

**MODELING DYNAMIC MICRO AND MACRO PANEL DATA WITH
AUTOCORRELATED ERROR TERMS**

By

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CERTIFICATION

This is to certify that this thesis is an original work carried out by UTHMAN, Kafayat Tolani with Matriculation Number 95/042865 in the Department of Statistics, Faculty of Physical Sciences, University of Ilorin. The thesis has been read and approved as meeting the requirements for the award of Doctor of Philosophy of the University of Ilorin, Ilorin, Nigeria.

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DEDICATION

“Say: Verily my prayer and my service of sacrifice, my life and death are for Allah; the Cherisher of the Universe; No partner has He, and this I have been commanded and I am the first among Muslim” (Surat Al-An ‘am verse 162-163).

In the light of the above verse, I hereby dedicate this work to Allah and my sweetheart, Uthman, Abdulfatai Olanrewaju.

DECLARATION

I, UTHMAN, Kafayat Tolani hereby declare that this thesis entitled “Modeling Dynamic Micro and Macro Panel Data with Autocorrelated Error Terms” is a record of my work. It has neither been presented nor accepted in any previous application for a higher degree. All sources of information have been specifically acknowledged.

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Some Acronyms used in the Thesis

Acronyms	Full meaning
OLS	Ordinary Least Squares
ABGMM1	First-step Arellano-Bond Generalized Method of Moments
GMM	Generalized Method of Moments
IV	Instrument Variable
SYS1	First-step Blundell-Bond System Generalized Method of Moments
P-est.	Proposed modified Estimator
AH (d)	Anderson-Hsiao Instrumental Variable in difference form
AH (l)	Anderson-Hsiao Instrumental Variable in levels form
NLS	National Longitudinal Survey
PSID	Panel Study of Income Dynamics
LM	Lagrangian Multilpie
GLS	Generalized Least Squares
WB	Wansbeek-Bekker (IV) estimator
RMSE	Root Mean Squares Error
MSE	Mean Squares Error
SGMM	System Generalized Method of Moments
MLE	Maximum Likelihood Estimator
RE	Random Effects
FE	Fixed Effects
LSDV	Least Squares Dummy Variable
LSDVC	Corrected Least Squares Dummy Variable
T	Time dimension or Time period
n	Sample size
2SLS	Two Stage Least Squares estimator
3SLS	Three Stage Least Squares estimator
G3SLS	Generalized Three Stage Least Squares estimator
MD	Minimum Distance estimator

FGLS	Feasible Generalized Least Squares
OLSD	OLS on the difference model
LIML	Limited Information Maximum Likelihood
WG	Within Group estimator
LEV	the levels Generalized Method of Moments
AS	the non-linear Generalized Method of Moments
EL	Empirical Likelihood
TML	Transformed Maximum Likelihood
FD-GMM	first difference Generalized Method of Moments
FD	first difference
BTW	Between estimator
TSYS GMM	Transformed System Generalized Method of Moments
LD-GMM	Long difference Generalized Method of Moments
LTS	Least Trimmed Squares estimator
ET	Exponential Tilting
REWLS	Reweighted Least Squares Estimator
DCML	Distance-constrained Maximum Likelihood
MA	Moving Average
AR	Autoregressive
ARMA	Autoregressive Moving Average
PW	Prais-Winsten
BQU	Best Quadratic Unbiased
OPEC	Organization of Petroleum Exporting Countries
PWT	Penn World Table

ABSTRACT

Estimation and Inference in dynamic panel data models is limited by the presence of autocorrelation of the error terms. This causes the traditional panel estimators to be biased and inconsistent. This study was aimed at investigating the sensitivity of some dynamic panel data estimators in the presence of autocorrelation. The objectives of the study were to:

(i) evaluate the performances of some dynamic panel data estimators in the presence of autocorrelation; (ii) evaluate the performances of the estimators with change in sample sizes and time periods; (iii) propose an estimator by modifying the existing estimators; and (iv) compare the performance of new estimator with some of the existing estimators at different sample sizes and levels of autocorrelation.

Simulation study was carried out using Monte-Carlo experiment in the environment of R statistical package to generate data with panel structure for sample sizes 10, 20, 50, 100 and 200 and time periods 5, 10, 15 and 20, autoregressive coefficients (λ and $\delta = 0.3, 0.5$ and 0.7) and autocorrelation coefficients (ρ and $\theta = 0.2, 0.5$ and 0.9). Five dynamic panel data estimators; Ordinary Least Square (OLS), Arellano-Bond Generalized Method of Moments one-step (ABGMM1), Blundell-Bond System Generalized Method of Moments one-step (SYS1), Anderson-Hsiao Instrumental Variable in difference form (AH (d)), Proposed modified estimator (P-est.) were compared. Two robust estimators (M and MM) were also used. Real life data from the Organization of Petroleum Exporting Countries was used to confirm the results from simulation study. Absolute bias and root mean square error were used to evaluate finite properties of the estimators

The following were the major findings from the study:

(i) AH (d) and ABGMM1 estimators performed well in the presence of autocorrelation.

- (ii) AH (d) estimator performed relatively well when the time period is small while ABGMM1 estimator outperformed all other estimators when sample size (n) is large for all the time periods considered;
- (iii) The results showed that all estimators (with the exception Blundell-Bond System GMM and OLS) generally performed better with small and large time periods (T). However, ABGMM1 seemed to show the largest improvement as sample size (n) and time periods (T) increases;
- (iv) The proposed modified estimator was obtained from the existing estimators;
- (v) The proposed modified estimator outperformed all other estimators in small and large samplesize irrespective of time period; and
- (vi) Results obtained from the analysis of the Real life data validated the findings from Monte-Carlo studies.

In conclusion, the proposed modified estimator is more appropriate in estimating the parameter of lagged dependent and exogenous variables when dealing with dynamic panel data models in the presence of autocorrelation. It is recommended to use the proposed modified estimator when dealing with dynamic panel data models in the presence of autocorrelation.

Word count: 451

CHAPTER ONE

GENERAL INTRODUCTION

1.1 Background

Panel data is a cross-section or group of entities that are surveyed periodically over a given time span. Panel data consist of repeated observations on some subjects at different occasions. It could be generated by pooling time-series observations across a variety of cross-sectional units. The units may be individuals, households, firms, regions or countries. Panel data analysis can be classified as Micro or Macro panels. Micro panel involve surveying a number of households or individuals and following them overtime. Macro panels usually involved a number of countries over time. Panel data have become increasingly available and popular in both developed and developing countries. For example, two very famous ones are the National Longitudinal Survey (NLS) of Labour Market Experience and the Michigan Panel Study of Income Dynamics (PSID) (Baltagi, 2000 and Hsiao, 2005).

Panel data sets possess several benefits over conventional cross-sectional and time-series data as described by Hsiao (2003) and Baltagi (2008). More accurate inference of model parameters is obtained when dealing with panel data. Panel data usually contain more degrees of freedom and more sample variability than cross-sectional data (which may be viewed as a panel with $T=1$) or time-series data (which is a panel with $N=1$), hence improving the efficiency of econometric estimates. Panel data involve at least two dimensions, cross-sectional dimension and time series dimension. Under normal circumstances, one would expect that the computation of panel data estimator or inference would be more complicated than cross-sectional or time series data. However, in certain cases, the availability of panel data actually simplifies computation and inference. Also, it is useful for controlling individual heterogeneity. Panel data suggest that individuals, firms, states or countries are heterogeneous. It gives more informative data, variability, degrees of freedom, efficiency and less collinearity among the variables. Panel data are better able to study the dynamics of adjustment. It also gives information on the time-ordering of events. Panel data are better able to identify and measure effects that are simply not detectable in pure cross-section or pure time-series data. Apart from the benefits that panel data allow for constructing and testing more complicated behavioural models than purely cross-section or time-series data, it also provides a means of resolving the econometric problems that arise in empirical studies, especially the common assertion that the real reason one finds (or does not

find) certain effects is the presence of omitted (mis-measure or unobserved) variables that are correlated with explanatory variables.

However, in spite of its notable benefits and advantages, panel data has some limitations such as design and data collection problems and distortions of measurement errors (Kasprzyk et.al, 1989). Design and data collection problems include coverage error (incomplete account of population of interest), nonresponse (due to lack of cooperation of the respondent or because of interviewer error), recall (respondent not remembering correctly), frequency of interview, interview spacing, reference period, the use of bounding and time-in-sample bias. A distortion of measurement errors is another limitation of panel data sets. Measurement errors may arise because of faulty responses due to unclear questions (including cultural differences, language translation etc.), memory errors and deliberate distortion of responses, inappropriate informants, wrong recording of responses and interviewer's effects. Although these problems can occur in cross-section studies, they are aggravated in panel data studies. Some other limitations are selectivity problems (these include self-selectivity, non-response and attrition), short time dimension and cross-section dependence.

The standard error component panel data model assumed that the disturbances have homoscedastic variances and constant serial correlation through the random individual effects (see Hsiao, 2003 and Baltagi, 2005).

The model is given as

$$y_{it} = \delta y_{i,t-1} + x'_{it}\beta + u_{it} \quad i = 1, \dots, N \quad t = 1, \dots, T \quad (1.1)$$

where δ is a scalar, x'_{it} is $1 \times k$ and $\beta = k \times 1$. It assumed that u_{it} follows a one-way error component model

$$u_{it} = \mu_i + v_{it} \quad (1.2)$$

According to Hsiao (2003), it is only by taking proper account of selectivity and heterogeneity biases in the panel data that one can have confidence in the results obtained. Often, however, researchers ignore the existence of these problems when carrying out panel data analysis. This is not uncorrelated with the fact that a standard panel data model assumes the regression disturbances are homoscedastic with the same variance across time and individuals (thereby ignoring the possibility of selectivity and heterogeneity biases).

The standard error components model has been extended to take into account serial correlation by Lillard and Willis (1978), Baltagi and Li (1995), Baltagi (2005), and Garba et.al (2013). Baltagi et.al (2008) considered a panel data regression model with

heteroscedastic as well as serially correlated disturbances in one way error component, and consequently derived a joint Lagrangian Multiplier (LM) test for homoscedasticity and no first order serial correlation, a conditional LM test for homoscedasticity given serial correlation, and a conditional LM test for no serial correlation given heteroskedasticity.

Panel data models can be specified as a static or dynamic panel. The inclusion of a lagged dependent variable on the right hand side of the equation of a panel data model leads to a dynamic panel data model. Among social and behavioural science researchers, panel data is increasingly becoming popular and exhibiting phenomenal growth. Its advantage over cross-section data in this context is obvious: we cannot estimate dynamic models from observations at a single point in time, and it is rare for single cross-section surveys to provide sufficient information about earlier time dimensions for dynamic relationships to be investigated. Its advantage over aggregate time series include the possibility that underlying micro-economic dynamics may be obscured by aggregation biases, and scope that panel data offers to investigate heterogeneity in adjustment dynamics between different types of individuals, households or firms. Dynamic models are of interest in a wide range of economic applications such as Euler equations for household consumption, democracy and education, empirical model of economic growth and so on (Bond, 2002). Even when the coefficients on lagged dependent are not of direct interest, allowing for dynamics in the underlying process may be crucial for recovering consistent estimates of other parameters.

Baltagi (2005) noted that many economic relationships are dynamic in nature and the panel data allow the researcher to have better knowledge about the dynamics of structural of adjustment exhibited by the data. The discussion of dynamic panel data modeling was opened by Balestra and Nerlove (1966) and Sevestre and Trognon (1985). However, the number of existing estimators to estimate dynamic models has increased steadily. As far as finite sample bias and efficiency is concerned, there is still some degree of progressive development.

Estimation of dynamic panel models is unfortunately problematic. They are difficult to estimate due to the presence of the Fixed Effects specification, the problem arises as a consequence of relatively short time period. Thus, the usual Hurwicz type bias is instigated into Ordinary Least Squares (OLS) estimation of a Fixed Effects dynamic panel model (Nickel, 1981). In the random effects specification, traditional (feasible) Generalized Least Squares (GLS) and Least Squares Dummy Variable (LSDV) estimators are similarly biased

due to a correlation between the equations disturbance terms (via the individual effect) and lagged dependent variable (Sevestre and Trognon, 1985).

The most common approach to consistent estimation (of both specifications) is that of Instrumental Variables (IV). This approach leads to the more general area of Generalized Method of Moments (GMM). GMM estimation has generated much interest in attempting to identify the maximum (optimal) number of such conditions (See Ahn Schmidt, 1993, 1995, 1997 and Crepon et al., 1997). Harris and Matyas (2010) provide a survey of the mainstream estimators, which consist of IV and GMM ones. They offered two new extensions to existing IV/GMM estimators (the new GMM estimator for the fixed effects specification and the Wansbeek-Bekker (WB) IV estimator for random effects specification), and their small sample performance compared to that of all of the existing mainstream ones.

The problems of autocorrelation due to the presence of lagged dependent variable among the regressors and individual effects characterizing the heterogeneity among the individuals leads to certain issues which are dealt with by different estimation techniques. The estimation of Fixed Effects dynamic panel data models has been one of the major challenges in Econometrics in the last three decades. A number of techniques for modeling dynamic panel data have been proposed and compared with Instrumental Variable (IV) and Generalized Method of Moments (GMM) estimators (Nickel, 1981; Anderson-Hsiao, 1981, 1982; Kiviet, 1995; Arellano-Bond, 1991; Ahn and Schmidt, 1995; Blundell-Bond, 1998; Judson and Owen, 1999; Islam, 1998; Harris and Matyas, 2010).

1.2 Aim and Objectives of the Study

The main aim of this research work is to compare the performance of some of the existing mainstream Instrumental Variable (IV)/ Generalized Method of Moments (GMM) estimators under small and large sample sizes when serial correlation is introduced into Random Effects One-way Error Component Model. The specific objectives are to:

- (i) evaluate the performances of some dynamic panel model estimators in the presence of autocorrelation;
- (ii) evaluate the performances of the estimators with changes in sample sizes and time periods;
- (iii) propose an estimator by modifying the existing estimators;
- (iv) compare the performance of new estimator with existing estimators at different sample sizes and levels of autocorrelation.

1.3 Motivation for the Study

The usual techniques for estimating panel data models are well known to be inconsistent in the dynamic setting. Apart from the general problems in obtaining the consistent estimates in the context of Dynamic Panel Data Models, a large time period often comes along with specific time issues like the autocorrelation due to the presence of lagged dependent variable among the regressors and individual effects characterizing the heterogeneity among individuals Baltagi (2008). Series of studies have been carried out on the estimation of dynamic panel data by ignoring the possibility of the presence of serial correlation in the disturbance term which may lead to misleading or invalid inferences.

Therefore, this study aimed at examining sensitivity of some Dynamic Panel Data estimators to varying degrees of autocorrelation in the error term. This study evaluates the performance of several estimators when the disturbance term is serially correlated.

1.4 Statement of the Problem

The area of panel data econometrics has become very popular over the last decade. This has become an increasingly obvious focus of many recent theoretical and simulation papers (for example Arellano and Bond, 1991; Arellano and Bover, 1995; Ahn and Schmidt, 1995 & 1997; Kiviet, 1995; Blundell and Bond, 1998; Crepon, Kramarz and Trognon, 1997; Harris and Matyas, 2004; Valentin, 2013; Olajide and Olubusoye, 2016).

The problem associated with estimation of dynamic panel models is connected with the fact that the disturbance terms of the equation and the lagged endogenous variable are correlated, causing panel estimators to be biased and inconsistent in the usual large N and fixed T case.

The most favoured approaches to consistent estimation are Instrumental Variables (IV) and Generalized Method of Moments (GMM).

This research surveyed the majority of the existing mainstream GMM and IV estimators and compares their small sample and large sample performances both in terms of Absolute bias and Root Mean Square Error (RMSE) using Monte Carlo experiments.

1.5 Significance of the Study

The focus of this study is to try to resolve the problem of autocorrelation in one-way error component model. Many existing Monte Carlo design in the Dynamic Panel Data literature do not allow autocorrelation in the error term. Therefore, the majority of the existing mainstream Generalized Method of Moments (GMM) and Instrumental Variables (IV)

estimators and two other estimators that are robust estimators, MM-estimator and M-estimator are surveyed. Various studies had been undertaken using Monte Carlo simulation and small samples, in an attempt to shed more light on the problems. This study examines the performance of different estimators from small samples to large samples.

1.6 Scope of the Study

It is established in literature that estimates of coefficients of lagged dependent variables in a Dynamic Panel Data models are biased due to the correlation between the disturbance term and the lagged dependent variable. Often, researchers ignore the existence of these problems when carrying out Dynamic Panel data analysis. Hence, the behavior of different estimators in the presence of autocorrelation needs to be well investigated.

In this research work, the following estimation methods were used in a Monte-Carlo experiment.

- (i) Ordinary Least Squares (OLS)
- (ii) Anderson-Hsiao using lagged differences as instrument (AH(d))
- (iii) First Step Arellano-Bond Generalized Method of Moment (ABGMM1)
- (iv) Blundell-Bond first step System estimator using level instrument (SSY1)
- (iv) M-estimator
- (vi) MM-estimator
- (vii) Proposed modified estimator (P-estimator).

The absolute bias and Root Mean Square Error (RMSE) were used as criteria to assess the performances of these estimators.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

In this chapter, the study reviewed related literature on Panel data. It also discussed the literature on static panel data model, dynamic panel data model and robust estimators.

2.2 Estimation Methods of Panel Data Models

It is well known that the use of the lagged dependent variable at the right hand side introduces a specific estimation problem, especially when the fixed estimator is becoming biased and inconsistent. The dynamic panel data model is given as

$$y_{it} = \delta y_{i,t-1} + x'_{it}\beta + u_{it} \quad i = 1, \dots, N \quad t = 1, \dots, T \quad (2.1)$$

where δ is a scalar, x'_{it} is $1 \times k$ and $\beta = k \times 1$. It assumed that u_{it} follows a one-way error component model

$$u_{it} = \mu_i + v_{it} \quad (2.2)$$

Ordinary Least Square (OLS) estimators are applied to the equation in level form. The OLS estimator of (δ, β') is inconsistent (asymptotically biased) even with time period (T) is large. OLS also gives an estimate of the composite error term but cannot provide its further decomposition parts, μ_i and v_{it} . Therefore OLS is biased and inconsistent.

M-estimator was proposed by Huber (1973). M-estimation for regression is an extension of M-estimation for location and scale. It was one of the first attempts at a compromise between the efficiency of the Least Squares estimator and a resistance estimator which is Least Absolute Value (LAV) estimators. M estimators are more robust to heavy tailed error distributions and non-constant error variance.

Anderson and Hsiao (1981) suggested an instrumental Variable (IV) estimator that is consistent for fixed time period(T) and sample size(N) as their values tends to infinity. Anderson and Hsiao (IV) estimate apply to the model in first differenced form.

$$y_{it} = \delta y_{i,t-1} + x'_{it}\beta + \mu_i + v_{it} \quad (2.3)$$

$$y_{it} - y_{i,t-1} = \delta(y_{i,t-1} - y_{i,t-2}) + (x'_{it} - x'_{i,t-1})\beta + v_{it} - v_{i,t-1} \quad (2.4)$$

It results in the “loss” of one cross-section from the actual estimation. They also suggested using level Instruments y_{t-2} or the lagged difference $y_{i,t-2} - y_{i,t-3}$ as an instrument for the

differenced lagged endogenous regressor $y_{i,t-1} - y_{i,t-2}$. The Anderson-Hsiao estimator is consistent but not efficient because it does not make use of all the available moment conditions.

