Q}$ is the imaginary part of the velocity. Q is the quality factor and w_c is the reference angular frequency. Substituting equation (3) into equation (2), the wave number, k , can be redefined as a complex quantity:

$$k = \frac{w}{\beta(w)} (1 - \frac{j}{2Q}). \quad (4)$$

For a case of normal incidence, where $k_x = 0$, the downward propagation of a plane wave (i.e. from the source to the reflector, in Fig. 2) in an attenuating medium can be written as:

$$A(z + \Delta z, w) = A(z, w) \exp(-j \frac{w}{\beta(w)} \Delta z) \exp(-\frac{w}{2\beta(w)Q} \Delta z). \quad (5)$$

Q-filtering is introduced to wave propagation by equations 4 and 5. The two exponential operators in (5) describe frequency-velocity-dispersion and amplitude reduction, respectively. To treat the wave for the undesirable effects of attenuation (i.e., amplitude reduction and phase dispersion), Q-compensation must be performed. Q-compensation in seismic records can be performed using an inverse Q filtering scheme. Inverse Q-filtering for the wave propagation defined in equation (5) can be implemented as:

$$A(z + \Delta z, w) = A(z, w) \exp(jk^*(w) \Delta z). \quad (6)$$

where the k^* in equation 6 is the conjugate of the k in equation 1. Substituting the conjugate of equation 4 in to 6, the inverse Q- filtering scheme for the down-going wave is defined thus:

$$A(z + \Delta z, w) = A(z, w) \exp(j \frac{w}{\beta(w)} \Delta z) \exp(\frac{w}{2\beta(w)Q} \Delta z). \quad (7)$$

Equation 7 represents the basic inverse Q filtering scheme in which the first and the second exponential operators, respectively, corrects for the effects of phase dispersion and amplitude diminution in the down-going waves.

To compensate for the effects of amplitude diminution and phase dispersion in the up-going waves (i.e. from the reflector to the receiver), equation 6 is used with the jk^* term replaced with $-jk^*$; k^* being the conjugate of k^* ; the negative sign represents a reverse direction. By this procedure, the wave amplitude grows with reversed travel time. The Q-compensated seismic traces would be similar to the traces from a perfectly elastic Earth medium. The basic concept of the Q-compensation scheme is to correct for the effects of attenuation from the source point to the receiver location. The procedure is analogous to starting from the receiver; backward-propagate the wave to the reflector; and to the source (Fig. 2).

Q-COMPENSATION SCHEME IN A STACK OF LAYERS

Algorithms for implementing inverse Q-filtering in a stack of layers are usually built using the theory of wavefield downward continuation proposed by Claerbout (1971). Full amplitude and phase compensation schemes using an inverse Q filter built on the downward continuation method usually suffer numerical instability or 'overflow' (Mittet *et al.*, 1995; Wang, 2002) in the estimation of the exponential function of the compensation term. The compensation term (see equation 7) is an exponential function of frequency, time, and Q. The consequence of the numerical instability is that artifacts are introduced and seismic noise is amplified at the expense of the seismic signals that are being compensated for attenuation (Fig 1).

We describe an efficient method of implementing inverse Q filtering algorithm in a stack of layers having similar or varying degrees of attenuation, using a layer-by-layer recursive Q-compensation scheme. The scheme does not use amplitude approximation or stabilization factor. The framework upon which the Q-compensation algorithm is built is the reflectivity algorithm of Müller, where the potential amplitude of the seismic signal in each layer is calculated by a recursive method. The recursive method is numerically stable for all frequencies and slownesses. In the correction scheme, attenuation compensation is introduced into the reflectivity algorithm of Müller (1985) through the phase operator-the wave number is now defined as a complex conjugate of the wave number in the attenuation scheme (equation 4), for the down going and up-going waves; and the direction of propagation is reversed by changing the sign preceding the imaginary number. Each layer is treated as a single homogenous entity; the attenuated signal corresponding to the layer boundary is isolated in the time domain using wave field extrapolation; treated with the inverse Q-filter in the frequency domain; and stored in the time domain.