MM estimation was introduced by Yohai (1987). It has a high breakdown point with a good efficiency (approximately 95%) relative to the Ordinary Least Squares estimator under the Gauss-Markov assumptions. MM estimator also combined the high asymptotic relative efficiency of M estimators with the breakdown of a class of estimators called S estimators. The MM estimation refers to the Multiple M-estimation procedure which was used to calculate the final estimates. MM estimator has also become most common robust regression method for linear regression.

Arellano and Bond (1991) suggested a Generalized Method of Moments (GMM) estimator that relies on first differencing the model and they also obtained additional instruments from the orthogonality conditions between the lagged values of y_{it} and the disturbances $v_i - v_{it-1}$. A very useful advantage of the Arellano and Bond estimator is that the independent variable in x_{it} need not be strictly exogenous, predetermined and endogenous covariates can be easily adapted. Arellano and Bond estimator are known to be inefficient when instruments are weak because of the information contained in differenced model only. Arellano and Bond estimator is not free of problems. In dynamic panel data models where the autoregressive parameter is relatively large and the number of the time periods is small, the Generalized Method of Moments (GMM) estimator obtained after first differencing has been found to have large finite sample bias and poor precision in simulation studies.

Arellano and Bover (1995) and Blundell and Bond (1998) developed another estimator called System Generalized Method of Moments estimator (SGMM) (or Blundell and Bond estimator). Their idea consists of using lagged differences of independent variables as instruments for equations in levels, and Arellano and Bover as lagged levels of independent variables as instruments for equations in first differences. Since Instrument Variables (IV) estimator is not generally efficient, Arellano and Bond (1991) suggested using a Generalized Method of Moments (GMM) estimator to improve the efficiency. The GMM estimator has subsequently been refined in a number of studies (See Arellano and Bover, 1995; Ahn and Schmidt, 1995; and Blundell and Bond, 1998). However, the GMM estimator is generally more efficient than the Instrumental Variable (IV) estimator in finite samples.

P-estimators were built on both Arellano-Bond GMM estimator and Anderson-Hsiao estimator in order to achieve a high consistent and efficient estimator.

2.3 Estimators on Dynamic Panel Data

The discussion of Dynamic Panel Data Models was opened by Balestra and Nerlove (1966). The authors suggested estimation of the model given in (2.5) with unobserved component using the Generalized Least Squares (GLS) estimators.

$$y = Z\alpha + Y_{\theta}\beta + u \quad (2.5)$$

The authors observed that Generalized Least Squares (GLS) or Maximum Likelihood (ML) and Random Effect (RE) estimator are not consistent if the observed individual effects are correlated with exogenous variable. That is, the Fixed Effects (FE) specification is preferred over other estimators.

Nerlove (1967) considered a simple auto-regressive model with no exogenous variable as given in (2.6)

$$y_{it} = \alpha y_{it-1} + u_t \quad (2.6)$$

Where the u_{it} are unobserved random variables such that $u_{it} = \mu_i + v_{it}$. He assumed that

$$E(\mu_i) = E(v_{it}) = 0, \text{ all } i \text{ and } t$$

$$E(\mu_i v_{it}) = 0, \text{ all } i, i' \text{ and } t,$$

When there is no exogenous variable in model, it does not only make it impossible to ascertain the properties of Instrumental Variables estimates, but may significantly affect the properties of those estimates. The author compared the performance of Ordinary Least Square (OLS), Least Square Dummy Variable (LSDV), Maximum Likelihood Estimator (MLE) and several variants of Generalized Least Squares (GLS) in estimating the model. In Nerlove (1971), the dynamic panel data model was extended to have exogenous variable. That is,

$$y = \alpha y_{it-1} + u_t \quad (2.7)$$

This allows consideration of instrumental variable (IV) estimator with lagged values of the explanatory variable as instrument and having another variant of two-stage Generalized Least Squares (GLS). Overall, Nerlove's simulation results favoured Generalized Least Squares (GLS) estimators over other estimators.

However, since Nerlove's work, there have been significant developments in the field of dynamic panel data estimators. Among these has been introduction of Anderson and Hsiao (IV) estimators which use further lagged values of the dependent variable as instruments. Arellano has also carried this idea forward and proposed using all possible further lagged variables as instruments within a GMM framework.

Arellano and Bond (1991) carried out a Monte-Carlo experiment based on dynamic panel data model given as

$$y_{it} = \delta y_{i,t-1} + x'_{it}\beta + \mu_i + v_{it} \quad (2.8)$$

They assumed that $E(u_{i2}x_{i1})=0$ and $E(u_{it}x_{iit})=0$ for $t = 2, \dots, T$. They made a deduction on the Anderson-Hsiao estimator against different Generalized Method of Moments (GMM) estimators and inferred that the Generalized Method of Moments (GMM) procedures produce substantial efficiency gains. Their results also showed that GMM1 performed better than GMM2 in both their bias and root mean square error.

Li and Stengos (1994) conducted a Monte Carlo experiments based on the sample regression model given as:

$$y_{it} = \alpha + \beta X_{it} + \mu_i + v_{it} \quad (2.9)$$

with $\mu_i \sim \text{IID}(0, \sigma_v^2)$ and assumed that $E[v_{it} | x'_{it}] = 0$ with $\text{var}[v_{it} | x'_{it}] = \gamma(x'_{it}) \equiv \gamma_{it}$.

They compared the OLS, Fixed effects or within estimator (WITHIN), the conventional GLS estimator for the one-way component model that assumed the error term u_{it} are homoskedastic (GLSH), and their own adaptive heteroskedastic estimator denoted by GLSAD. They found that their adaptive estimator outperformed all the other estimators in terms of Mean Square Error with respect to true GLS for $N= 50, 100$ and $T=3$ and for moderate and severe degree of heteroscedasticity. The GLSH performed better than WITHIN and Ordinary Least Square (OLS). The OLS estimator performed worse in the homoscedastic case since it ignores both the individual effects μ_i and the heteroscedasticity in v_{it} . Also as N is doubled, the performance of GLSAD, GLSH and WITHIN all improved slightly. However there was no systematic improvement for the OLS estimators. They reported a 5% size performance of the estimated t-ratios of the slope coefficient and found that only GLSAD had the correct size while OLS, GLSH and WITHIN over-rejected substantially for the chosen sample sizes and parameter values.

Kiviet (1995) conducted a Monte-Carlo experiment on the simple regression model given as

$$y_{it} = \alpha + x'_{it}\beta + \mu_i + v_{it} \quad (2.10)$$

With the assumption that $E(\mu_i) = 0$, $E(v_{it}) = 0$, $E(v_{it}\mu_i) = 0$ for $i = 1, \dots, N$ and $t = 2, \dots, T$.

$E(v_{it}v_{is}) = 0$ for $i = 1, \dots, N$ and $\forall t \neq s$. He compared the Anderson-Hsiao estimator against various Generalized Method of Moments (GMM) estimators. He found that the Anderson-Hsiao estimator compares favourably to GMM and concludes that no estimator has been found to be the appropriate choice in all circumstances. He also derived an approximation for the bias of the within estimator in a dynamic panel data model with serially uncorrelated disturbances and exogenous regressors. He also proposed a bias corrected fixed effect estimator that subtracts a consistent estimator of this bias from the original fixed effect estimator. He also found out that GMM1 is more efficient than GMM2 in terms of finite samples.

Wansbeek and Bekker (1996) used a simple dynamic panel data model with no exogenous regressors and disturbances u_{it} and random effects μ_i that are independent and normally distributed. They derived an expression for the optimal instrumental variable estimator, that is, one with minimal asymptotic variance. Their result showed the difference in the efficiency between the Instrumental Variable and Maximum Likelihood estimators. They found that for the regions of the autoregressive parameter δ which are likely in practice, Maximum Likelihood estimator (MLE) is superior to Instrumental Variable estimator. The gap between IV (and Generalized Method of Moments) and ML can be narrowed down by adding moment restrictions and warn against the loss in efficiency as compared with MLE by ignoring them. Judson and Owen (1996) conducted a Monte Carlo experiments based on dynamic fixed effect model given as:

$$y_{i,t} = \mathcal{Y}_{i,t-1} + x'_{i,t}\beta + \eta_i + \varepsilon_{i,t} \quad (2.11)$$

They assumed that

$$\begin{aligned} \sigma_\varepsilon^2 &\geq 0, \\ E(\varepsilon_{i,t}, \varepsilon_{j,s}) &= 0 \text{ } i \neq j \text{ or } t \neq s \\ E(\eta_i, \varepsilon_{j,t}) &= 0 \text{ } \forall i, j, t \\ E(x_{i,j}, \varepsilon_{j,s}) &= 0 \text{ } \forall i, j, t, s \end{aligned} \quad (2.12)$$

They investigated the performance of Ordinary Least Squares (OLS), Least Square Dummy Variable (LSDV), Anderson-Hsiao (AH), Corrected Least Squares Dummy Variable (LSDVC) and they focus on two restricted GMM estimators. Their results showed that the

bias of the least squares dummy variable (LSDV) method can be significant even when the time dimension (T) is as large as 30. But for panels with small time periods, they found that a corrected least squares dummy variable (LSDVC) was the best choice among all the estimators. They also found that when time periods (T) is large or increases, Anderson-Hsiao preformed equally well. They also showed that all the estimators generally performed better with a larger N except the Ordinary Least Squares (OLS) and One-Step GMM procedure which outperformed the Two-Step GMM estimator both in terms of bias and standard deviation.

Ahn and Schmidt (1997) considered a simple linearized Generalized Method of Moments estimator that is asymptotically as effective as the nonlinear Generalized Method of Moments estimator. The additional moment conditions pointed out by Ahn and Schmidt are

$$E[u_{iT}(v_{it} - v_{i,t-1})] = 0, \quad t = 2, \dots, T-1.$$

They also provided simple moment tests for the validity of these nonlinear GMM restrictions. They investigated the circumstances under which the optimal GMM estimator is equivalent to linear instrumental variable estimators. Their results showed that these situations were quite restrictive and go beyond uncorrelatedness and homoscedasticity of the errors. Their result showed that the linearized GMM estimator is asymptotically more efficient than nonlinear GMM estimator.

Hahn (1997) suggested the asymptotic efficient estimation of the dynamic panel data model with sequential moment restrictions in an environment with independently and identically normal disturbance observations. He showed that Generalized Method of Moments estimator with an increasing set of instruments as the sample size grows attains the semi parametric efficiency bound of the model. He also explained how Fourier series or polynomials may be useful as the set of instruments for efficient estimation in order to have a better result. His Monte Carlo result showed that this estimator had similar finite sample properties as the Keane and Runkle (1992) and/ or Schmidt et al. (1992) estimators when the latter estimators were efficient. In cases where the latter estimator was not efficient, the Hahn efficient estimator outperformed both estimators in finite samples.

Islam (1998) investigated the small sample properties of dynamic panel data estimators as applied for estimation of the growth convergence equations using Summers-Heston data set. The individual effect term is assumed to be a linear function of the mean (over time) of the exogenous variable for the individual concerned. He compared Ordinary Least Squares, Least Squares Dummy Variable, Anderson-Hsiao, and Generalized Method of Moments, 2SLS,

3SLS, G3SLS and Minimum Distance estimators. His Monte-Carlo result showed that estimates from different estimators do change with time. He also showed that those dynamic panel estimators which do not use further lagged dependent variable as instruments performed better than AH and GMM that uses further lagged variable as instruments. He also showed that AH (d) performed relatively well than AH (L) under all different generation of V_{it} and for all sample sizes, while the performance of AH(L) is found to be unsatisfactory under all different generation of V_{it} . 3SLS and G3SLS estimators performed better than 2SLS. Their result together suggested that investigators or researchers using panel estimators need to check the possible small sample bias of their estimators.

Harris and Matyas (1998) performed a Monte-Carlo experiment on the two new estimators with two exogenous variables. They compared Ordinary Least Squares (OLS), OLS on the differenced model (OLSD), WITHIN, Feasible Generalized Least Squares (FGLS), Wansbeek and Bekker (WB^+) and Arellano-Bond estimators. They found that in small T samples, GMM estimators dominate when there is no misspecification. They also found that WB estimators continue to performed well at all levels of serial correlation and GMM estimators also performed at low levels of serial correlation in the disturbance, followed by WITHIN and OLS estimators. They also showed that as T increased and the assuming data process is true, the performance of GMM and WB estimators are significantly reversed. Now both variants of the latter outperformed those of the former, with the best performance of all being achieved by WB^+ . WB^+ dominates all other estimators.

Blundell and Bond (1998) conducted a Monte-Carlo experiment to consider the two types of restrictions with model below:

$$y_{it} = \alpha y_{i,t-1} + \eta_i + v_{it} \quad (2.13)$$

They assumed that $E(\eta_i) = 0$, $E(v_{it}) = 0$, $E(v_{it}\eta_i) = 0$ for $i = 1, \dots, N$ and $t = 2, \dots, T$

$$E(v_{it}v_{is}) = 0 \text{ for } i = 1, \dots, N \text{ and } \forall t \neq s. E(y_{it}v_{it}) = 0 \text{ for } i = 1, \dots, N \text{ and } t = 2, \dots, T$$

They compared Conditional Generalized Least Squares with three GMM estimators: (i) The standard first-difference estimator based on moment condition only GMM (DIF), (ii) The System GMM based on moment conditions GMM (SYS) and (iii) The system GMM estimator which also exploits the further T-2 moment restriction GMM (ALL). They showed that there is a serious problem of weak instruments for the GMM (DIF) at values of serial correlation (α) around 0.8. They also showed that the finite sample performance of conditional Generalized Least Square is generally found to be efficient than that of the system

GMM estimator. Then, the first differenced GMM (GMM DIF) improved dramatically when the autoregressive parameter is moderately large and the number of time periods is moderately small. They also suggested that the system GMM estimator can be more efficient than Non-linear GMM.

Alonso-Borrego and Arellano (1999) carried out a Monte-Carlo experiment based on dynamic panel data model given as

$$y_{it} = \alpha y_{i,t-1} + \eta_i + v_{it} \quad (2.14)$$

They assumed that $E(\eta_i) = 0$, $E(v_{it}) = 0$, $E(v_{it}\eta_i) = 0$ for $i = 1, \dots, N$ and $t = 2, \dots, T$

$$E(v_{it}v_{is}) = 0 \text{ for } i = 1, \dots, N \text{ and } \forall t \neq s. E(y_{it}v_{it}) = 0 \text{ for } i = 1, \dots, N \text{ and } t = 2, \dots, T$$

Their paper is motivated by the finite sample bias in panel data instrumental variable estimators when the instruments are weak. The dynamic panel model produces many over identifying restrictions even for the moderate values of time periods (T). Also, the number of instruments increases with T, but the quality of these instruments is poor because they tend to be only weakly correlated with the first-differenced endogenous variables that appear in the equation. Limited Information maximum likelihood (LIML) performed the best compare to 2SLS if the number of instruments gets large as the sample size (N) tends to infinity. They also derived a symmetrically normalized GMM (SNM) and compared it with ordinary GMM and LIML. Their simulation and empirical result showed that GMM can exhibit large biases when the instruments are poor, whereas LIML and SNM remain essentially unbiased. Then, LIML and SNM always had a large interquartile range than GMM. For $T=4$, $N=100$, $\sigma^2\mu=0.2$ and $\sigma^2v=1$, the bias for $\delta=0.5$ was 6.9% for GMM, 1.7% for SNM and 1.7% for LIML. The bias increases to 17.8% for GMM, 3.7% for SNM and 4.1% for LIML for $\delta=0.8$.

Wansbeek and Knaap (1999) performed a Monte-Carlo study on a simple dynamic panel data model with heterogeneous coefficients on the lagged dependent variable and time trend, that is

$$y_{it} = \delta y_{i,t-1} + \xi_{it} + \mu_i + u_{it} \quad (2.15)$$

They assumed that u_{it} are stationary. They showed that double differencing gets rid of the individual country effects (μ_i) on the first round of differencing and heterogeneous coefficient on the time trend (ξ_i) on the second round of differencing. They compared Ordinary Least Square (OLS), Instrumental Variable (IV) and Generalized Method of Moments (GMM) methods and Limited Information Maximum Likelihood estimator. Their result showed that LIML is the superior estimator for $T \geq 10$ and $N \geq 50$ and OLS estimator

was the only estimator that delivered a distribution reminiscent of the normal distribution in their small samples. Macroeconomic data are subject to measurement error and they also showed how these estimators can be modified to account for measurement error that is white noise.

Judson and Owen (1999) performed a Monte Carlo experiments based on dynamic fixed effect model given as:

$$y_{i,t} = \gamma y_{i,t-1} + x_i' t \beta + \eta_i + \varepsilon_{i,t} : |\gamma| < 1 \quad (2.16)$$

They assumed that

$$\begin{aligned} \sigma_\varepsilon^2 &> 0 \\ E(\varepsilon_{i,t} \varepsilon_{j,s}) &= 0 \text{ if } i \neq j \text{ or } t \neq s \\ E(x_{i,t} \varepsilon_{j,s}) &= 0 \quad \forall i, j, t, s. \end{aligned} \quad (2.17)$$

They considered four estimators: an instrumental variables estimator proposed by Anderson and Hsiao (1981), two Generalized Method of Moments estimator proposed by Arellano and Bond (1991) and a corrected Least Square Dummy Variables estimator (LSDVC) derived by Kiviet (1995). Their result confirmed some research work conclusions about OLS and LSDV estimators: (1) in both cases, the bias of γ are more severe than that of β . (2) OLS showed biased estimates even for large T and (3) the bias of the LSDV estimator increases with γ and decrease with T. Their result also showed that the bias of LSDV estimate is not unsubstantial when T =20, but when T increase to 30, the average bias become smaller although the LSDV does not become efficient. However, all the estimators' performed better with a larger N and T, and the one-step GMM also performed better than the two-step GMM estimator. Their result also showed that LSDV performed just as well as the viable alternatives when T=30, GMM is the best when $T \leq 10$ and GMM or AH may be chosen when T=20.

Blundell, Bond and Windmeijer (2000) considered an autoregressive panel data model of the form:

$$y_{it} = \alpha y_{i,t-1} + \beta' x_{it} + \eta_i + v_{it} \quad (2.18)$$

They assumed that η_i and v_{it} have the familiar error components structure in which $E(\eta_i) = 0$, $E(v_{it}) = 0$, $E(v_{it} \eta_i) = 0$ for $i = 1, \dots, N$ and $t = 2, \dots, T$

$$E(v_{it} v_{is}) = 0 \text{ for } i = 1, \dots, N \text{ and } \forall t \neq s. E(y_{it} v_{it}) = 0 \text{ for } i = 1, \dots, N \text{ and } t = 2, \dots, T$$

They compared the Ordinary Least Squares (OLS), Within Groups estimator (WG), the GMM first-differenced estimator (DIF), the non-linear GMM estimator (AS), the levels GMM (LEV), and the system GMM estimator (SYS). They observed that the OLS estimates are biased upward and WG estimates biased downwards. They also reported that when x_{it} is not highly persistent and the instruments available for the equations in the first-differences are not weak in this experiment, all four GMM estimators are virtually unbiased. The AS, LEV and SYS estimators all provide an improvement in precision compared to the standard DIF estimator. When $\alpha=0.5$ and $\rho=0.5$, The OLS estimates for α are biased upwards, while for β are biased downward. The WG estimates for α and β are both biased downwards. Then the four GMM estimators performed quite well when y and x series has a low degree of persistence. The system estimator has the smallest RMSE for both parameters, but the gain are dramatic at $T=8$. The estimates for ρ showed that OLS is upward biased, WG is downward biased, and DIF is downward biased towards WG as a result of weak instruments. The AS estimator provided an improvement in both bias and precision. However, the LEV and SYS estimators provide more dramatic gains when $T=4$. They found that both a large bias and very low precision for the standard first-differenced estimator when the individual series are highly persistent. The system GMM estimator can improve the precision and reduced the finite bias when these additional moment conditions are valid, by exploiting instruments available for the equations in levels.

Alvarez and Arellano (2003) considered the asymptotic properties of Fixed Effect(FE), one-step Generalized Method of Moments (GMM) and non-robust Limited Information Maximum Likelihood (LIML) estimators for a first order autoregressive model when N and T tend to infinity with $(N/T) \rightarrow c$ for $0 \leq c < 2$. They assumed that v_{it} for $t=1, \dots, T$ and $i=1, \dots, N$ are independent and identically distributed across time and individuals and independent of μ_i and y_{i0} , with $E(v_{it})=0$, $\text{var}(v_{it})=\sigma^2$ and finite fourth moments.

Additionally they assumed that the initial observations are generated as $y_{i0} = \frac{\mu_i}{1-\phi} + \sum_{j=0}^{\infty} \phi^j v_{i,-j}$

For this autoregressive model, the FE estimator is inconsistent for T fixed and N large, but becomes consistent as T increases. GMM is consistent for fixed T but the number of orthogonality conditions increases with T . They showed that GMM bias is extremely smaller than FE bias and LIML bias is smaller than FE and GMM when T is less than 5. For a fixed T , GMM and LIML are asymptotically equal, but as T increases, LIML has a smaller bias

than GMM. Their results also provided some theoretical support for LIML over GMM. They also derived the asymptotic properties of the FE, GMM and LIML estimators of a dynamic panel data model with random effects. When both T and N tend to infinity, GMM and LIML are consistent and asymptotically equal to the FE estimator. When $(T/N \rightarrow 0)$, the results for GMM and LIML remain valid, while FE is consistent, and still showed an asymptotic bias term in its asymptotic distribution. All the three estimators are consistent, when $T/N \rightarrow c$, where $0 \leq c < 2$.

Behr (2003) conducted a Monte-Carlo study on dynamic panel model given as:

$$y_{it} = \rho y_{i,t-1} + x'_{i,t-1} \alpha_i + \varepsilon_{it} \quad (2.19)$$

With the assumption that $E(x_{it} \varepsilon_{is}) = 0$ for $s > t$ but $E(x_{it} \varepsilon_{is}) \neq 0$ for $s < t$. He investigated the performance of Pooled estimator, Least Square Dummy Variable, Anderson-Hsiao, Bias corrected estimator proposed by Hansen (Bc, K_1), Kiviet (Bc, K_2), Arellano and Bond (GMM1, GMM2) and Blundell and Bond (SYS1 and 2). His simulation result showed that in small samples, the bias corrected method proposed by Hansen seem, superior to the other estimators. Also, the GMM and System estimators are less efficient than the other estimators in the small samples, while the different narrows when turning to large samples. However, the system estimator proposed by Blundell and Bond is more efficient than the GMM estimators proposed by Arellano and Bond in the large samples. His empirical result also showed that the use of the system estimators as well as bias corrected estimators seems appropriate among all the estimators.

Bruno (2004) conducted a Monte-Carlo experiment based on the standard autoregressive panel data model given as:

$$y_{it} = \gamma y_{i,t-1} + x'_{it} \beta + \eta_i + z_{it} \quad (2.20)$$

$$E(\eta_i) = 0, E(z_{it}) = 0, E(z_{it} \eta_i) = 0 \text{ for } i = 1, \dots, N \text{ and } t = 2, \dots, T$$

$$E(z_{it} z_{is}) = 0 \text{ for } i = 1, \dots, N \text{ and } \forall t \neq s. E(y_{it} z_{it}) = 0 \text{ for } i = 1, \dots, N \text{ and } t = 2, \dots, T$$

He compared the performance of bias corrected least square dummy variable (LSDVC) against Anderson-Hsiao, Arellano-Bond and Least Square Dummy Variable estimators. His result showed that LSDVC estimator outperformed Anderson-Hsiao and Arellano-Bond in small samples in terms of both bias and root mean square error.

Hujer, Rodrigues and Zeiss (2005) performed a Monte-Carlo study to assess the finite sample performance of the Generalized Method of Moments (GMM) and the transformed maximum likelihood estimator in the presence of serial correlation and weakly exogenous regressors.

They assumed that the explanatory variable is to be weakly exogenous forcing the variables with $E(\omega_{it}x_{is}) \neq 0$ for $s \geq t$ and zero otherwise. Their result showed that the presence of serial correlation leads to severe finite sample bias. They found that the maximum likelihood estimator to be superior relatively to the GMM estimator under serial correlation in terms of Root Mean Square Error. They also found that the advantage of the maximum likelihood estimator over the GMM increases with a higher autoregressive coefficient. They also found that there is a smaller difference between maximum likelihood estimators and Generalized Method of Moments with respect to the coefficient of the explanatory variables. Their empirical result showed that the consideration of serial correlation changes the estimate of autoregressive parameter, where the changes are more distinctive for the Maximum Likelihood Estimators and GMM.

Bun and Kiviet (2006) considered a dynamic panel data models with individual effects and an additional regressor. The additional regressor may be correlated with the individual effects and is predetermined. They compared the finite sample behaviour of the Fixed Estimator (FE), Generalized Least Squares (GLS) and a range of Generalized Method of Moments (GMM) estimators. They indicated how the order of magnitude of bias of these estimators depends on N and T . For instance, they showed that the Fixed Estimator (FE) and Generalized Linear Squares (GLS) are biased of the order $O(1/T)$ irrespective of the Value of N , while the Generalized Method of Moments (GMM) estimators are the order $O(1/N)$ for T fixed. They also revealed how the bias of the Generalized Methods of Moment (GMM) estimators tends to increase with the number of moment conditions exploited. They study Generalized Methods of Moments (GMM) based on both the levels of equation and forward orthogonalization procedure. They also provided evidence on how the bias of the various estimators depends on the feedbacks and some model characteristics such as individual effects and correlation between observed and unobserved heterogeneity. The simulation result showed that none of the estimators examined dominates in terms of bias and mean squared error over all parameterizations examined. When sample sizes (N) and time periods (T) are moderate, all estimators showed substantial bias and poor root mean square estimator (RMSE) performance leading the authors to conclude that"Standard first-order asymptotic theory is of little use indeed to establish and rank the qualities of the estimators."

Bun and Caress (2006) performed a Monte Carlo experiment based on dynamic panel data model given as

$$y_{it} = \gamma y_{i,t-1} + \beta'x_{it} + \eta_i + \varepsilon_{it} \quad (2.21)$$

They assumed that conditional on the observables y_{i0}, x_i' and unobservable η_i , the disturbance term ε_i is independently distributed across individuals with mean zero and variance-covariance matrix Σ_i . They investigated the performance of additive and nonlinear bias-corrected estimators with Least Squares Dummy Variables (LSDV) and the Generalized Method of Moments estimator by Arellano and Bond (1991). They showed that the inconsistency of the Least Squares Dummy Variable (LSDV) estimator for finite T and large N were derived under both time series and cross-section heteroscedasticity and expressions are used in extending existing additive and nonlinear bias correction procedures. Their result showed that the proposed bias-corrected estimators behaved better in finite samples and also showed that GMM performed well especially when T is small and N is large. Their result also showed that based on a root mean squared error criterion bias-corrected Least Square Dummy Variable (LSDV) estimators performed well against Generalized Method of Moments (GMM) estimators.

Oguzoghu (2006) conducted a Monte Carlo experiment based on dynamic panel data model given as

$$y_{it} = \alpha_i + \gamma y_{i,t-1} + x_{it}'\beta + \eta_i + v_{it} \quad (2.22)$$

He assumed that $E(\eta_i) = 0, E(v_{it}) = 0, E(v_{it}\eta_i) = 0$ for $i = 1, \dots, N$ and $t = 2, \dots, T$

$$E(v_{it}v_{is}) = 0 \text{ for } i = 1, \dots, N \text{ and } \forall t \neq s. E(y_{it}v_{it}) = 0 \text{ for } i = 1, \dots, N \text{ and } t = 2, \dots, T$$

He compared the performance of Transformed Maximum Likelihood (TML), Minimum Distance (MD), Bias Corrected Least Square Dummy Variable (LSDVC), Generalized Method of Moments (GMM) and Empirical Likelihood (EL). His result showed that EL outperformed all other estimator in terms of bias, LSDVC also performed well when T is greater 5 and MLE estimator also improved its performance when T increases. His simulation result showed that EL performed quietly better without showing any significant fluctuations over N while LSDVC is the second best. In term of Root Mean Square Errors, LSDVC and ML performed better than other estimators, except when T=5, while the EL showed a poor results. His result also showed that EL suffers from its high variance, whereas performance of GMM is dominated by its high bias in term of Root Mean Square Error. Then LSDVC performed the best, due to its low dispersions for all sample sizes.

Hayakawa (2008) considered the following panel Auto regression (p) model

$$y_{it} = \alpha_1 y_{i,t-1} + \alpha_2 y_{i,t-1} + \dots + \alpha_p y_{i,t-1} + \eta_i + v_{it} \quad (2.23)$$

He assumed that $E(v_{it}) = 0$, $\text{var}(v_{it}) = \sigma_v^2$, $E(\eta_i) = 0$ and $\text{var}(\eta_i) = \sigma_\eta^2$. Hayakawa showed that the infeasible optimal IV estimator is slightly less efficient than the feasible optimal IV estimators and the proposed IV estimator are less efficient than infeasible optimal IV estimators; the difference becomes negligible as T increases. He also found that GMM has the smallest median absolute error in many cases, the difference in median absolute error between GMM and IV is fairly small. The simulation result suggested that, in terms of bias and median absolute error, the IV and GMM using the proposed instruments performed better than the commonly used GMM estimator even when T is as large as 10.

Harris and Matyas (2010) performed a Monte Carlo experiment based on the fixed effects dynamic panel model given as

$$y_{it} = \alpha_i + \delta y_{it-1} + x'_{it}\beta + u_{it} \quad (2.24)$$

They assumed that the variable y_{it} is a linear function of the individual previous realization of the variable, and of the contemporaneous personal characteristics of x_{it} with unknown coefficients. Firstly, they compared the majority of the existing IV/GMM estimators with a small sample performance. Secondly, two new extensions to existing estimators (the new GMM estimator for the fixed and WB (IV) estimator for random effects specifications). Their simulation result suggested that in the fixed effects setting, asymptotically inconsistent estimators (such as the within) often performed better than consistent ones. For the consistent ones, those proposed by Arellano (1988) and Arellano and Bond (1991) generally performed the best. They result also showed that Instrumental Variable (IV) and Generalized Method of Moments (GMM) estimators exploiting more moment conditions generally outperformed those based on fewer conditions for the random effects model.

Santos and Barrios (2011) conducted a Monte Carlo experiment based on dynamic model as:

$$y_{it} = \alpha y_{i,t-1} + x'_{it}\beta + \mu_{it} \quad (2.25)$$

They assumed that $(1/N) \sum_{i=1}^N \eta_i^2 = 0(1)$ and $(1/T) \sum_{i=1}^T \lambda_i^2 = 0(1)$. They investigated both the small and large sample properties of the Within Group (WG) and the first difference Generalized Method of moments (FD-GMM). Their result showed that both the bias of WG estimator and FD-GMM estimator decreases as T increases in most cases. Their result also showed that WG estimator performed best than FD-GMM estimator for long panels such as those with time period as large as 50. The benefit of the FD-GMM estimator however, is obtained in panels that are long and wide with time period at least 25 and cross-section dimension of at least 30.

They also showed that WG and FD-GMM are downward biased, the bias increases with α . However, the bias as a percentage of the true of α decreases as α increases. Their result also showed that the WG estimator performed best when the sample size as low as 10 and when time period is as large as 50. Also, the WG estimator performed better than the FD-GMM estimators based on the interquartile range.

Hayakawa and Pesaran (2012) conducted a Monte Carlo simulation based on dynamic panel data model given as

$$y_{it} = \alpha_i + \gamma y_{i,t-1} + \beta x_{it} + u_{it} \quad (2.26)$$

They assumed that $E(u_{it}) = 0$, $E(u_{it}^2) = \sigma_i^2$ and $E(u_{it}^4 / \sigma_i^4) = k$ such that $0 < \sigma_i^2 < k < \infty$, and $0 < k < K < \infty$, for $i = 1, 2, \dots, N$ and $t = 1, 2, \dots, T$. They evaluated the finite sample properties of the transformed log likelihood approach and compared them with various Generalized Method of Moments (GMM) estimators. Their result showed that the transformed likelihood estimator has a smaller bias than the GMM estimators in all cases with exception of CU-GMM in the small sample result. The transformed likelihood estimator outperformed the GMM estimator in terms of mean absolute error. As sample sizes is increased, the performance of the GMM estimator reduced in some cases as variance ratio (τ) increased from 1 to 5 and the transformed likelihood estimator performed best. Their results also compared with power of the tests, they observed that the transformed likelihood estimator is more powerful than the GMM estimators. They also found that the system GMM estimators have smaller biases and mean absolute errors than transformed likelihood estimator in some cases. When variance ratio (τ) is equal to five, the transformed likelihood performed better than the GMM estimators in terms of bias and mean absolute error. They also considered the size and power of the alternative procedures, their results showed that the transformed likelihood almost correct for all experiments. Some sizes distortions when $N = 50$, the empirical sizes become close to nominal value as N is increased for the GMM estimator. When $T = 5, 10$ and $N = 500$ and $\tau = 1$, the size distortions of the GMM are smaller. However, when $\tau = 5$, there are severe size distortion for the system GMM estimator even when N is 500.

Garba, Oyejola and Yahya (2013) conducted a Monte Carlo experiments based on panel data model that has two exogenous and one endogenous variable as shown as

$$y_{it} = \alpha_i + \beta_1 X_{1it} + \beta_2 X_{2it} + U_{it} \quad (2.27)$$

They assumed that when estimating panel data model the assumption of homoscedasticity, no autocorrelation and no collinearity are jointed violated.

They compared the Pooled Ordinary Least Squares (OLS), First-Differenced (FD), Between Estimator (BTW) and Feasible Generalized Least Squares (FGLS). Their findings from Monte Carlo experiments showed that in small sample, irrespective of time periods, FGLS is performed better when heteroscedasticity is severe regardless of autocorrelation level. But when heteroscedasticity is low and mild with moderate autocorrelation level, both FD and FGLS are efficient. BTW estimator performed better only when there is no autocorrelation with low degree of heteroscedasticity. Their simulation results showed that in large sample with short time periods both FD and BTW could be used when there is no autocorrelation with low degree of heteroscedasticity. Also when the degree of heteroscedasticity is mild and there is no autocorrelation, either of the FD and FGLS would produce efficient results. However, FGLS outperformed all other estimators when severe degree of heteroscedasticity is present and autocorrelation is apparent in large sample regardless of time periods. Again, both FD and FGLS are suitable when there is low heteroscedasticity despite the existence of autocorrelation and multicollinearity.

Valentin (2013) considered the estimation of dynamic panel data models when the data are suspected to exhibit cross-sectional dependence based on model given as

$$y_{it} = \rho_0 y_{i,t-1} + c_i + u_{it}, t = 1, \dots, T \quad (2.28)$$

He assumed that $E(u_{it}/Y_{t-1}) = 0, t = 1, \dots, T$ and $\rho_0 \neq 1$. He compared the small sample properties of three estimators (i) The estimator defined in Arellano and Bond (1991) as AB estimator, (ii) Feasible Optimal estimator as Estimator 1, (iii) Feasible optimal estimator with estimated within cluster correlations replaced by zero as Estimator 2. Non-feasible optimal estimator was used as a benchmark for comparison. The Estimator 1 and 2 makes use of cross-sectional dependence for efficiency while being robust to misspecification of the form of the cross-sectional dependence. Their simulation results revealed that when there is cross-sectional dependence, Estimator 1 and 2 performed better than the existing estimators in terms of accuracy and quality of inference.

Sun and Ashley (2014) conducted a Monte Carlo experiment on dynamic panel model given below:

$$y_{it} = \theta y_{i,t-1} + x_{it}^1 \beta + \mu_i + v_{it} \quad (2.29)$$

They assumed that $E(\mu_i) = 0, E(v_{it}) = 0, E(v_{it}\mu_i) = 0$ for $i = 1, \dots, N$ and $t = 2, \dots, T$

$$E(v_{it}v_{is})=0 \text{ for } i=1,\dots,N \text{ and } \forall t \neq s. E(y_{it}v_{it})=0 \text{ for } i=1,\dots,N \text{ and } t=2,\dots,T$$

They compared System Generalized Method of Moments (SYS GMM) and Transformed System Generalized Method of Moments (TSYS GMM). In term of finite sample bias, Mean Squares Errors and the 95% confidence interval coverage, their results showed that the transformed system GMM outperformed the System GMM when the fixed effects have a large variation relative to the disturbance errors and when the lagged dependent variable is highly persistent. Their simulation results indicated that the SYS GMM estimator tends to over-estimate the lagged dependent variable coefficient in the case of high variance ratio while the TSYS GMM estimator does not over estimate. They also compared the estimators with Sargan/Hansen test, when the TSYS GMM is applied; the test is well-sized while SYS GMM was over-rejecting a true null hypothesis. Their results suggested that the Transformed System GMM estimator is more reliable than the System GMM estimator in terms of Sargan/Hansen test based on inferences in validity of instruments and the overall model specifications.

Viet, Minjoo and Yongcheol (2014) conducted a Monte-Carlo simulation experiments based on a dynamic panel data models given as

$$y_{it} = \mathcal{W}_{i,t-1} + \beta X_{it} + \eta_i + v_{it} \quad (2.30)$$

They assumed that $E(v_{it}v_{is})=0 \forall t \neq s$ and a special case where any correlation between η_i and x_{it} , i.e. $E(x_{it}\eta_i)=0$. They compared (i) the instrumental variable approach (AH-IV), the first-difference GMM (FD-GMM), system GMM (SYS-GMM) and long difference GMM (LD-GMM), (ii) the bias corrected estimators based on an analytical approach, an iterative bootstrap procedure (BC) or a simulation- based indirect inference technique (II), the estimators either develop bias correction formulas in the (fixed effects) least squares dummy variable model (LSDVC). Their findings revealed that the bias- corrected estimators, LSDVC, BC and II are generally most appropriate and robust for estimating dynamic panel models in empirical corporate research than the Instrumental Variables and GMM estimators.

Olajide and Olubusoye (2016) performed a Monte-Carlo experiment based on dynamic panel data models given as

$$y_{it} = \delta y_{i,t-1} + X'_{it}\beta + \mu_i + u_{it} \quad (2.31)$$

They assumed that the explanatory variable is to be weakly exogenous forcing the variables with $E(\omega_{it}x_{is}) \neq 0$ for $s \geq t$ and zero otherwise.

They compared the Anderson-Hsiao estimator using lagged levels as instrument (AH (l)), Anderson-Hsiao using lagged differences as instrument (AH (d)), Arellano-Bond GMM estimator (first and second step), Blundell-Bond GMM estimator (first and second step). Their simulation result revealed that AH(l) and AH(d) performed reasonably when the time period is small and when time period is moderate while first-step Arellano-Bond GMM estimator performed better than all other estimators when the time period is large. Meanwhile, the first-step Blundell-Bond system GMM estimators do not perform well when the panel sample size is large.