The output of the compensation scheme is a trace formed by the summation of compensated signals representing all layers in the stack. The cumulative inverse Q-filter at a layer step contains the Q factor that is responsible for the attenuation the wave experienced in the previous and the current layer. Given the diagram shown in figure 3, the compensated signal A_3 from reflector 3 is:

$$A_3 = E_3 A_2 E_{u3} \quad (8)$$

$$E_3 = e^{-jk\Delta Z_3}; E_{u3} = e^{+jk\Delta Z_3}; \Delta Z_3 = Z_3 - Z_2$$

E_3 and E_{u3} correct the effects of attenuation in the down-going and up-going waves, respectively, in layer 3. $A_2 = E_2 A_1 E_{u2}$; $A_1 = E_1 A_0 E_{u1}$; and

Where A_2 and A_1 are the matrices containing the compensated signals from reflectors 2 and 1, respectively. B_0 is the matrix containing the potential amplitude of the attenuated signal at the base of layer 0. E_i and E_{u_i} are the phase operators for the down-going and up-going waves in layer i . $E_i = e^{-jk\Delta Z_i}$; $E_{u_i} = e^{+jk\Delta Z_i}$, i is the layer number ($i = 0, 1, 2, \dots, n$). In each case, E_i is an exponential function containing the compensation factor, the thickness, and velocity of each layer. The terms j, k and Z_i represent the imaginary number, wave-number, and the depth of layer i , respectively. k contain the Q-factor for correcting amplitude reduction and phase dispersion (see equation 4) experienced by the waves in layer i . The isolated attenuated event at every layer step is treated with the corresponding inverse Q-filter in the Fourier domain, and stored in the time domain. To circumvent numerical instability that often mars the result of inverse Q-filtering: (i) wave-numbers are replaced with

$$u = \frac{k_x}{w} = \frac{\sin(\theta)}{v(w)} \quad (\text{horizontal slowness}) \quad (10a)$$

$$a = \frac{k}{w} = (v(w)^{-2} - u^2)^{\frac{1}{2}}, (\text{vertical slowness}) \quad (10b)$$

where k_x and k are defined in equation 2; (ii) Q-compensation is implemented layer by layer in a recursive manner; (iii) and the compensation scheme avoids working with exponential functions as much as possible - in the algorithm, exponential function only appears in the phase operator.

The compensated plane wave at a layer step 3 can be defined as:

$$A_3(T_3, w) = A_2(T_2, w) \exp\left(j \frac{w \Delta T_3}{\beta_3(w)}\right) \exp\left(\frac{w \Delta T_3}{2 \beta_3(w) Q_3}\right), \quad (11)$$

where T is the traveltime, and $\Delta T_3 = T_3 - T_2$. The time-domain seismic signal at layer step 3 can be given as shown in equation 12.

$$a_3(T_3) = \frac{1}{2\pi} \int A_3(T_3, w) dw. \quad (12)$$

Similar to Wang (2002); Montana and Margrave, 2005; Knut Sorsdal (2009), the complete seismic record across the stack of layers is the summation of the Q-compensated seismic events (in the time domain) representing the individual layer in the stack. Each Q-compensated seismic event is the plane wave reflection at each layer boundary. One advantage of the algorithm is that it can be used for attenuation compensation in multi-component seismic data. Another advantage of the method is that it is built on a recursive algorithm which is unconditionally stable at all frequencies and slownesses, see Müller (1985). Some algorithms, see Kennett (1983) Sec.6.2.2, suffer from 'overflow' problem in the calculation of exponential functions for high frequency and slownesses.