Moreover, both Arellano and Bond and Blundell and Bond estimator can be obtained using one or two-step procedures, comparing the one-step and two-step procedure to know the one that is better among the two procedures. Some studies as shown that the two-step estimator is asymptotically more efficient, but simulation studies also showed that its asymptotic standard errors tend to be severely downward biased in small samples, when the disturbances are non-normal or heteroskedastic while the one- step estimator does not have this problem. Then, the one-step estimator weights the moment conditions by a matrix that is independent of the parameter of the model, while the matrix used by the two-step estimator depends on model parameters. To obtain the two-step procedures, the true parameters are substituted with a consistent estimate coming from the first step. Therefore, the Anderson-Hsiao estimator also has two procedures as well, Anderson-Hsiao estimator using lagged levels as instrument (AH(l)) and Anderson-Hsiao estimator using lagged differences as instrument (AH (d)). Some studies showed that AH (d) estimator performed better than AH (l).

However, for the robust estimator, M and MM estimators performed better all other robust estimator under Monte Carlo experiments.

The assumption considered were

$$E(\omega_{it} | x_{it}) = 0$$

$$E(\omega_{i,t-s} y_{i,t-j}) = 0 \quad s < j$$

$$V(\omega_{it} | x_{it}) = \sigma_v^2 \Rightarrow \text{var}(v_{it}) = \sigma_v^2 = (1 - \rho^2) \sigma_\omega^2$$

$$\text{Cov}(\omega_{it}, v_{it}) = 0, \text{Cov}(v_{it}, \mu_i) = 0, \text{Cov}(v_{it}, x_{it}) = 0$$

$$\text{Cov}(u_{it}, x_{it}) = 0 \quad (\text{Random effects only}).$$

2.4 Robust Estimators

Barrera and Yohai (2006) conducted a Monte Carlo experiment to compare the performance of the fast-S and fast- Least Trimmed Squares. They used two measures of performance to compare the two estimators: a) the percentage of samples for which each algorithm converged to a wrong local minimum: and b) The Mean Squares Errors. Their result showed that S-estimator are more efficient at the non-normal model and has the smaller bias than the Least Trimmed Squares estimator for all the values of ε . Their results also showed that the fast-s estimator performed better than the fast- Least Trimmed squares estimator and its performance was attributed to the better asymptotic efficiency and bias of S- estimators.

Bramati and Croux (2007) conducted a Monte Carlo experiment to compare the performance of Within Group (WG), Within-GM (WGM), and Within-MS (WMS) estimators under linear panel data with a fixed effect. They observed that the efficiency of the two robust estimators (Within- GM and MS) is very close to Within Group estimator. Also, when Mean Squares Error has high value, the Within Group performed very badly when there are vertical outliers and WG also showed worse in the presence of bad leverage points. Their result also showed that Within-MS and Within-GM estimates yield similar results. In the presence of bad leverage points, the WMS estimator gives slightly better results, while the WGM also shows a better result with concentrated vertical outliers. They also suggested that WGM and WMS estimators yield a large gain in Mean Squares Error in the presence of outliers and very small efficiency loss in the absence of outliers. They also observed that there was a large difference between the Within Group estimates of β_1, β_2 and β_3 .

Verardi and Croux (2009) conducted an experiment to investigate the performance of the Least squares estimator, L, M, S and MM-estimator in Stata. Their result showed that the MM-estimator has less bias among the estimators with an efficiency of 70% while the bias of L, M, S and Least Squares are similar. Also, L and M –estimators performed poorly and have a bias and Mean Squares Error similar to that of Least Squares. Their results also showed that the higher the values of MM-estimator, the more the efficiency of the estimator and when the breakdown point is $\frac{1}{2}$, its bias also increased with its efficiency. However, MM- estimator outperformed the other estimators because they combine a high resistance to the outliers and high efficiency of regression models with normal errors.

Alma (2011) conducted a Monte Carlo experiment based on the multiple regression models. He compared the Least Trimmed Squares estimators (LTS), M-estimator, Yohai MM-

estimator, S-estimator and Ordinary Least Squares. The simulation results showed that the S-estimator, M-estimator methods performed better than Least Trimmed Squares and MM-estimator methods. The result also showed that the S-estimator has a reasonable efficiency, with the influence of high leverage outliers, and demonstrate high breakdown. For 10% breakdown S-estimator increases its efficiency. MM-estimation performed the best overall against a comprehensive set of outlier conditions. However, his results also showed that when the percentages of outliers are increased, the performances of the estimators were reduced.

Maronna and Yohai (2013) performed a Monte Carlo experiment to compare the following estimator : a) Least Squares Estimator, b) the regression S-estimator with bi-square scale(S-E), c) the MM estimator with bi-square loss function and 85% asymptotic efficiency, the Gervini-Yohai (2002) estimator (G-Y), d) the Bondell- Stefanski (2013) estimator (S-E) and e) the proposed estimator Distance-Constrained Maximum Likelihood (DCML). Their results showed that B-S has the highest efficiencies in all cases; S-E has low efficiency while G-Y has the worst efficiency. DCML performed better than both its initial estimator MM and G-Y. Their results also showed that the G-Y and DCML are similar in term of Mean Square Error. However, G-Y and DCML performed better than MM-estimator, which in turn outperformed S-E while B-S has high efficiency due to the high contamination bias.

Moreover, when a real life data was applied to experiment, their results also showed that the G-Y, B-S and DCML as fully efficient, while S-E is rather inefficient and MM-estimator has a high efficiency. However, the S-E, MM and DCML are the most robust estimator, followed by G-Y has the least robust one. Therefore, the DCML estimator performed better than the S-E estimator in all cases.

Bondell and Stefanski (2013) conducted a simulation study to investigate the finite-sample efficiency and robustness of S-estimator using Turkey's bi-weight function, MM-estimator and Reweighted Least Squares estimator (REWLS). Empirical Likelihood (EL) and Exponential Tilting (ET) are used for comparison. Their simulation results showed that the proposed Exponential Tilting Estimator is highly efficient with sample size as low as 20 and as large as 50. Both REWLS and the ET estimator are asymptotically efficient under the normal distribution. Furthermore, when Empirical Likelihood estimator was used for comparison, the efficiency of the EL is slightly better than that of the Exponential Tilting. They also showed that the ET was higher efficiency than OLS and MM-estimators. Their results also showed that ET was more robust than EL in term of contamination bias under

more extreme contamination. However, when considering high efficiency of the Empirical Likelihood approach, the ET has a best performance.

Moreover, they also compared the coverage and width of 95% confidence interval constructed using the asymptotic normality of Exponential Tilting with that of the MM-estimator and the OLS estimator. Their results showed that the asymptotic distribution were close to nominal coverage when the sample size is for ET and MM-estimator while the OLS has exact intervals under normality, but asymptotic under the t-distribution. Also, ET greatly outperformed EL in a real data, in terms of the robustness and efficiency.

Hence, various studies have been carried out on the estimation of dynamic panel data by ignoring the possibility of the presence of autocorrelation in the disturbance term which may lead to misleading or invalid inferences.

Therefore, this study examined the sensitivity of some Dynamic Panel Data estimators to varying degrees of autocorrelated error term. This study evaluate the performance of several estimators when the disturbance term is serially correlated through a Monte-Carlo study..

CHAPTER THREE

METHODOLOGY

3.1 Introduction

In this chapter, the attention is focused on the theoretical framework of the model under consideration. This study considered various forms of autocorrelation in linear static and dynamic panel data models. Details of the procedures for estimating dynamic panel data models were discussed.

3.2 Autocorrelation in Panel Data Model

The classical error component disturbances given by $u_{it} = \mu_i + v_{it}$ assumed that the only correlation overtime is due to the presence of individual across the panel. The equicorrelation coefficient is shown to be $\text{correlation}(u_{it}u_{is}) = \sigma_\mu^2 / (\sigma_\mu^2 + \sigma_v^2)$ for $t \neq s$ (Baltagi, 2008). Note that it is the same no matter how far t is from s . This may be a restrictive assumption for economic relationships, like investment or consumption, where an observed shock at a period will affect the behavioural relationship for, at least, the next few periods. This type of serial correlation is not allowed for in the simple error component model. Ignoring serial correlation when it is present results in consistent but inefficient estimates of the regression coefficients and biased standard errors. This section introduces serial correlation in the v_{it} , as an autoregressive process of order one AR (1) and as a first order moving average MA (1) process.

3.3 The AR (1) Process

Lillard and Willis (1978) generalized the error component to serially correlated case, by assuming that the remainder disturbances (v_{it}) follow an AR (1) process. In this case $\mu_i \sim \text{IIND}(0, \sigma_\mu^2)$, whereas

$$v_{it} = \rho v_{i,t-1} + \varepsilon_{it} \quad (3.1)$$

$|\rho| < 1$ and $\varepsilon_{it} \sim \text{IIND}(0, \sigma_\varepsilon^2)$. The μ_i 's are independent of the v_{it} 's and $v_{it} \sim (0, \sigma_\varepsilon^2 / (1 - \rho^2))$.

Baltagi and Li (1991a) derived the corresponding Fuller and Battese (1974) transformation matrix.

$$C = \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 & 0 & \dots & 0 & 0 & 0 \\ -\rho & 1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\rho & 1 & 0 \\ 0 & 0 & 0 & \dots & 0 & -\rho & 1 \end{bmatrix}$$

to transform the remainder AR (1) disturbances into serially uncorrelated errors. For panel data, this has to be applied for N individuals (see Baltagi and Li, 1995). The transformed regression disturbances are in vector form:

$$\dot{U} = (I_N \otimes C)u = (I_N \otimes C_{IT})\mu + (I_N \otimes C)v \quad (3.2)$$

Using the fact that $C_{IT} = (1-\rho)l_T^\alpha$ where $l_T^{\alpha'} = (\alpha, l_{T-1}')$ and $\alpha = \sqrt{\frac{1+\rho}{1-\rho}}$, equation (3.2) can be expressed as

$$u^* = (1-\rho)(I_N \otimes l_T^\alpha)\mu + (I_N \otimes C)v \quad (3.3)$$

Therefore, the variance-covariance matrix of the transformed disturbances is

$$\Omega^* = E(u^*u^{*\prime}) = \sigma_\varepsilon^2(1-\rho)^2[I_N \otimes l_T^\alpha l_T^{\alpha'}] + \sigma_\varepsilon^2(I_N \otimes I_N)$$

Since $(I_N \otimes C)E(vv') = \sigma_\varepsilon^2(I_N \otimes I_T)$. Alternatively, this can be rewritten as

$$\Omega^* = d^2\sigma_\omega^2(1-\rho)^2[I_N \otimes l_T^\alpha l_T^{\alpha'} / d^2] + \sigma_\varepsilon^2(I_N \otimes I_T) \quad (3.4)$$

where $d^2 = l_T^{\alpha'} l_T^\alpha = \alpha^2 + (T-1)$. This replaces $J_T^\alpha = l_T^\alpha l_T^{\alpha'}$ by $d^2 \bar{J}_T^\alpha$, its idempotent counterpart, where $\bar{J}_T^\alpha = l_T^\alpha l_T^{\alpha'} / d^2$. Extending the Wansbeek and Kapteyntrick I_T by $E_T^\alpha + \bar{J}_T^\alpha$ was replaced, where $E_T^\alpha = I_T - \bar{J}_T^\alpha$. Collecting terms with the same matrices, to obtained the spectral decomposition of Ω^* .

$$\Omega^* = \sigma_\alpha^2(I_N \otimes \bar{J}_T^\alpha) + \sigma_\varepsilon^2(I_N \otimes E_T^\alpha) \quad (3.5)$$

where $\sigma_\alpha^2 = d^2\sigma_\mu^2(1-\rho)^2 + \sigma_\varepsilon^2$.

Therefore

$$\sigma_\varepsilon \Omega^{*-1/2} = (\sigma_\varepsilon / \sigma_\alpha)(I_N \otimes \bar{J}_T^\alpha) + (I_N \otimes E_T^\alpha) = I_N \otimes I_T - \theta_\alpha (I_N \otimes \bar{J}_T^\alpha) \quad (3.6)$$

where $\theta_\alpha = 1 - (\sigma_\varepsilon / \sigma_\alpha)$.

Premultiply the Prais-Winsten (PW) transform observations $y^* = (I_N \otimes C)y$ by $\sigma_\varepsilon \Omega^{*-1/2}$ to gets $y^{**} = \sigma_\varepsilon \Omega^{*-1/2} y^*$. The typical elements $y^{**} = \sigma_\varepsilon \Omega^{*-1/2} y^*$ are given by

$$(y_{i1}^* - \theta_\alpha \alpha b_i, y_{i2}^* - \theta_\alpha b_i, \dots, y_{iT}^* - \theta_\alpha b_i)' \quad (3.7)$$

where $b_i = [(\alpha y_{i1}^* + \Sigma_2^T y_{iT}^*)/d^2]$ for $i = 1, \dots, N$. The first observation gets special attention in the AR (1) error component model. First, the PW transformation gives it a special weight $\sqrt{1-\rho^2}$ in y^* . Second, the Fuller and Battese (1974) gives it a special weight $\alpha = \sqrt{(1+\rho)/(1-\rho)}$ in computing the weighted average b_i and the pseudo-difference in (3.7). Note that (i) if $\rho = 0$, then $\alpha = 1$, $d^2 = T$, $\sigma_\alpha^2 = \sigma_1^2$ and $\theta_\alpha = \theta$. Therefore, the typical element of y_{it}^{**} reverts to the familiar $(y_{it} - \bar{y}_i)$ transformation for the one-way error component model with no serial correlation. (ii) If $\sigma_\mu^2 = 0$, then $\sigma_\alpha^2 = \sigma_\varepsilon^2$ and $\theta_\alpha = 0$. Therefore, the typical element of y_{it}^{**} reverts to the PW transformation y_{it}^* .

The Best Quadratic Unbiased (BQU) estimators of the variance components arise naturally from the spectral decomposition of Ω^* .

$$\text{In fact, } (I_N \otimes E_T^\alpha) \mathbf{u}^* \sim (0, \sigma_\varepsilon^2 [I_N \otimes E_T^\alpha]) \text{ and } (I_N \otimes \bar{J}_T^\alpha) \mathbf{u}^* \sim (0, \sigma_\alpha^2 [I_N \otimes \bar{J}_T^\alpha]) \text{ and} \\ \hat{\sigma}_\varepsilon^2 = \mathbf{u}^* (I_N \otimes E_T^\alpha) \mathbf{u}^* / N(T-1) \text{ and } \hat{\sigma}_\alpha^2 = \mathbf{u}^* (I_N \otimes J_T^\alpha) \mathbf{u}^* / N \quad (3.8)$$

Provide the BQU estimators of σ_ε^2 and σ_α^2 , respectively, Baltagi and Li (1991a) suggested estimating ρ from within residuals $\tilde{v}_{it}$ as $\tilde{\rho} = \sum_{i=1}^N \sum_{t=1}^T \tilde{v}_{it} \tilde{v}_{i,t-1} / \sum_{i=1}^N \sum_{t=2}^T \tilde{v}_{i,t-1}^2$. Then, $\hat{\sigma}_\varepsilon^2$ and $\hat{\sigma}_\alpha^2$ are estimated from (3.8) by substituting OLS residuals $\hat{u}^*$ from the PW transformed equation using $\tilde{\rho}$. Using Monte Carlo experiments, Baltagi and Li (1997) found that $\tilde{\rho}$ performed poorly for small T and recommended an alternative estimator of ρ which is based on the auto-covariance function $Q_s = E(u_{it} u_{i,t-s})$. For AR (1) model given in (3.1), it is easy to showed that $Q_s = \sigma_\mu^2 + \sigma_v \rho^s$. From Q_0, Q_1 and Q_2 , it can easily be shown that $\rho + 1 = (Q_0 - Q_2)/(Q_0 - Q_1)$ (Baltagi, 2008). Hence, a consistent estimator of ρ (for large N) is given by

$$\tilde{\rho} = \frac{\tilde{Q}_0 - \tilde{Q}_2}{\tilde{Q}_0 - \tilde{Q}_1} - 1 = \frac{\tilde{Q}_1 - \tilde{Q}_2}{\tilde{Q}_0 - \tilde{Q}_1}$$

where $\tilde{Q}_s = \sum_{i=1}^N \sum_{t=s+1}^T \hat{u}_{it} \hat{u}_{i,t-s} / N(T-s)$ and $\hat{u}_{it}$ denotes the OLS residuals. $\hat{\sigma}_\varepsilon^2$ and $\hat{\sigma}_\alpha^2$ are estimated from (3.8) by substituting OLS residual $\hat{u}^*$ from the PW transformed equation using $\hat{\rho}$ rather than $\tilde{\rho}$.

Therefore, the estimation of an AR (1) serially correlated error component model is considered by (i) applying the PW transformation in the first step, as is usually done in the time-series literature, and (ii) subtracting a pseudo-average from these transformed data as in (3.7) in the second step.

3.4 The MA (1) Process

For the MA (1) model, defined by

$$v_{it} = \varepsilon_{it} + \lambda \varepsilon_{i,t-1} \quad (3.9)$$

Where $\varepsilon_{it} \sim IIN(0, \sigma_\varepsilon^2)$ and $|\lambda| < 1$, Balestra (1980) gave the following C matrix, $C = D^{-1/2}P$ where $D = \text{diag} \{ a_t, a_{t-1} \}$ for $t = 1, \dots, T$

$$P = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ \lambda & a_1 & 0 & \dots & 0 \\ \lambda^2 & a_1 \lambda & a_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \lambda^{T-1} & a_1 \lambda^{T-2} & a_2 \lambda^{T-3} & \dots & a_{T-1} \end{bmatrix}$$

and $a_t = 1 + \lambda^2 + \dots + \lambda^{2t}$ with $a_0 = 1$. For this C matrix, it can be shown that the new

$l_T^\alpha = C_{IT} = (\alpha_1, \alpha_2, \dots, \alpha_T)'$ where α_i can be solved for recursively as follows:

$$\alpha_1 = (a_0/a_1)^{1/2} \quad (3.10)$$

$$\alpha_t = \lambda(a_{t-2}/a_{t-1})^{1/2} \alpha_{t-1} + (a_{t-1}/a_t)^{1/2}$$

Therefore, $d^2 = l_T' l_T^\alpha = \sum_{t=1}^T \alpha_t^2$, $\sigma_\alpha^2 = d^2 \sigma_\mu^2 + \sigma_\varepsilon^2$ and the spectral decomposition of Ω^* is the same as that given in (3.5), with the newly defined l_T^α and σ_α^2 . The typical elements of

$y^{**} = \sigma_\varepsilon \Omega^{*1/2} y^*$ are given by

$$(y_{i1}^* - \theta \alpha_1 b_i, \dots, y_{iT}^* - \theta \alpha_T b_i) \quad (3.11)$$

where $b_i = [\sum_{t=1}^T \alpha_t y_{it}^* / d^2]$. Therefore, for MA (1) error component model, one applied the recursive transformation given in (3.10) in the first step and subtracts a pseudo-average described in (3.11) in the second step (Baltagi and Li, 1991a).