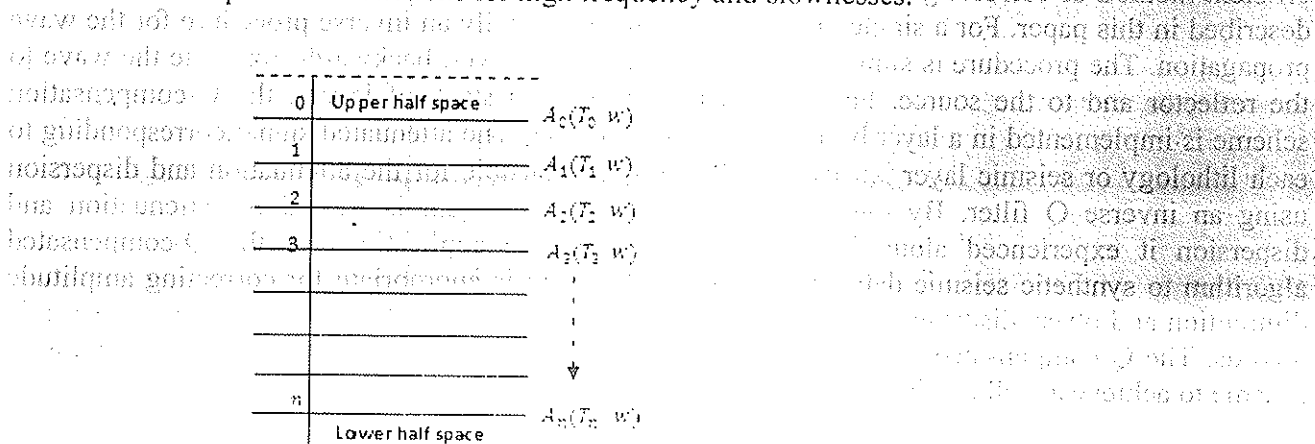


Figure 3. A schematic diagram of a stack of homogeneous absorbing layers between two half spaces

APPLICATION TO SYNTHETIC SEISMIC DATA

The Q-compensation algorithm is tested on synthetic seismic data from a stack of horizontal homogenous absorbing Earth layers. The Earth models consist of eight layers as shown in figures 4 and 5. The layer properties are selected such that seismic events representing the top layer are separated in time by 300ms from the event representing the next lower layer (see the reference plot in figures 4c & 5c). The layer parameters for the Earth model are listed in the appendix. The reference figure is a seismic trace from perfectly elastic layers, with no attenuation and dispersion.

Figure 4a shows seismic records from constant Q layers before attenuation compensation, while figure 4b shows the traces after attenuation compensation. The difference plot between the reference trace and the Q-compensated traces are shown in figure 4d & e. Figure 5a shows synthetic seismic records from a stack of layers having various degrees of attenuation. Figures 5b and c show the seismic records after Q-compensation and the reference plot, respectively. The difference plot between the reference and the compensated traces are shown in figure 5d & e. Overall, the Q-compensation algorithm efficiently corrects the phase dispersion and amplitude diminution. It returns the seismic events to their correct positions with high resolution. The compensated traces compare favourably with the reference plot - as if the Earth layers are perfectly elastic. The Q-compensation process improved the interpretability of the seismic traces compared to the original (attenuated) traces.

DISCUSSION

Seismic wave propagation in real Earth is different in many ways from wave propagation in an ideal solid. The Earth is neither a perfectly elastic medium nor an ideal solid medium. The amplitude, waveform, and arrival time of waves propagating in the Earth are influenced by anelasticity and inhomogeneity in the rocks through which they travel. These cause attenuation which degrades the quality of seismic data and affects the resolution of subsurface structures. An efficient method of correcting the undesirable effects of the Earth's attenuation on seismic waves is described in this paper. For a single layer, the scheme is basically an inverse procedure for the wave propagation. The procedure is similar to starting from the receiver, backward-propagate the wave to the reflector and to the source. For seismic records from a stack of layers, the Q-compensation scheme is implemented in a layer by layer recursive manner. The attenuated signal corresponding to each lithology or seismic layer is treated with the Q responsible for the attenuation and dispersion using an inverse Q filter. By this approach, every seismic event is treated for attenuation and dispersion it experienced along its propagation path. The applications of the Q-compensated algorithm to synthetic seismic data show that the algorithm is appropriate for correcting amplitude diminution and phase dispersion in seismic waves, and thus improves the interpretability of seismic records. The Q-compensated scheme described in this chapter can be incorporated into a migration scheme to achieve a well resolved migrated image