In order to implement the estimation of an error component model with MA (1) remainder errors, Baltagi and Li (1997) proposed an alternative transformation that is simple to compute and requires only least squares. This can be summarized as follows:

Let $\gamma_s = E(v_{it}v_{i,t-s})$ denote the autocovariance function of v_{it} and $\tau = \gamma_1/\gamma_0$. Note that when v_{it} follows an MA (1) process, it can be expressed as $Q_s = \sigma_\mu^2 + \gamma_s$ for $s = 0, 1$ and $Q_s = \sigma_\mu^2$ for $s > 1$. Hence, it can also be expressed as $\gamma_\tau = Q_s - Q_\tau$ ($\tau = 0, 1$) for $s > 1$.

Step 1: Compute $y_{i1}^* = y_{i1}/\sqrt{g_1}$ and $y_{it}^* = [y_{it} - (\tau y_{i,t-1}^*)/\sqrt{g_{t-1}}]/\sqrt{g_t}$ for $t = 2, \dots, T$

Where $g_t = 1$ and $g_t = 1 - \tau^2/g_{t-1}$ for $t = 2, \dots, T$. Note that this transformation depends only on τ , which can be estimated by $\hat{\tau} = \hat{\gamma}_1/\hat{\gamma}_0 = (\tilde{Q}_1 - \tilde{Q}_s)/(\tilde{Q}_0 - \tilde{Q}_s)$ for some $s > 1$.

Step 2: Compute y^{**} using the result that $l_T^\alpha = C l_T = (\alpha_1, \dots, \alpha_T)'$ with $\alpha_1 = 1$ and $\alpha_t = [1 - \tau/\sqrt{g_{t-1}}]/\sqrt{g_t}$ for $t = 2, \dots, T$. Note that in this case $\sigma^2 = \gamma_0$. The estimators of σ_α^2 and σ^2 are simply given by $\hat{\sigma}_\alpha^2 = (\sum_{t=1}^T \hat{\alpha}_t^2) \hat{\sigma}_\mu^2 + \hat{\sigma}^2$, and $\hat{\sigma}^2 = \hat{\gamma}_0 = \tilde{Q}_0 - \tilde{Q}_s$ for some $s > 1$ with $\hat{\sigma}_\mu^2 = \tilde{Q}_s$ for some $s > 1$. Finally, $\hat{\delta} = 1 - \sqrt{\hat{\gamma}_0/\hat{\sigma}_\alpha^2}$. Again, the OLS estimator on the (y^{**}) transformed equation is equivalent to GLS on this model

$$y_{it} = \beta x'_{it} + u_{it}$$

In summary, a simple transformation for one-way error component model with serial correlation can be easily generalized to any error process generating remainder disturbances, v_{it} as long as there exists a simple $(T \times T)$ matrix C such that the transformation $(I_N \otimes C)v$ has zero mean and variance $\sigma^2 I_{NT}$.

Step 1: Perform the C transformation on the observations of each individual $y'_i = (y_{i1}, \dots, y_{iT})$ to obtain $y_i^* = C y_i$ free of serial correlation.

Step 2: Perform another transformation on the y_{it}^* 's obtained in step 1, which subtracts from y_{it}^* a fraction of a weighted average of observations on y_{it}^* , i.e.,

$$y_{it}^{**} = y_{it}^* - \theta_\alpha \alpha_t \left(\sum_{s=1}^T \alpha_s y_{is}^* / \sum_{s=1}^T \alpha_s^2 \right)$$

Where the α_t 's are the element of $l_T^\alpha = C l_T \equiv (\alpha_1, \alpha_2, \dots, \alpha_T)'$ and $\theta_\alpha = 1 - (\sigma/\sigma_\alpha)$ with $\sigma_\alpha^2 = \sigma_\mu^2 \left(\sum_{t=1}^T \alpha_t^2 \right) + \sigma^2$.

3.5 Dynamic Panel Data Models

All panel models are dynamic, in so far as they exploit the longitudinal nature of panel data (Baltagi, 2008). Dynamic models include a lagged dependent variable on the right-hand side of the equation. A widely used modeling approach is:

$$y_{it} = \delta y_{i,t-1} + X'_{it}\beta + u_{it}; i = 1, \dots, N, t = 1, \dots, T \quad (3.12)$$

Where y_{it} is the dependent variable, $y_{i,t-1}$ is the one-period lagged value of the dependent variable with parameter δ , X'_{it} is the row vector of exogenous variable and β is the parameter of the exogenous variable. This study assumed that the u_{it} follow a one way error component model.

$$u_{it} = \mu_i + v_{it} \quad (3.13)$$

where μ_i is the unobserved individual specific effect for which $\mu_i \sim IIND(0, \sigma^2_\mu)$ and v_{it} is the disturbance error term ($v_{it} \approx IIND(0, \sigma^2_\mu)$) independent of each other and among themselves. The dynamic panel regression described in (3.12) and (3.13) are characterized by two sources of persistence over time; autocorrelation due to the presence of a lagged dependent variable among the regressors and individual effects characterizing the heterogeneity among the individuals (Baltagi, 2008). This study assumed that the disturbance term, v_{it} is serially correlated.

3.6 The Dynamic Fixed Effects Model

It is assumed that the variable of interest y_{it} is a linear function of the individuals' previous realization of this variable, and of their contemporaneous personal characteristics x_{it} with unknown coefficient, δ and β respectively:

$$y_{it} = \alpha_i + \delta y_{i,t-1} + X'_{it}\beta + u_{it}, i = 1, 2, \dots, N, t = 1, 2, \dots, T \quad (3.14)$$

where initial condition y_{i0} , $i = 1, 2, \dots$ are assumed known. This study assume that $E(u_{it}) = 0 \forall it$, $E(u_{it}u_{js}) = \sigma^2_\mu$ If $i = j$, $t = s$ and 0 otherwise, $E(\alpha_i u_{it}) = 0 \forall it$ and α_i is the individual effects (constant for each i) and u_{it} are the usual white noise disturbance terms, or in matrix form:

$$y_{it} = D\alpha + \delta y_{i,t-1} + X'\beta + u_{it} \quad (3.15)$$

and $D = I_N \otimes l_T$ and l_T is the $T \times 1$ unit vector.

The common method of estimating equation (3.15), i.e. when there is no Lagged Dependent Variable (LDV), consists of estimating equation directly by OLS (the Least Squares Dummy Variable Estimator-LSDV), which also leads to the well-known within estimator. However, given the short time period typical of panel data sets, the OLS and within estimators are well known to be biased and inconsistent as $N \rightarrow \infty$ with finite T (see Nickel, 1981; Sevestre and Trognon, 1985 for a theoretical approach, and Nerlove, 1967, 1971 for a simulation based only).

3.7 The Dynamic Random Effects

Under the random effects specification, the α_i terms of (3.14) are treated as independent random drawings from a particular distribution and disturbance term becomes “composite”, $u_{it} = \alpha_i + v_{it}$. The dynamic random effects model is given as: $y_{it} = \alpha_i + X'_{it}\beta + u_{it}$, $i = 1, 2, \dots, N$ $t = 1, 2, \dots, T$

(3.16)

where $\alpha_i \sim IIND(0, \sigma_\mu^2)$ and $v_{it} \sim IIND(0, \sigma_v^2)$

As with the fixed effects specification, the traditional estimators (Within and GLS) of the static random effects panel model are semi-inconsistent in the dynamic setting (Sevestre and Trognon, 1985). Again semi-consistent estimators for dynamic random effects model rely on certain maintained hypothesis, which are violated by the inclusion of a lagged dependent variable. The assumptions concerning the equation's disturbances imply that variance-covariance matrix of the composite disturbance term will be

$$\Omega_v = V(v) = I_N \otimes E(vv') = I_N \otimes \Sigma_v \quad (3.17)$$

$$\Sigma_v = \sigma_\alpha^2 J_T + \sigma_u^2 I_T = \sigma_v^2 \begin{bmatrix} 1 & \rho & \dots & \rho \\ \rho & 1 & \ddots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \dots & \rho & 1 \end{bmatrix} \quad (3.18)$$

Where ρ is the intra-class correlation coefficient $\rho = \frac{\sigma_\alpha^2}{(\sigma_\alpha^2 + \sigma_u^2)}$

3.8 Estimation of Random-Effects models

There are various ways to estimate the unknown parameters. Here, the two methods will be discussed; the Instrumental Variable (IV) and Generalized Methods Moment (GMM) method

3.9 Generalized Method of Moments (GMM) Estimator

The main idea behind GMM is that from a set of basic assumptions about a Data Generating Process, population moment conditions was established and then use sample analogs of these moment conditions to compute parameter estimates. The population moment conditions typically involve expectations of functions of the disturbance term and explanatory variables, while the sample analogues of the populations take the form of sample moments (for example, sample means).

Considering the Cross- Sectional regression and fix ideas about how GMM works

$$y_i = X_i \beta + U_i \quad (3.19)$$

where the key identifying assumption was adopted

$$E[X_i^l U_i] = 0 \quad (3.20)$$

X_i is a $1 \times K$ matrix of explanatory variables, β is a $K \times 1$ vector of parameters to be estimated and U_i is the disturbance. For the disturbance term, this study assumed that

$$E[U_{i,t} | y_{i,t-1}, \dots, y_{i,1}, X_{i,t}, X_{i,t-1}, \dots, X_{i,1}] = 0 \quad (3.21)$$

Again, this assumption simply means that the explanatory variables and disturbance term are uncorrelated. Substituting in for U_i , and rewrite equation (3.20) as

$$E[X_i^l (y_i - X_i \beta)] = 0 \quad (3.22)$$

to get the moment conditions in terms of observables and parameters. Population moments are estimated consistently with sample moments, the next step is to write down the sample analog of equation (3.22)

$$\frac{1}{N} \sum_{i=1}^N X_i^l (y_i - X_i \hat{\beta}) = 0 \quad (3.23)$$

where $\hat{\beta}$ is our estimator? Multiply this out solving for $\hat{\beta}$ gives

$$\hat{\beta} = \left(\sum_{i=1}^N X_i^l X_i \right)^{-1} \left(\sum_{i=1}^N X_i^l y_i \right) \quad (3.24)$$

this is identical to equation for the OLS estimator of β . Equation (3.24) can be written as $(X^l X)^{-1} X^l y$ by stacking the x_i and y_i for observations $i = 1, \dots, N$ into $N \times K$ matrix X and $N \times 1$ vector y respectively.

Now suppose that the assumption given by equation (3.22) does not hold, for example, because $X_{i,k}$ in X_i is correlated with U_i . If there are some variables Z_i available for which

$$E[Z_i^l U_i] = 0 \quad (3.25)$$

And if the element of Z_i are partially correlated with $X_{i,k}$, then the Z_i can serve as instrumental variables. The population moment conditions for the GMM estimator of β are

$$E[Z_i'(y_i - x_i\beta)] = 0 \quad (3.26)$$

Which have the sample analog

$$\frac{1}{N} \sum_{i=1}^N Z_i' (y_i - X_i \hat{\beta}) = 0 \quad (3.27)$$

If the number of columns in Z_i (i.e. the number of moment conditions) is greater than the number of parameters to be estimated (which is typically the case), then our equation is over-identified and there is not a closed-form solution as with Equation (3.23) (which was just identified). To get around this problem, $\hat{\beta}$ is chosen so that it minimizes the quadratic

$$\left(\sum_{i=1}^N Z_i' (y_i - X_i \hat{\beta}) \right)' W \left(\sum_{i=1}^N Z_i' (y_i - X_i \hat{\beta}) \right) \quad (3.28)$$

where W is a positive semi-definite weighting matrix. The solution to the minimization problem does have a closed form, and with a little manipulation to get

$$\hat{\beta} = (X'ZWZ'X)^{-1} (X'ZWZ'y) \quad (3.29)$$

It can be shown that the asymptotic variance of $\sqrt{N}(\hat{\beta} - \beta)$ is

$$\Omega = (E[X_i'Z_i]WE[Z_i'X_i])^{-1} E[X_i'Z_i]WVWE[Z_i'X_i] (E[E(X_i'Z_i) | X_i]WE[Z_i'X_i])^{-1} \quad (3.30)$$

Where $V = Var[Z_i'U_i] = E[Z_i'U_iU_i'Z_i]$

To make the GMM estimator efficient we choose $W = V^{-1}$, which makes Ω as small as possible. Substituting V^{-1} for W in Equation (3.23) given

$$\Omega = (X_i'Z_iV^{-1}Z_i'X_i)^{-1} \quad (3.31)$$

A Consistent estimator for V^{-1} , which produces standard errors that are robust to non-spherical disturbances, is

$$\hat{W} = \hat{V}^{-1} = \left[\frac{1}{N} \sum_{i=1}^N Z_i' \hat{U}_i \hat{U}_i' Z_i \right]^{-1} \quad (3.32)$$

Where the $\hat{U}_i$ are estimated residual produced using a first-stage, consistent estimator of β . Plugging in $\hat{W}$ and $\hat{V}^{-1}$ in Equations (3.29) and (3.31) produces the asymptotically optimal GMM estimator.

3.10 The Instrumental Variable (IV) Estimator

The method of instrumentation is one possible way to prevent the bias resulting from correlation between the regressor X and the error term ε . The idea of instrumentation can be stated as follows:

To find a variable Z that is highly correlated with X , but does not correlate with ε . Use as the new regressor only that part of the observable variable X which correlates with Z and is orthogonal to ε ."

Starting with the problem of correlation between the observable and the error term.

$$p \lim \left(\frac{1}{n} X'Y \right) = 0 \quad (3.33)$$

In the linear regression case

$$y = X\beta + Y \text{ with } \text{var}(v) = \sigma^2 I \quad (3.34)$$

The bias can be circumvented using an instrumental Z that correlates with X but is orthogonal to v .

$$p \lim \left(\frac{1}{n} Z'X \right) = \Sigma ZX \neq 0 \quad (3.35)$$

$$p \lim \left(\frac{1}{n} Z'V \right) = 0 \quad (3.36)$$

Premultiplying the regression with Z leads to the residual $Z'V$ and the following variance:

$$Z'y = Z'X\beta + Z'Y \quad (3.37)$$

$$\text{var}(Zv) = Z' \text{var}(\varepsilon) Z = \sigma^2 Z'Z$$

Making use of Generalized Least Square-Estimators (GLS) with $v^{-1} = (Z'Z)^{-1}$ and dropping σ^2 leads to the following instrumental variable estimator:

$$b_{IV} = \left[(Z'X) V^{-1} Z X \right]^{-1} (Z'X) V^{-1} Z'y = \left(X'Z(Z'Z)^{-1} Z'X \right)^{-1} X'Z(Z'Z)^{-1} Z'y \quad (3.38)$$

$$b_{IV} = (X'PX)^{-1} X'Py \text{ with } P = Z(Z'Z)^{-1} Z' \quad (3.39)$$

Inserting $y = X\beta + \varepsilon$ results in

$$b_{IV} = (X'PX)^{-1} X'Py(X\beta + \varepsilon) = Z(X'PX)^{-1} X'PX\beta + (X'PX)^{-1} X'PV \quad (3.40)$$

$$= \left(X'Z(Z'Z)^{-1} Z'X \right)^{-1} X'Z(Z'Z)^{-1} Z'X\beta + \left(X'Z(Z'Z)^{-1} Z'X \right)^{-1} X'Z(Z'Z)^{-1} Z'V \quad (3.41)$$

$$= \beta + \left(\frac{1}{n} X'Z(Z'Z)^{-1} Z'X \right)^{-1} \frac{1}{n} X'Z(Z'Z)^{-1} Z'V \quad (3.42)$$

Now taking probability limits shows the estimators being unbiased:

$$p \lim b_{IV} = \beta + p \lim \left[\frac{1}{n} \left[X'Z(Z'Z)^{-1}Z'X \right]^{-1} \frac{1}{n} X'Z(Z'Z)^{-1}Z'V \right] \quad (3.43)$$

$$p \lim b_{IV} = \beta + \left(\Sigma_{XZ} \Sigma^{-1}_{ZZ} \Sigma_{ZX} \right)^{-1} \Sigma_{XZ} \Sigma^{-1}_{ZZ} \Sigma_{Z\varepsilon} = \beta \quad (3.44)$$

Making use of $\Sigma_{Z_y} = 0$

Therefore it is evident, that basic to idea of instrumenting is the assumed uncorrelated of Z and Y .

The instrumentation can be made transparent through the exposition as a two stage procedure. In the first step the explanatory variable X is regressed on the instrument Z . The regression values $\hat{X}$ containing the linear dependent part of X are used as explanatory variables in the second step.

Based on the first auxiliary regression

$$X = Z\gamma + \varepsilon \quad (3.45)$$

Regression values are obtained

$$\hat{X} = Z\hat{\gamma} = Z(Z'Z)^{-1}Z'X, \quad (3.46)$$

which will be used as new regressors in the second stage

$$y = \hat{X}b_{2S} + Y \quad (3.47)$$

$$b_{2S} = \left(\hat{X}'\hat{X} \right)^{-1} \hat{X}'y \quad (3.48)$$

Inserting $Z(Z'Z)^{-1}Z'X$ for the regression values $\hat{X}$ leads to the instrumental variable estimator b_{IV} :

$$b_{2S} = \left(Z(Z'Z)^{-1}(Z'X)'Z(Z'Z)^{-1}Z'X \right)^{-1} \left(Z(Z'Z)^{-1}Z'X \right)'y \quad (3.49)$$

$$\begin{aligned} &= \left(XZ(ZZ')^{-1}Z'Z(Z'Z)^{-1}Z'X \right)^{-1} XZ(ZZ')^{-1}Z'y \\ &= \left(XZ(ZZ')^{-1}Z'X \right)^{-1} X'Z(ZZ')^{-1}Z'y \end{aligned} \quad (3.50)$$

$$b_{IV} = (X'PX)^{-1}X'Py \quad (3.51)$$

3.11 GMM Estimation for Dynamic Panel Data

The model under consideration is the same as equation (3.12)

where y_{it} and x_{it} denote the (scalar) dependent variable and $1 \times k$ vector of exogenous variables corresponding to cross sectional unit i in period t , δ and β represent corresponding 1×1 and $k \times 1$ parameters, and $u_{it} = \mu_i + v_{it}$ denotes the overall disturbance term consisting of individual effects μ_i and an innovation v_{it} . Under different assumptions on the disturbance process different possible moment restrictions that are exploited by the estimator were obtained. The proposed estimator also differs under different exogeneity assumptions on the $k \times 1$ vector of explanatory variables.

Arellano and Bond (1991) assumed that the errors terms are distributed as

$$\mu_i \sim IID(0, \sigma^2_\mu), \quad (3.52)$$

and

$$v_{it} \sim IID(0, \sigma^2_v), \quad (3.53)$$

Independent of each other. Because the disturbances as well as the endogenous variable contain individual effects, they will be correlated when interacted in levels. Therefore, the moment conditions involve first differences of the disturbances and in particular they are

$$E[(u_{it} - u_{i,t-1})y_{i,t-k}] = 0, \quad t = 2, \dots, T, \quad k = 2, \dots, t-1 \quad i = 1, \dots, N, \quad (3.54)$$

And with strictly exogenous variables also

$$E[x^1_{is}(u_{it} - u_{i,t-1})] = 0 \quad p \times 1, \quad t = 2, \dots, T, \quad s = 1, \dots, T \quad i = 1, \dots, N, \quad (3.55)$$

While with the variables being only predetermined these conditions hold only for $s = 1, \dots, t-1$.

Stacking the model by grouping the observation first by time and then by individuals, the first differenced model were written (after dropping the initial observation) as

$$\begin{matrix} \Delta \delta \\ (T-1) \times N \times 1 \end{matrix} + \begin{matrix} \Delta Z \delta \\ (T-1) \times N \times 2^2 \times 1 \end{matrix} + \begin{matrix} \Delta \varepsilon \\ (T-1) \times N \times 1 \end{matrix} \quad (3.56)$$

Where $\Delta Z = [\Delta_{y-1}, X]$ with

$$\begin{aligned}
\Delta y &= \begin{pmatrix} y_{i2} - y_{i1} \\ \vdots \\ y_{iT} - y_{i,T-1} \\ \vdots \\ y_{N2} - y_{N1} \\ \vdots \\ y_{NT} - y_{N,T-1} \end{pmatrix}, & \Delta y_{-1} &= \begin{pmatrix} y_{i1} - y_{i0} \\ \vdots \\ y_{i,T-1} - y_{i,T-2} \\ \vdots \\ y_{N1} - y_{N0} \\ \vdots \\ y_{N,T-1} - y_{N,T-2} \end{pmatrix}, \\
\Delta X &= \begin{pmatrix} x_{i2} - x_{i1} \\ \vdots \\ x_{iT} - x_{i,T-1} \\ \vdots \\ x_{N2} - x_{N1} \\ \vdots \\ x_{NT} - x_{N,T-1} \end{pmatrix}, & \Delta \varepsilon &= \begin{pmatrix} \varepsilon_{i2} - \varepsilon_{i1} \\ \vdots \\ \varepsilon_{iT} - \varepsilon_{i,T-1} \\ \vdots \\ \varepsilon_{N2} - \varepsilon_{N1} \\ \vdots \\ \varepsilon_{NT} - \varepsilon_{N,T-1} \end{pmatrix}
\end{aligned} \tag{3.57}$$

The matrix of instruments was defined as $H = (H'_1, \dots, H'_N)'$ where for the case of strictly exogenous variables. It is given as

$$H'_i = \begin{bmatrix} \begin{pmatrix} y_{i0} \\ x_{i1} \\ \vdots \\ x_{i,T} \end{pmatrix} & \begin{pmatrix} y_{i0} \\ y_{i2} \\ x_{i1} \\ \vdots \\ x_{i,T} \end{pmatrix} & \dots & \begin{pmatrix} y_{i0} \\ \vdots \\ y_{i,T-2} \\ x_{i1} \\ \vdots \\ x_{i,T} \end{pmatrix} \end{bmatrix} \tag{3.58}$$

The proposed estimator is of the form

$$\hat{\delta} = \left(\Delta Z' H A^{-1} H' \Delta Z \right)^{-1} \Delta Z' H A^{-1} H' \Delta y, \quad (3.59)$$

Where A is some weights matrix for the moments. More specifically, the first step of the estimation uses a simple weighting matrix

$$\begin{aligned} A &= \sum_{i=1}^N H_i' D D' H_i \\ &= H' (I_N \otimes D D') H, \end{aligned} \quad (3.60)$$

Where D is a $(T-1) \times T$ first difference operator matrix:

$$D = \begin{pmatrix} -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -1 & 1 \end{pmatrix}_{(T-1) \times T} \quad (3.61)$$

In the second step the moment conditions are weighted by their estimated variance covariance matrix and the authors proposed to use

$$\begin{aligned} A &= \sum_{i=1}^N H_i' \Delta \hat{u}_i \Delta \hat{u}_i' H_i \\ &= H' \left[I_N \otimes D \left(\sum_{i=1}^N \hat{u}_i \hat{u}_i' \right) D' \right] H, \end{aligned} \quad (3.62)$$

Where $\Delta \hat{u}_i = (\Delta \hat{u}_{i2}, \dots, \Delta \hat{u}_{iT})'$ and $\hat{u}_i = (u_{i1}, \dots, u_{iT})'$ are the fitted residuals from the first step estimator.

Arellano and Bover (1995) considered a general nonsingular transformation of the model that removes the individual effects. Consider again the model in equation (3.12) and let K be any $(T-1) \times T$ transformation matrix of the rank $(T-1)$ such that $Ke_T = 0_{T-1}$, where e_T is a $T \times 1$ vector of ones. That is, the transformation by K is nonsingular and removes the individual effects. Hence K can, for example, be the matrix D considered above, or be equal to the ‘Within Group’ operator, with

$$K = \begin{pmatrix} \left[1 - \frac{1}{T}\right] & -\frac{1}{T} & \cdots & -\frac{1}{T} & -\frac{1}{T} \\ -\frac{1}{T} & \left[1 - \frac{1}{T}\right] & \cdots & -\frac{1}{T} & -\frac{1}{T} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\frac{1}{T} & -\frac{1}{T} & \cdots & \left[1 - \frac{1}{T}\right] & -\frac{1}{T} \end{pmatrix} \quad (3.63)$$

Arellano and Bover (1995) also suggested the orthogonal deviations operator defined as:

$$K = \begin{pmatrix} 1 & -\frac{1}{(T-1)} & -\frac{1}{(T-1)} & \cdots & -\frac{1}{(T-1)} & -\frac{1}{(T-1)} & -\frac{1}{(T-1)} \\ 0 & 1 & -\frac{1}{(T-2)} & \cdots & -\frac{1}{(T-2)} & -\frac{1}{(T-2)} & -\frac{1}{(T-2)} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & \cdots & 0 & 1 & -1 \end{pmatrix} \quad (3.64)$$

This transformation subtracts the mean of future observations available in the sample from the first $T - 1$ observations.

The transformed model is then

$$(I_N \otimes K)y = (I_N \otimes K)Z\delta + (I_N \otimes K)\varepsilon, \quad (3.65)$$

If the transformation matrix is upper triangular and the disturbances v_{it} are not serially correlated, then the same moment conditions as consider by Arellano and Bond (1991) remain valid for the transformed model. Arellano and Bover (1995) then showed that the resulting GMM estimator is in fact invariant to the choice of the transformation matrix.

If the exogenous variables are uncorrelated with the individual effects, Arellano and Bover (1995) also suggested the use of additional moment conditions in the form of

$$E\left[\left(\frac{1}{T} \sum_{t=1}^T u_{it}\right) x_{is}\right] = 0_{p \times 1}. \quad (3.66)$$

In this case the transformation matrix is appended with a row consisting of e_T/T and can be denoted as:

$$C = \begin{pmatrix} K \\ e_T/T \end{pmatrix}, \quad (3.67)$$

The instrument matrix Hi becomes

$$H'_i = \begin{bmatrix} \begin{pmatrix} y_{i0} \\ x_{i1} \\ \vdots \\ x_{iT} \end{pmatrix} & \begin{pmatrix} y_{i0} \\ y_{i2} \\ x_{i1} \\ \vdots \\ x_{iT} \end{pmatrix} & \ddots & \begin{pmatrix} y_{i0} \\ \vdots \\ y_{iT-2} \\ x_{i1} \\ \vdots \\ x_{iT} \end{pmatrix} & \begin{pmatrix} x_{i1} \\ \vdots \\ x_{iT} \end{pmatrix} \end{bmatrix} \quad (3.68)$$

The GMM estimator of Arellano and Bover (1995) can then be expressed as

$$\hat{\delta} = [Z'(I_N \otimes C')HA^{-1}H'(I_N \otimes C)Z]^{-1}Z'(I_N \otimes C')HA^{-1}H'(I_N \otimes C)y. \quad (3.69)$$

The preliminary estimates are obtained with $A = H'(IN \otimes CC')H$ and the second stage estimator uses consistently with (3.62)

$$A = H' \left[IN \otimes C \left(\sum_{i=1}^N \hat{u}_i \hat{u}_i' \right) C' \right] H, \quad (3.70)$$

where $\hat{u}_i$ are the fitted residuals from the preliminary estimation. Given that the estimator is invariant to the choice of the transformation matrix, the filtering is in fact irrelevant and the estimator can be obtained by performing three stage least squares (3SLS).

Ahn and Schmidt (1995) showed that there are additional moment conditions that can be exploited. Ahn and Schmidt also make weaker assumptions that lead to the set of moment restriction utilized by the Arellano and Bond (1991) and Arellano and Bover (1995) estimators. In particular, Ahn and Schmidt assumed that the disturbances satisfy

$$\begin{aligned}
Cov(v_{it}, y_{i0}) &= 0, t = 1, \dots, T \\
Cov(v_{it}, \mu_i) &= 0, t = 1, \dots, T \\
Cov(v_{it}, v_{is}) &= 0, t, s = 1, \dots, T; t \neq s
\end{aligned} \tag{3.71}$$

The additional moment conditions pointed out by Ahn and Schmidt are

$$E[u_{iT}(v_{it} - v_{i,t-1})] = 0, \quad t = 2, \dots, T-1. \tag{3.72}$$

These restrictions, together with the moment conditions utilized by the Arellano and Bond (1991) estimator, represent all the moment conditions implied by the assumption that the innovation v_{it} are mutually uncorrelated among themselves and with μ_i and y_{i0} .

Ahn and Schmidt also pointed out that further restrictions can be derived from homogeneity and stationary assumptions. The assumption that the innovations v_{it} have a variance that does not change over time implies the following additional moment restrictions:

$$E[y_{i,t-2}\Delta v_{i,t-1} - y_{i,t-1}\Delta v_{it}] = 0 \quad t = 4, \dots, T \tag{3.73}$$

In model without exogenous variables the homogeneity restrictions can be implemented by utilizing the extended instrument set defined as

$$H_i^+ = \begin{pmatrix} H_i & & & & \\ & y_{i2} & -y_{i3} & & \\ & & y_{i3} & -y_{i4} & \\ & & & \ddots & \ddots \\ & & & & y_{i,T-2} & -y_{i,T-3} \end{pmatrix}, \tag{3.74}$$

Where H_i is the Arellano and Bond instrument matrix for the case without exogenous variables, i.e.

$$H_i = \begin{pmatrix} y_{i0} & & & & \\ & y_{i0} & y_{i1} & & \\ & & \ddots & \ddots & \\ & & & y_{i0} & \dots & y_{i,T-2} \end{pmatrix}, \tag{3.75}$$

Ahn and Schmidt showed that the GMM estimator based on the full set of moment restrictions is asymptotically equivalent to Chamberlain's (1982, 1984) optimal minimum distance estimator and that it reaches the semi parametric efficiency bound.

Blundell and Bond (1998) documented a potential gain in efficiency arising from exploiting restrictions on the initial observations when the time period of the panel is small and degree of autocorrelation is high. The estimation approaches discussed so far usually drop the first

observation. With N going to infinity and T fixed this amounts to ignoring information from a fixed proportion of the sample and thus can lead to sizeable inefficiency.

In their simulation study Blundell and Bond consider two types of additional restrictions. The first type of restriction justifies the use of an extended linear GMM estimator that uses lagged differences of y_{it} as instruments for equations in levels (in addition to lagged differences of y_{it} as instruments for equations in first differences). The second type of restriction validates the use of the error components GLS estimator on an extended model that conditions on the observed initial values. This provides a consistent estimator under homoscedasticity which, under normality, is asymptotically equivalent to conditional maximum likelihood.

In a model without exogenous variables, Blundell and Bond showed that after removing redundant restrictions the extended GMM estimator they considered utilizes the following instrument matrix:

$$H_i^{++} = \begin{pmatrix} H_i^+ & & & \\ & \Delta y_{i2} & & \\ & & \ddots & \\ & & & \Delta y_{i,T-1} \end{pmatrix} \quad (3.76)$$

Where H_i^+ is the instrument matrix employed by the Ahn and Schmidt estimator and is defined in (3.76) above.

Their Monte Carlo simulations and asymptotic variance calculations showed that this extended GMM estimator offers considerable efficiency gains in situations where the basic GMM estimator performed poorly. The GLS estimator that conditions on the initial values is also found to have good finite sample properties. However, the conditional GLS estimator requires homoscedasticity, and only extends to a model with regressors are strictly exogenous which is not the case is for the GMM estimators.

The efficiency gain from incorporating the information in the initial observation is also documented by a simulation study of Hahn (1999)

Alvarez and Arellano (2002) considered the same model (3.12) with $|\phi| < 1$ and $E(v_{it} | \mu_i, y_{i0}, \dots, y_{it-1}) = 0$. They assumed that y_{i0} is also observed. To derived asymptotic results, they assumed that v_{it} for $t = 1, \dots, T$ and $i = 1, \dots, N$ are independent and identically distributed across time and individuals and independent of μ_i and y_{i0} , with

$E(v_{it}) = 0, \text{Var}(v_{it}) = \sigma^2$ and finite fourth moments. Additionally they assumed that the initial observation are generated as

$$y_{i0} = \frac{\mu_i}{1-\phi} + \sum_{j=0}^{\infty} \phi^j v_{i,-j} \quad (3.77)$$

The articles than establishes asymptotic properties of the ‘Within Group’ estimators, the GMM estimator, and Limited Information Maximum Likelihood (LIML) estimator when both T and N tend to infinity. The WG estimator can be obtained by OLS estimation on the model transformed by forward orthogonal means transformation.

The LIML estimator is what has been suggested by Alonso-Borrego and Arellano (1999) as a symmetrically normalized GMM estimator. It can also be regarded as a ‘continuously updated GMM estimator’ in terminology of Hansen, Heaton and Yaron(1996). The estimator is only an analogue LIML estimator in the sense of the minimax instrumental variable interpretation given by Sargan (1958) to the original LIML estimator. It is defined as

$$\hat{\delta} = \arg \min_{\delta} \frac{(y - Z\delta)' (I_N \otimes C') H (H'H)^{-1} H' (I_N \otimes C) (y - Z\delta)}{(y - Z\delta)' (I_N \otimes C') (I_N \otimes C) (y - Z\delta)}, \quad (3.78)$$

where H is an instrument matrix.

Alvarez and Arellano showed that the asymptotic bias of the WG estimator only disappears when $N/T \rightarrow 0$. When N/T tends to a positive constant, all three estimators are asymptotically biased with negative asymptotic biases of $1/T, 1/N$, and $1/(2N - T)$ respectively. When N/T tends to infinity, the fixed T results assumed by the GMM literature remain valid. They also considered a random effects maximum likelihood estimator that leaves the mean and variance of the initial conditions unrestricted and showed that this estimator is asymptotically unbiased for all cases.

Keane and Runkle (1992) suggested an alternative estimation procedure that takes into account the covariance structure of the disturbances. First the model is estimated by an initial procedure, such as the instrumental variables (IV) with instruments that could, for example, be the instruments suggested by Arellano and Bond (1991). Then an estimate of the inverse of the variance covariance matrix and its cholesky decomposition is calculated. The model is then transformed and estimated with original (untransformed) instruments, i.e.

$$\hat{\delta} = [Z'(I_N \otimes \hat{P}') H A^{-1} H' (I_N \otimes \hat{P}) Z]^{-1} \quad (3.79)$$

$$.Z'(I_N \otimes \hat{P}') H A^{-1} H' (I_N \otimes \hat{P}) y, \quad (3.80)$$

Where $\hat{P}$ is cholesky decomposition of the estimated inverse of the variance covariance matrix and A is moment weighting matrix that is chosen analogously to the standard GMM estimators.

3.12 Dynamic Panel Data Estimators

Here, this study will considered the estimators of dynamic panel data models.

3.13 Ordinary Least Square (OLS) Estimator

In the static case in which all the explanatory variables are exogenous and are correlated with effects, the error-component structure was ignored and applied the OLS method. The OLS estimator, although less efficient, is still unbiased and consistent. But this is no longer true for dynamic error component models. The correlation between the lagged dependent variable and individual specific effects would seriously bias the OLS estimator. OLS, the simplest of all estimators considered, is applied to the equation in the level form. Since the initial values of y_{it} are known, OLS can use, in actual estimation, all of the cross-sections. The following simple models were used to illustrate the extent of bias. Let

$$y_{it} = \delta y_{i,t-1} + \alpha_i + u_{it}, \quad |\delta| < 1, \quad 1, \dots, N, \quad t = 1, \dots, T \quad (3.81)$$

Where u_{it} is independently, identically distributes over i and t . The OLS estimator of δ by substitution is

$$\delta^{OLS} = \frac{\sum_{i=1}^N \sum_{t=1}^T y_{it} y_{i,t-1}}{\sum_{i=1}^N \sum_{t=1}^T y_{i,t-1}^2} = \delta + \frac{\sum_{i=1}^N \sum_{t=1}^T (\alpha_i + u_{it}) y_{it}}{\sum_{i=1}^N \sum_{t=1}^T y_{i,t-1}^2} \quad (3.82)$$

3.14 The Anderson-Hsiao Estimator

The estimator suggested by Anderson and Hsiao (1982) is based on the differenced form of the original equation.

$$y_{it} = \delta y_{i,t-1} + X'_{it} \beta + \mu_i + v_{it}$$

$$y_{it} - y_{i,t-1} = \delta (y_{i,t-1} - y_{i,t-2}) + (x'_{it} - x'_{i,t-1}) \beta + v_{it} + v_{i,t-1} \quad (3.83)$$

which cancels the individual fixed effects assumed to possibly correlate with the exogenous variables ($E(x'_{it} - \mu_i) \neq 0$). Using matrix notation, it could be written as

$$y = y_1 \delta + X' \beta + D \mu + V$$

$$Fy = Fy_1 \delta + FX \beta + Fv$$

Where

$$F = I_N \otimes F_T \text{ and } F_T = \begin{bmatrix} -1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -1 & 1 \end{bmatrix} \text{ with dimension } (T-1) \times T$$

Because $FD = 0$, the individual fixed effects cancel out. But the difference of the lagged endogenous variable $y_{it-1} - y_{it-2} = \delta(y_{it-1} - y_{it-3})\beta + v_{it-1}v_{it-2}$ is now obviously correlated with the error term $v_{it} - v_{it-1}$.

Therefore $E(dy_{it-1}dv_{it}) \neq 0$ and the estimator will be biased. Anderson and Hsiao proposed using level instruments y_{t-2} or the lagged difference $y_{it-2} - y_{it-3}$ as an instrument for the differenced lagged endogenous regressor $y_{it-1} - y_{it-3}$. Both are valid instruments, since they are correlated with $y_{it-1} - y_{it-2}$ and uncorrelated with $v_{it} - v_{it-1}$. The use of y_{it-2} is usually recommended (see Arellano (1989) and Arellano and Bond (1991)). These instruments can be expected to be uncorrelated with the differenced error term:

$$E(y_{it-2}dv_{it}) = 0 \text{ and } E(dy_{it-2}dv_{it}) = 0 \quad (3.84)$$

When analyzing the properties of the two possible instruments.