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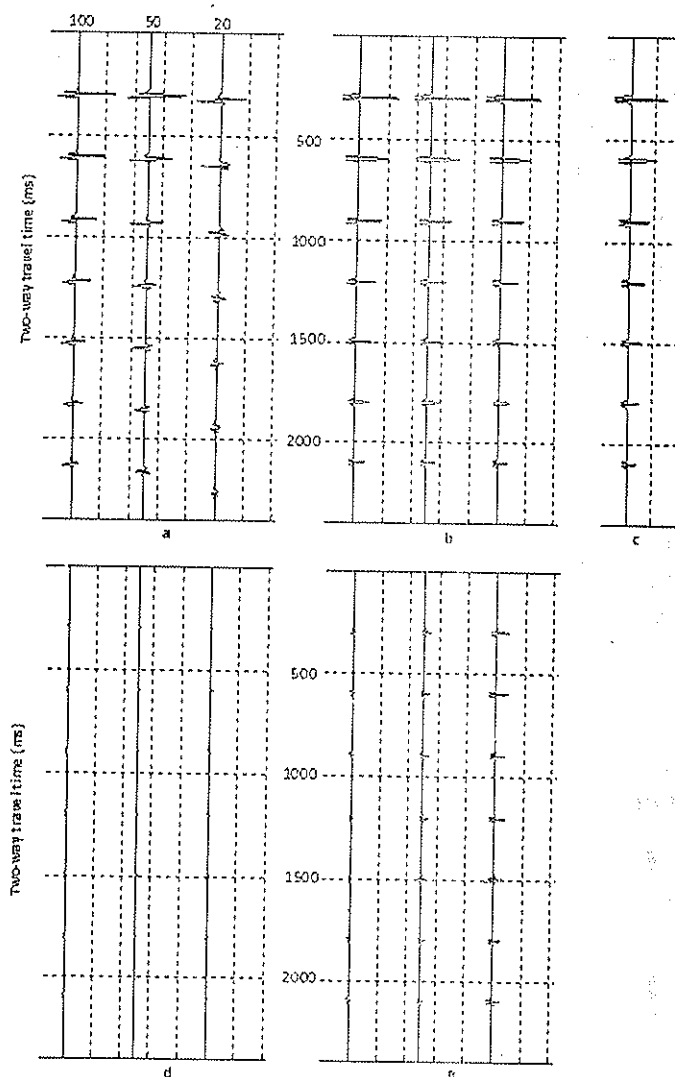


Figure 4. Synthetic seismic traces before and after Q-compensation: (a) constant Q attenuated traces. Q values are 100, 50 and 20 for the first, second and third trace, respectively. (b) The respective trace after Q-compensation. (c) The reference trace -representing seismic record from 'perfectly elastic' layers. (d) The difference plot between the reference trace (c) and the compensated traces (b). (e) The difference plot magnified by 5. The Earth model consists of eight layers such that seismic events are separated in time by 300ms.