Arellano (1989) found the estimator using level instruments superior because of having smaller variances and no points of singularities. Furthermore the use of the levels as instruments has the advantage of losing one year less what can be relevant in the practical use, especially when using data files with a large number of individuals and few years. The differencing imposes a MA (1) structure on the error term, even when the errors ε_{it} originally where not correlated over time. Estimation will then make use of the following matrices in the case of levels as instruments

$$\tilde{Z}_i = \begin{bmatrix} y_{i,1} & x'_{i3} - x'_{i2} \\ y_{i,2} & x'_{i4} - x'_{i3} \\ \vdots & \vdots \\ y_{i,T-2} & x'_{iT} - x'_{iT-1} \end{bmatrix}$$

$$\tilde{X}_i = \begin{bmatrix} y_{i2} - y_{i1} & x'_{i3} - x'_{i2} \\ y_{i3} - y_{i2} & x'_{i4} - x'_{i3} \\ \vdots & \vdots \\ y_{iT-1} - y_{iT-2} & x'_{iT} - x'_{iT-1} \end{bmatrix}$$

$$\tilde{y}_i = \begin{bmatrix} y_{i3} - y_{i,2} \\ y_{i4} - y_{i,3} \\ \vdots \\ y_{iT} - y_{i,T-1} \end{bmatrix}$$

And as follows for the use of differenced instruments

$$\tilde{X}_i = \begin{bmatrix} y_{i3} - y_{i2} & x'_{i4} - x'_{i3} \\ y_{i4} - y_{i3} & x'_{i5} - x'_{i4} \\ \vdots & \vdots \\ y_{iT-1} - y_{i,T-2} & x'_{iT} - x'_{iT-1} \end{bmatrix}$$

$$\tilde{Z}_i = \begin{bmatrix} y_{i2} - y_{i,1} & x'_{i4} - x'_{i3} \\ y_{i3} - y_{i,2} & x'_{i5} - x'_{i4} \\ \vdots & \vdots \\ y_{iT-2} - y_{i,T-3} & x'_{iT} - x'_{i,T-1} \end{bmatrix}$$

$$\tilde{y}_i = \begin{bmatrix} y_{i4} - y_{i,3} \\ y_{i5} - y_{i,4} \\ \vdots \\ y_{iT} - y_{i,T-1} \end{bmatrix}$$

Stacking the observations for all individuals' results in the estimators:

$$Z = \begin{bmatrix} \tilde{Z}_1 \\ \tilde{Z}_2 \\ \vdots \\ \tilde{Z}_N \end{bmatrix}, X = \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \vdots \\ \tilde{X}_N \end{bmatrix}, y = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_N \end{bmatrix}$$

$$\hat{\gamma}^{AH} = (X'PX)^{-1} X'Py \quad \text{Where } P = Z(Z'Z)^{-1} Z' \quad (3.85)$$

The symbol L or D were added to indicate the use of lagged levels or lagged differences as instruments. $\hat{\gamma}^{AH,L}, \gamma^{AH,D}$.

3.15 The Arellano – Bond Estimator

Arellano and Bond (1991) proposed a generalized method of moments (GMM) estimator and it has become increasingly popular. The estimator is similar to the estimator suggested by

Anderson and Hsiao but exploits additional moment restrictions, which enlarges the set of instruments. The dynamic equation to be estimated in levels is $y_{it} = \delta_{y_{i,t-1}} + x'_{it} \beta + \mu_i + v_{it}$ where differencing eliminates the individual effects μ_i

$$y_{it} - y_{i,t-1} = \delta(y_{i,t-1} - y_{i,t-2}) + (x'_{it} - x'_{i,t-1})\beta + v_{it} - v_{i,t-2}$$

Looking for instruments available for instrumenting the difference equation for each year.

For $t = 3$ the equation to be estimated is

$$y_{i3} - y_{i2} = \delta(y_{i2} - y_{i1}) + (x'_{i3} - x'_{i2})\beta + v_{i3} - v_{i2}$$

where the instruments (again assuming x being at least predetermined) y_{i1} , x'_{i2} and x'_{i3} are available. For $t = 4$ the equation is

$$y_{i4} - y_{i3} = \delta(y_{i3} - y_{i2}) + (x'_{i4} - x'_{i3})\beta + v_{i4} - v_{i3}.$$

Where the instruments y_{i1} , y_{i2} , x'_{i1} , x'_{i2} and x'_{i3} are available

For $t=5$ the equation is

$$y_{i5} - y_{i4} = \delta(y_{i4} - y_{i3}) + (x'_{i5} - x'_{i4})\beta + v_{i5} - v_{i4}$$

And the instruments y_{i1} , y_{i2} , y_{i3} , x_{i1} , x_{i2} , x_{i3} and x_{i4} are available. As can be seen, the time periods valid for instrumentation enlarge and for the equation in the final Period T

$$y_{iT} - y_{iT-1} = \delta(y_{iT-1} - y_{iT-2}) + (x'_{iT} - x'_{iT-1})\beta + v_{iT-1}.$$

The instruments y_{i1} , y_{i2} , ..., y_{iT-2} , x'_{i1} , x'_{i2} , ..., $x'_{i,T-1}$ are available.

The instrumented equation is

$$W'Fy = W'FX\beta + W'Fv \quad (3.86)$$

Where

$$X_i = \begin{bmatrix} y_{i2} - y_{i1} & x'_{i3} - x'_{i2} \\ y_{i3} - y_{i2} & x'_{i4} - x'_{i3} \\ \vdots & \vdots \\ y_{iT-1} - y_{iT-2} & x'_{iT} - x'_{iT-1} \end{bmatrix}$$

$$X = (y_{-1}, X), \hat{\gamma} = (\delta, \beta'), W = (w_1, w_2, \dots, w_N)'$$

$$Wi = \begin{bmatrix} [y_{i1}, x'_{i1}, x'_{i2}] & 0 & \dots & 0 \\ 0 & [y_{i1}, y_{i2}, x'_{i1}, x'_{i2}, x'_{i3}] & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & [y_{i1}, y_{i2}, \dots, y_{iT-2}, x'_{i1}, x'_{i2}, \dots, x'_{iT-1}] \end{bmatrix}$$

As has been shown for the simple case of the instrumental variable estimation, the estimation procedure can be seen as two-step estimation. First a cross-section auxiliary equation

$$y_{it} - y_{i,t-1} = \hat{a}_{1t} y_{i,t-2} + \hat{a}_{2t} y_{i,t-3} + \dots + x'_{it} \hat{b}_{1t} + x'_{i,t-2} \hat{b}_{2t} + \dots + \varepsilon_{it}$$

is and in the second step the resulting estimates are used as explanatory variables in the equation of the original interest.

In the k-explanatory variable case the maximal number of parameters to be estimated $T - 2 + k (T - 1) = (k + 1) (T - 1) - 1$ which determines the number of the individuals which has to be available to allow estimation. Because the differencing operation introduces first order autocorrelation into the error term, the first-step estimator makes use of a covariance matrix taking this autocorrelation into account.

$$V = W'GW = \sum_{i=1}^N W_i' G_T W_i \quad (3.87)$$

$$\text{Where } G = (I_N \otimes G_T) = \begin{bmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & \ddots & \dots & 0 \\ \vdots & \ddots & \ddots & \dots & -1 \\ 0 & \dots & \dots & -1 & 2 \end{bmatrix} \text{Premultiplying the matrix}$$

F results in transforming the original observations into differences. Because $\text{Var}(Fu) = F\sigma^2 F'$, the covariance matrix $V = FF'$ is used as a first step approximation to the covariance matrix.

The two-step GMM estimator uses the residuals of the first-step estimation to estimate the covariance matrix as suggested by White (1980):

$$\hat{V} = \sum_{i=1}^N W_i' F_T v_i v_i' F_T W_i \quad (3.88)$$

The resulting estimator finally is

$$\hat{\gamma}^{GMM} = (XW\hat{V}^{-1}W'X)^{-1} X'W\hat{V}^{-1}W'y \quad (3.89)$$

3.16 The Blundell – Bond Estimator

The GMM estimator which was suggested by Arellano and Bond (1991) is known to be rather inefficient which instruments are weak because making the use of the information contained in differences only. Blundell and Bond (1998) suggested making use of additional level information beside the differences. The combination of moment restrictions for differences and levels results in an estimator which was called GMM system estimator by Arellano and Bond. There are $T - 2$ orthogonally restrictions in levels which are exploited. The observation t in levels

$$y_{it} = \delta_{y_{i,t-1}} + x'_{it}\beta + \mu_i + v_{it} \quad (3.90)$$

will be used for estimation, where differences are used as valid instruments (again assuming x being at least predetermined). Take for example the last observation T .

$$y_{iT} = \delta y_{i,T-1} + x'_{iT}\beta + \mu_i + v_{iT} \quad (3.91)$$

where use is made of the instruments $dy_{i,1}, dy_{i,2}, \dots, dy_{i,T-1}, dx'_{i1}, dx'_{i2}, \dots, dx'_{iT}$. The matrices used for estimation are then defined as:

$$y_i = \begin{pmatrix} y_{i3} - y_{i,2} \\ y_{i4} - y_{i3} \\ \cdot \\ \cdot \\ \cdot \\ y_{iT} \end{pmatrix} \quad X_i = \begin{pmatrix} y_{i2} - y_{i,1} & x'_{i3} - x'_{i2} \\ y_{i3} - y_{i,2} & x'_{i4} - x'_{i3} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ y_{i,T-1} - y_{i,T-2} & x'_{iT} - x'_{i,T-1} \\ y_{i,2} & x'_{i2} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ y_{i,T-1} & x'_{iT} \end{pmatrix}$$

$$\hat{X} = (y^{-1}, X), \quad \gamma'(\rho, \beta'), \quad W = (W'_1 W'_2 \dots, W'_N)$$

$$W_i^D = \begin{pmatrix} y_{i1}, x'_{i1}, x'_{i2} & 0 & \dots & 0 \\ 0 & (y_{i1}, y_{i2}, x'_{i1}, x'_{i2}, x'_{i3}) & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & (y_{i1}, y_{i2} \dots y_{i,T-2}, x'_{i1}, x'_{i2} \dots x'_{iT-1}) \end{pmatrix}$$

$$W_i^L = \begin{pmatrix} (dy_{i2}, dx'_{i2}, dx'_{i2}) & 0 & \cdot & 0 \\ 0 & (dy'_{i2}, dy'_{i3}, dx'_{i2}, dx'_{i3}, dx'_{i4}) & \cdot & 0 \\ 0 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & (dy'_{i2} \dots, dy_{i,T-2}, dx'_{i2} \dots, dx'_{iT-1}) \end{pmatrix}$$

$$W_i = \begin{pmatrix} W_i^D & 0 \\ 0 & W_i^L \end{pmatrix}$$

The first-step estimator makes use of a covariance matrix taking this auto-correlation into account enlarge for the level equations

$$V = W'GW = \sum_{i=1}^N W_i G_T W_i \quad (3.92)$$

Where $G = (I_N \otimes G^{D,L})$ and $G^D = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

$$G^L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$G^{D,L} = \begin{pmatrix} W_i^D & 0 \\ 0 & W_i^L \end{pmatrix}$$

The two-step GMM estimator uses the residuals of the first-step estimation to estimate the covariance matrix as suggested by White (1980).

$$\hat{V} = \sum_{i=1}^n W_i' F_T \hat{\varepsilon} \hat{\varepsilon}' F_T' W_i \quad (3.93)$$

The resulting estimator finally is

$$\hat{\gamma}^{GMM-SYS} = \left(XW' \hat{V}^{-1} W' X \right)^{-1} X' W \hat{V}^{-1} W' y. \quad (3.94)$$

3.17 M-Estimator

M-estimator was proposed by Huber (1973). M-estimation for regression is a relatively extension of M- estimation for location and scale. It represents one of the first attempts at a compromise between the efficiency of the Least Squares estimator and a resistance estimator (Least Absolute Value estimators).The M-estimator minimizes some function of the residuals.

If a normal distribution of the errors is assumed, the Maximum Likelihood Estimator of β is simply the Least Squares estimator found by minimizing the sum of squares function

$$\text{Min} \sum_{i=1}^n \left(y_i - \sum X_i^1 \beta \right)^2 = \text{Min} \sum_{i=1}^n (e_i)^2 \quad (3.95)$$

However from M-estimator, instead of minimizing the sum of squared residuals, a robust regression M-estimator minimizes a function of the residuals.

$$\text{Min} \sum_{i=1}^n \rho \left(y_i - \sum X_i^1 \beta \right) = \text{Min} \sum_{i=1}^n \rho(e_i) \quad (3.96)$$

Then the M-estimation function is

$$\text{Min} \sum_{i=1}^n \rho \left(\frac{y_i - \sum X_i^1 \beta}{t} \right) = \text{Min} \sum_{i=1}^n \rho \left(\frac{e_i}{t} \right) \quad (3.97)$$

Where t is an estimate of scale formed from linear combination of the residual and the function ρ gives the contribution of each residual to the M-estimation function.

In order to minimize (equation 3.85) to find an M-estimate, it requires partial differentiation with respect to each of the ρ parameters in turn, resulting in a system of ρ equations:

$$\sum_{i=1}^n X_{ik} \Psi \left(\frac{y_i - \sum X_i^1 \beta}{t} \right) = \sum_{i=1}^n x_i \Psi \left(\frac{e_i}{t} \right) = 0 \quad k = 1, \dots, \rho \quad (3.98)$$

Where $\Psi(u) = \frac{\partial \rho}{\partial u}$ is called the ‘Score’ function. A ‘weight’ function is then defined as

$$w(u) = \Psi(u)$$

Yielding $w_i = w \left(\frac{e_i}{t} \right)$ for $i, \dots, n$, with $w_i = 1$ if $e_i = 0$. Substituting into equation (3.98)

$$\begin{aligned} \sum_{i=1}^n x_i w_i \frac{e_i}{t} &= \sum_{i=1}^n x_{ik} w_i \left(y_i - \sum X_i^1 \beta \right) \frac{1}{t} = 0 \quad k = 1, \dots, \rho \\ \sum_{i=1}^n x_{ik} w_i \left(y_i - \sum X_i^1 \beta \right) &= 0 \quad k = 1, \dots, \rho \\ \sum_{i=1}^n x_{ik} w_i x_i \beta &= \sum_{i=1}^n x_{ik} w_i y_i \quad k = 1, \dots, \rho \end{aligned} \quad (3.99)$$

Since $t \neq 0$, defining the weight matrix $W = \text{diag} \{ (w_i : i = 1, \dots, n) \}$ as follows

$$W = \begin{pmatrix} w_1 & & & 0 \\ & w_2 & & \\ & & \ddots & \\ 0 & & & w_n \end{pmatrix}$$

yielding the following matrix form of equation (3.87)

$$X^1 W X \beta = X^1 W Y$$

$$\hat{\beta} = (X^1 W X)^{-1} X^1 W Y \quad (3.100)$$

This equation is similar to the solution for the least squares estimator but with the introduction of a weight matrix to reduced the influence of outliers. Therefore, since Weight (W) depends on the residuals, the residuals depend on the estimated coefficient and the estimated coefficients depend upon the weight. An Iterative solution called Iteratively Reweighted Least Squares (IRLS) is therefore employed.

By setting the iteration at $I = 0$, the least squares method is used to fit the data, yielding the initial estimates of the regression coefficient $\beta^{(0)}$.

The residuals are extracted from the preliminary least squares estimator, $e_i^{(0)}$ and used to calculate initial estimates for the weights.

A weight function is then chosen and applied to the initial least squares residuals to create preliminary weights $w_i^{(0)}$.

Setting $I = 1$, using $w^{(0)}$ in equation (3.100), then the estimate $\beta^{(1)}$ is obtained in matrix form

$$\beta^{(1)} = (X^1 W^{(0)} X)^{-1} X^1 W^{(0)} Y$$

The process continues by using the residuals from the initial Weight Least Squares (WLS) to calculate new weights, $w^{(1)}$

If I is set at 2, A new estimate for $\beta^{(2)}$ is found using $w^{(1)}$ and the next weight matrix $w^{(2)}$ are also found.

This process is now iterated such that at $I = m$.

$$\hat{\beta}^{(m)} = (X^1 W^{(m-1)} X)^{-1} X^1 W^{(m-1)} Y \text{ Where } W^{(m-1)} = \text{diag} \{ w_i^{(m-1)} \}$$

Until the estimate of β converge, at which point the final M-estimate has been found. However, M-estimator are not robust to x-axis outlier data values, their breakdown point is $1/n$.

3.18 MM-Estimator

MM estimation was introduced by Yohai (1987). It combines a high breakdown point with a good efficiency (approximately 95%) relative to the Ordinary Least Square under the Gauss Markov assumptions. The MM refers to the fact that Multiple M-estimation procedure is used to calculate the final estimate. It has also become most common robust regression technique for linear regression. MM estimator has five step procedures. The procedures are

The first step is find an initial estimate of the coefficients $\beta^{(1)}$ and corresponding $e_i^{(1)}$ are taken from a highly resistant regression (that is, a regression with a breakdown point of 50%.

The estimator must be consistent but not necessary to be efficient. Then S-estimation with Huber or bisquare weights which is in form of M-estimation is employed.

Step 2: By using the residuals $e_i^{(1)}$ from the initial estimation at step 1 are used to compute an M-estimation of the scale of the residual, $\hat{\sigma}_e$.

Step 3: The MM-estimator is now defined as M-estimator of estimate coefficient (β) using a redescending score function, $\Psi_1(u) = \frac{\partial \rho(u)}{\partial u}$, and the scale estimate $\hat{\sigma}_e$ obtained from step 2.

Therefore, an MM-estimator $\hat{\beta}$ is defined as a solution to

$$\sum_{i=1}^n x_{ik} \Psi_i \left(\frac{y_i - x_i^1}{\hat{\sigma}_e} \right) = 0 \quad k = 1, \dots, \rho \quad (3.101)$$

Step 4: The new weights are calculated $w_i^{(2)}$ using the residual from the initial Weight Least Squares.

Step 5: The measure of the scale of the residuals are kept constant from step 2, step 3 and step 4 are continually reiterated until convergence.

3.19 The Proposed Modified Estimator

The model under consideration is the same as equation (3.12)

For simplicity, considering $\beta = 0$

$$y_{it} = \delta y_{i,t-1} + \mu_i + v_{it} \quad (3.102)$$

Subtracting $y_{i,t-1}$, to get

$$\Delta y_{i,t} = \delta \Delta y_{i,t-1} + \Delta v_{it} \quad (3.103)$$

Firstly, equation (3.103) was differentiated to eliminate the individual effects.

The first period for which this relationship was observed is when $t=3$.

$$\Delta y_{i,3} = \delta \Delta y_{i,2} + \Delta v_{i,3}$$

In this case y_{i1} is valid instrument: that is correlated with

$$\Delta y_{i,2} \equiv y_{i,2} - y_{i,1} \text{ but not with } \Delta v_{i,3} \equiv v_{i,3} - v_{i,2}$$

For $t = 4$, the equation becomes

$$\Delta y_{i,4} = \delta y_{i,3} + \Delta v_{i,4}$$

But in the case y_{i2} and y_{i1} are valid instruments

For $t=5$, the equation is

$$\Delta y_{i,5} = \delta y_{i,4} + \Delta v_{i,5}$$

And the instruments y_{i3} and y_{i2} are available.

Using this logic, the valid instruments for period T are $y_{i1}, y_{i2}, \dots, y_{i,T-2}$

The instrumented equation is $W'Fy = W'Fx + W'Fv$

$$\text{where } X_i = \begin{bmatrix} y_{i2} - y_{i1} & x'_{i3} - x'_{i2} \\ y_{i3} - y_{i2} & x'_{i4} - x'_{i3} \\ \vdots & \vdots \\ y_{iT-1} - y_{iT-2} & x'_{iT} - x'_{iT-1} \end{bmatrix}$$

$$X = [y_{-1}, X] \hat{\gamma} = (\delta, \beta') W = [W'_1, W'_2, \dots, W'_N]$$

For each individual, define W_i as follows

$$W_i = \begin{bmatrix} [y_{i1}] & \dots & \dots & 0 \\ 0 & [y_{i1}, y_{i2}] & & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & [y_{i1}, \dots, y_{iT-2}] \end{bmatrix} \text{ is } (T-2) \times (T-2)(T-1)/2 \text{ matrix.}$$

$$y_i = \begin{bmatrix} y_{i,4} - y_{i,3} \\ y_{i,5} - y_{i,4} \\ \vdots \\ y_{iT} - y_{iT-1} \end{bmatrix}$$

$$V = W'GW = \sum_{i=1}^N W_i' G_T W_i$$

$$\text{Where } G = (I_N \otimes G_T) = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \text{ Premultiplying the matrix } F \text{ results in transforming}$$

the original observations into differences. Because $\text{Var}(Fu) = F\sigma^2 F'$, the covariance matrix $V = FF'$ is used as a first step approximation to the covariance matrix.