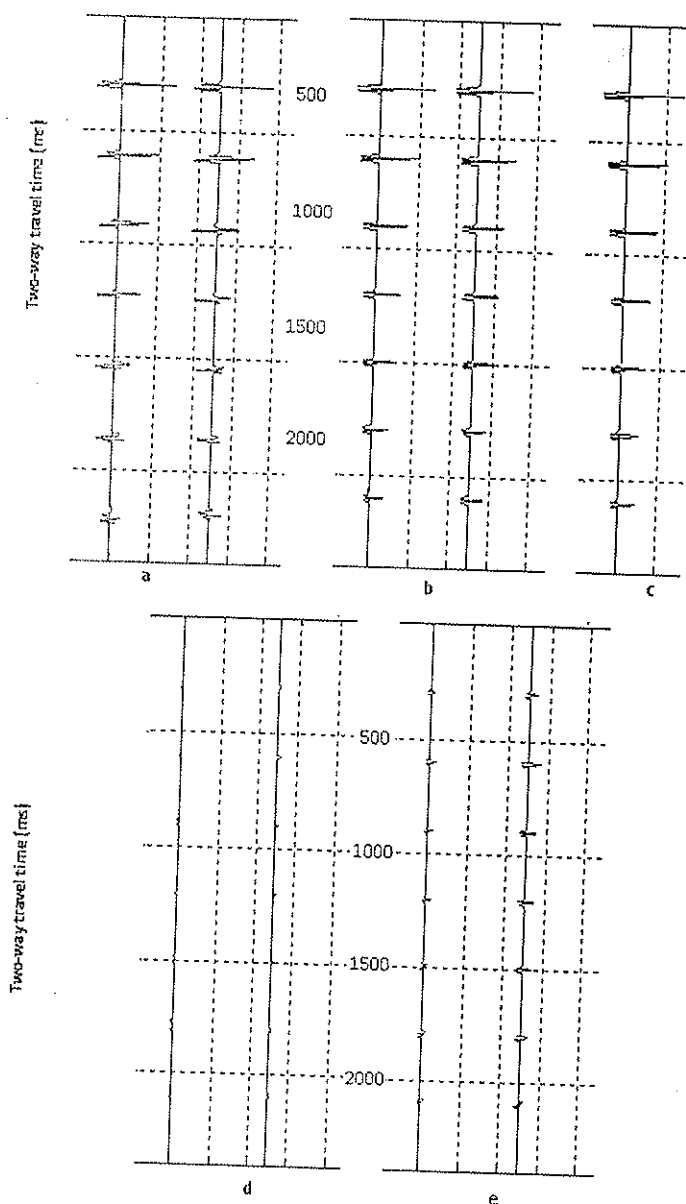


Figure 5. Synthetic traces before and after Q-compensation. (a) Seismic traces from varying Q earth layers. In the first race, Q values decreases progressively as 150, 120, 100, 50, 30, 20, 20, 10. In the second trace, Q value varies as 100, 50, 30, 100, 50, 30, 20, 100 - typical of partially saturated reservoir rocks surrounded by less attenuating rocks. (b) The respective trace after Q-compensation. (c) The reference trace - seismic record from 'perfectly elastic' layers. (d) Difference plot- the difference between the reference (c) and the compensated traces (b). (e) The difference plot magnified by 5. The Earth model consists of eight layers such that seismic events are separated in time by 300ms

CONCLUSION

A new method for compensating the undesirable effects of amplitude diminution, wavelet distortion and velocity dispersion in seismic waves has been presented. The Q-compensation scheme circumvents the issue of numerical instability and prevents the introduction of artifacts that usually mar the results of an inverse Q-filtering process. The method proved appropriate when applied to synthetic but realistic seismic data computed from various Earth Q models. The Q-compensation process improves the quality of seismic images and returns subsurface reflectors to their correct position, by correcting amplitude diminution, waveform distortion and phase delay. Generally, the process improves the interpretability of seismic records and thus will reduce the risks in seismic interpretation.

APPENDIX

Table 1: Layer parameters for the Earth model used to generate the attenuated earth traces.

Parameters	Density kg/m ³	P-velocity km/s	Thickness km
Layer1	0.0025	3.0	0.45
Layer 2	0.0025	4.0	0.60
Layer 3	0.0025	5.0	0.75
Layer 4	0.0025	6.0	0.90
Layer 5	0.0025	7.0	1.05
Layer 6	0.0025	8.0	1.20
Layer 7	0.0025	9.0	1.35
Layer 8	0.0025	10.0	inf.

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