The two-step GMM estimator uses the residuals of the first-step estimation to estimate the covariance matrix as suggested by White (1980):

$$\hat{V} = \sum_{i=1}^N W_i' F_T v_i v_i' F_T W_i \quad (3.104)$$

The resulting estimator finally is

$$\hat{\gamma} = \left(X'W\hat{V}WX \right)^{-1} X'WV^{-1}Wy \quad (3.105)$$

Integrating the blinded model/ estimation with the instrument form

$$\hat{\gamma} = \left(X'PX \right)^{-1} X'Py, \text{ where } P = Z(Z'Z)^{-1}Z'$$

Premultiply to get and replacing in the formula for A-H estimator

$$\hat{\gamma} = \left(X'PWV^{-1}WPX \right)^{-1} X'PWV^{-1}WP_y \quad (3.106)$$

3.20 The Monte Carlo approach

The Monte Carlo method (or the Monte Carlo simulation) can be used to describe any technique that approximates solutions to quantitative problems through statistical sampling. Monte Carlo simulation is more specifically used to describe a method for propagating uncertainties. It relies on the process of implicitly representing uncertainties by specifying inputs as probability distributions. In econometrics, while asymptotic properties of estimators obtained by various econometric techniques are deduced from postulates or self-evident assumptions an approach that is often described as analytical, small sample properties of such estimators have always been studied from simulated data in what are known as Monte Carlo studies. The use of this approach is due to the fact that the real life observations on economic variables are in most cases plagued by one or all of the problems of non-spherical disturbances and measurement errors.

3.21 The design of the sampling experiments

For simulation study, Monte-Carlo experiment was used to simulate and estimate the parameters with behaviour of different estimators under different scenarios. The values of the data set as well as various key parameters are varied in the simulation setting. The design of the Monte-Carlo experiments closely follows Nerlove (1971) and Arellano- Bond (1991). The differences are that the error term v_{it} is assumed to be correlated and different generating mechanism of error term is considered (i.e. AR (1), MA (1), ARMA (1, 1)), this study modifies the parameter values of autoregressive coefficients by varying it to be 0.2, 0.5 and 0.9. The parameter of exogenous variable is held fixed throughout the entire sets of experiments. However, cross-section units and time periods were considered. The simulation is based on the model in equations (3.12) and (3.13).

3.22 Choice of parameter and specification of explanatory variables

The row vector explanatory variable x'_{it} is generated by the process: $X_{it} = \lambda x_{i,t-1} + \varepsilon_{it}$, where ε_{it} is uniformly distributed in the interval $[-0.5, 0.5]$. For the random effect specification, we generate $u_{it} = \mu_i + v_{it}$ where $\mu_i \sim N(0, \sigma^2_\mu)$, σ^2_μ is normalized to unity and error term, v_{it} is generated by incorporating various degrees of autocorrelation.

Autocorrelation of the disturbance term is specified as:

$$\text{AR (1):} \quad v_{it} = \rho v_{i,t-1} + \omega_{it}$$

$$\text{MA (1):} \quad v_{it} = \omega_{it} + \theta \omega_{i,t-1}$$

$$\text{ARMA (1, 1):} \quad v_{it} = \rho v_{i,t-1} + \omega_{it} + \theta \omega_{i,t-1}$$

Where ω_{it} is assumed to be independently and identically normally distributed with mean zero and variance σ^2_ω .

The parameter choices are summarized as follows: δ takes the values 0.3, 0.5, 0.7, the autocorrelation coefficients ρ and θ takes the values 0.2, 0.5, 0.9 and the variance of the disturbance term, σ^2_ω is normalized to one. While generating data for x , λ takes the values 0.3, 0.5 and 0.7. The parameter of exogenous variable was chosen to be fixed at one ($\beta = 1$) throughout the simulation experiments.

The following Dynamic Panel Data estimators were considered: Ordinary Least Square (OLS) estimator; Anderson-Hsiao (AH (d)) estimator; First step Arellano-Bond Generalized Method of Moment (ABGMM1) estimator; Blundell-Bond first step system (SSY1) estimator; M estimator; MM estimator and the proposed modified estimator (P-estimator).

Cross-sectional units (n) were chosen to be 10, 20, 50, 100 and 200 and the time periods, T to be 5, 10, 15 and 20 for each combination of n and T . 1000 replication was conducted for the simulation. In the simulation study, the initial observations x_{i0} and y_{i0} was set to zero and discard the first observations before choosing the appropriate samples. The exclusion of initial observations avoids an unnecessary impact of the initial values on ε_{it} and y_{it} . x_{it} is a covariance stationary process and thus y_{it} is also stationary if $|\delta| < 1$.

The estimators were ranked from 1 to 7 with rank 1 having lowest value of the absolute bias and root mean square error which are regarded as best estimator. A rank 2 is assigned to the second best estimator, rank 3 is assigned to the third best estimator, and rank 4 is assigned to the fourth best estimator and so on.

The whole Monte Carlo experiments were carried out in the environment of R Statistical package which is downloadable freely on www.cran.org.

The part of the code used is given as

```
cat(c("Job started at:",date()),fill=TRUE)
library(plm)
library(quantmod)
library(mvtnorm)
data(EmplUK)
myd<- EmplUK
#####
nsim=1000
#####
# ORDINARY LEAST SQUARES ESTIMATOR#
#####
y=myd[,4]
x1=myd[,5]
ylag1<- Lag(y,k=1)
ylag1[ylag1 %in% NA]=0
y1<-ylag1
#unname(y1)
X<-cbind(1,x1,y1)
N<- length(y)
a=array(0,dim=c(nsim,3))
#LEAST SQUARES ESTIMATOR
for (i in 1:nsim) {
  Y=y+rnorm(N)
  bet<- solve(t(X)%*%X)%*%t(X)%*%Y
  a[i,] = t(bet)
}
##### OLS Results#####
mols=apply(a,2,mean)
vols=apply(a,2,sd)
```

```

ols<-solve(t(X)%*%X)%*%t(X)%*%y
bias1=mols-t(ols)
mse1=bias1^2+ vols
rmse1<-sqrt(mse1)
ols1<-(rbind(mols, bias1,vols,mse1,rmse1))
#####
#####Arellano and Bond (1991) one step.#####
#####
c=array(0,dim=c(nsim,10))
for (i in 1:nsim) {
  Y=y+rnorm(N )
  areb<- pgmm(Y ~ lag(emp, 1:2) + lag(wage,0:1) | lag(emp,2:99),
data =EmplUK, effect = "twoways", model = "onestep")

  c[i,] = coef(areb)
}
#####Results#####
  mbc=apply(c,2,mean)
  vbc=apply(c,2,var )
arebc<- pgmm(emp ~ lag(emp, 1:2) + lag(wage,0:1) |
  lag(emp,2:99), data =EmplUK, effect = "twoways", model = "onestep")
  biasc=mbc-coef(arebc)
msec=biasc^2+vbc
rmsec<-sqrt(msec)
arella<-(cbind(mbc,biasc,msec,rmsec))
#####
##Blundell and Bond (1998):onestep#####
#####
e=array(0,dim=c(nsim,11))
for (i in 1:nsim) {
  Y=y+rnorm(N*T)
  blunde <- pgmm(Y ~ lag(emp, 1:2) + lag(wage,0:1) |
  lag(emp,2:99), data =EmplUK, effect = "twoways", model = "onestep",

```

```

transformation = "ld")
e[i,] = coef(blunde)
}
#####Results#####
mbe=apply(e,2,mean)
ve1=apply(e,2,var)
blundel<- pgmm(emp ~ lag(emp, 1:2) + lag(wage,0:1) |
               lag(emp,2:99), data =EmplUK, effect = "twoways", model = "onestep",
               transformation = "ld")
               biase=mbe-coef(blundel)
               msee=biase^2+ve1
               rmsee<-sqrt(msee)
blun<-(cbind(mbe,biase,msee,rmsee))
#####
###Anderson-Hsiao IV estimators in Difference DATA#####
#####
wd<-cbind(1,yb,xa)
y2t<-Lag(y,k=3)-Lag(y,k=4)
yt<- yc-y2t ; yt[yt %in% NA]=0
zd<-cbind(1,yt,xa)
hd=array(0,dim=c(nsim,3))
for (i in 1:nsim) {
  Y=ya+rnorm(N*T)
  ahd<- solve(t(z)%*%w)%*%t(z)%*%Y
  hd[i,] = ahd
}

mbh=apply(hd,2,mean)
vh1=apply(hd,2,var)
AHd<-solve(t(zd)%*%wd)%*%t(zd)%*%ya
gd<-t(AHd)
unnname(gd)
biash=mbh-gd

```

```

mseh=biash^2+vh1
rmseh<-sqrt(mseh)
anderd<-(rbind(mbh ,biash,mseh,rmseh))

#####
#
write.csv(cbind(ols1),"uthm1.csv") ## OLS
write.csv(cbind(arella),"uthm2.csv") ##Arelona Bond one step
write.csv(cbind(blun),"uthm3.csv") ## Blundel one step
write.csv(cbind(anderd),"uthm4.csv") ## Anderson difference
cat(c("Job stopped at:",date()),fill=TRUE)

```

3.23 Sampling properties of the estimators

In this study, the absolute bias and Root Mean Square Error criteria were employed to evaluate the performance of the estimators.

Bias: The bias of an estimator is the difference between the estimator's expected value and the true value of the parameter being estimated. The bias of an estimator (relative to the parameter β) is defined to be:

$$Bias[\hat{\beta}] = E[\hat{\beta}] - \beta = E[\hat{\beta} - \beta]$$

$$Absolute\ Bias = E[\hat{\beta} - \beta]$$

Mean Squared Error: The mean squared Error (MSE) of an estimator measures the average of the squares of the errors, that is, the difference between the estimator and what is estimated. The MSE of an estimator $\hat{\beta}$ with respect to the unknown parameter is defined as

$$MSE = E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)]$$

$$= Var(\hat{\beta}) + Bias$$

Therefore, Root Mean Squared Error

$$RMSE = \sqrt{Var(\hat{\beta}) + (Bias)^2}$$

CHAPTER FOUR

PRESENTATION AND DISCUSSION OF RESULTS

4.1 Introduction

The results of the investigations of the finite properties of seven dynamic panel data model estimators with autocorrelated error terms are reported in this chapter. 1000 replications (R) of the experiments are performed on cross-sections, $n= 10, 20, 50, 100$ and 200 and time periods, $T= 5, 10, 15$ and 20 , for different degrees of autocorrelation and coefficient values ρ and θ : $0.2, 0.5$ and 0.9 . The parameter of lagged dependent variable, δ and lagged exogenous variable, λ were specified as $0.3, 0.5$ and 0.7 .

Tables 4.1 to 4.4 show the simulation result of bias and RMSE of δ and β when $N=10$ and $T=5$ when the design of serial correlation assumes MA (1). The simulation results of the models were presented in Tables 4.1 to 4.4.

The summary and the ranking of the results obtained for the model, estimator types, degree of autocorrelation of the error terms and each lagged variables are presented below in Table (4.5) to (4.20)

TABLE 4.1: Absolute Bias of various Estimators when n=10 and T=5 for δ

			MA(1) ABSOLUTE BIAS (δ)			T=5	n=10		
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.053389	0.010639	0.01211	0.025929	0.017856	0.000303	0.000295
		0.5	0.057173	0.005626	0.017409	0.137612	0.017491	0.019469	0.004524
		0.9	0.060468	0.001287	0.016046	0.090396	0.041595	0.030969	0.001185
	0.5	0.2	0.055971	0.010637	0.002557	0.01681	0.030388	0.00811	0.002127
		0.5	0.059616	0.005588	0.012872	0.086408	0.000233	0.015479	0.000157
		0.9	0.062713	0.001224	0.014325	0.096013	0.030609	0.03031	0.001152
	0.7	0.2	0.057844	0.010603	0.003335	0.010361	0.014279	0.014591	0.010252
		0.5	0.061378	0.005538	0.009793	0.12091	0.014013	0.015895	0.005112
		0.9	0.064302	0.001168	0.013281	0.101922	0.04207	0.029395	0.001112
0.5	0.3	0.2	0.056155	0.005871	0.014879	0.612642	0.012753	0.015627	0.004522
		0.5	0.059493	0.002081	0.017923	0.102066	0.02504	0.030061	0.001521
		0.9	0.062252	0.00069	0.016746	0.080697	0.041997	0.032686	0.000452
	0.5	0.2	0.058658	0.005907	0.010563	0.580997	0.007919	0.009217	0.003206
		0.5	0.061789	0.002104	0.015783	0.112611	0.038379	0.026709	0.002051
		0.9	0.064311	0.000663	0.016036	0.08187	0.041944	0.032837	0.000521
	0.7	0.2	0.060458	0.005937	0.007605	1.701769	0.017633	0.000638	0.000428
		0.5	0.06341	0.002129	0.01422	0.122027	0.03788	0.024428	0.002012
		0.9	0.065722	0.000632	0.015532	0.083277	0.047303	0.030631	0.000412
0.7	0.3	0.2	0.057926	0.002379	0.01403	0.101619	0.020944	0.023008	0.002154
		0.5	0.060891	0.000161	0.016692	0.084509	0.034388	0.032397	0.000125
		0.9	0.063269	0.00173	0.016054	0.080917	0.051174	0.031222	0.00128
	0.5	0.2	0.060327	0.002411	0.011968	0.107977	0.024117	0.01942	0.002311
		0.5	0.063054	0.000105	0.015685	0.087093	0.024169	0.031826	0.000099
		0.9	0.065182	0.001654	0.015866	0.080777	0.050366	0.031485	0.001154
	0.7	0.2	0.062025	0.002455	0.010555	0.112164	0.008975	0.015828	0.002123
		0.5	0.064542	4.29E-05	0.014917	0.089858	0.03394	0.030638	0.000035
		0.9	0.066449	0.001578	0.015687	0.080759	0.047646	0.031419	0.001352

TABLE 4.2: Absolute Bias of Various Estimators when n=10 and T=5 for β

			MA(1) ABSOLUTE BIAS (β)			T=5	n=10		
λ	δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.027254	0.025079	0.037046	0.897182	0.019273	0.001683	0.001551
		0.5	0.026474	0.023044	0.030842	4.021409	0.000604	0.00727	0.000425
		0.9	0.025962	0.019726	0.026684	1.661634	0.010848	0.0125	0.001525
	0.5	0.2	0.025765	0.024613	0.046006	0.505319	0.024729	0.005045	0.004525
		0.5	0.025045	0.02255	0.041588	2.579974	0.007484	0.004928	0.004315
		0.9	0.024578	0.019299	0.037678	1.781536	0.008052	0.011353	0.004124
	0.7	0.2	0.023964	0.024152	0.049239	0.629029	0.014732	0.007185	0.005425
		0.5	0.023304	0.022053	0.048188	3.507057	0.01325	0.00724	0.005226
		0.9	0.022884	0.018864	0.046088	1.888841	0.01344	0.01033	0.005124
0.5	0.3	0.2	0.026786	0.023318	0.03237	17.56395	0.002663	0.005091	0.004916
		0.5	0.026168	0.020403	0.025998	2.03326	0.004691	0.012872	0.004962
		0.9	0.025789	0.016691	0.021588	1.172894	0.014082	0.013426	0.004962
	0.5	0.2	0.025343	0.02272	0.042419	16.50565	0.01165	0.002179	0.003542
		0.5	0.024777	0.019853	0.037196	2.256472	0.009259	0.009926	0.005125
		0.9	0.024431	0.016253	0.032618	1.201691	0.014935	0.012591	0.001249
	0.7	0.2	0.023586	0.022114	0.048271	46.67418	0.015091	4.75E-05	0.000045
		0.5	0.023072	0.019297	0.045557	2.428921	0.011052	0.008391	0.007321
		0.9	0.022761	0.015812	0.042031	1.223787	0.014401	0.013676	0.012175
0.7	0.3	0.2	0.0266	0.021022	0.029738	2.062658	0.002157	0.008653	0.002075
		0.5	0.026082	0.017762	0.023057	1.323824	0.011112	0.013299	0.010015
		0.9	0.02577	0.014118	0.018544	0.977146	0.013282	0.012694	0.011654
	0.5	0.2	0.025187	0.020382	0.039789	2.173822	0.005054	0.006542	0.005045
		0.5	0.024712	0.017211	0.033959	1.368103	0.007859	0.012229	0.005358
		0.9	0.024425	0.013698	0.028984	0.985933	0.017266	0.011987	0.010585
	0.7	0.2	0.023458	0.019728	0.046665	2.21336	0.001087	0.00479	0.001053
		0.5	0.023026	0.016654	0.04279	1.402541	0.011064	0.011027	0.010025
		0.9	0.022764	0.013276	0.038424	0.988721	0.015501	0.011222	0.010022

TABLE 4.3: RMSE of Various Estimators when n=10 and T=5 for δ

MA(1) RMSE (δ) T=5, n=10									
Λ	δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-Est
0.3	0.3	0.2	0.419941	0.276676	0.463063	0.131645	0.327931	0.316082	0.28762
		0.5	0.437039	0.243886	0.322433	0.188089	0.329379	0.319226	0.25726
		0.9	0.451934	0.207021	0.228301	0.156395	0.343256	0.327945	0.23726
	0.5	0.2	0.437279	0.273968	0.443274	0.130087	0.348499	0.325602	0.23025
		0.5	0.452943	0.241836	0.316051	0.154606	0.348656	0.327944	0.22011
		0.9	0.466481	0.2055	0.226366	0.159699	0.360219	0.337344	0.22004
	0.7	0.2	0.452913	0.270904	0.419248	0.129324	0.365477	0.333699	0.23021
		0.5	0.467414	0.23959	0.306701	0.176218	0.365105	0.337216	0.22001
		0.9	0.47981	0.203947	0.222424	0.163313	0.376392	0.345021	0.21099
0.5	0.3	0.2	0.434507	0.247408	0.320533	0.625923	0.329227	0.31844	0.25072
		0.5	0.45047	0.214912	0.246155	0.163517	0.339329	0.321816	0.24125
		0.9	0.46341	0.182001	0.187006	0.150739	0.354139	0.336831	0.23103
	0.5	0.2	0.451551	0.2455	0.312088	0.594981	0.349161	0.327674	0.25096
		0.5	0.465911	0.213241	0.242976	0.170295	0.358496	0.334853	0.25001
		0.9	0.477412	0.180499	0.18629	0.151357	0.372343	0.346918	0.24071
	0.7	0.2	0.467045	0.243331	0.301385	1.706593	0.366133	0.338074	0.24511
		0.5	0.480034	0.211478	0.237806	0.176661	0.374199	0.342494	0.23072
		0.9	0.490271	0.179051	0.184005	0.152112	0.387845	0.355174	0.22108
0.7	0.3	0.2	0.444635	0.220118	0.244519	0.16328	0.336259	0.324214	0.23018
		0.5	0.459228	0.190988	0.198618	0.15292	0.348858	0.333499	0.22185
		0.9	0.470529	0.162099	0.157875	0.150662	0.363511	0.346567	0.21007
	0.5	0.2	0.461566	0.218572	0.239488	0.167308	0.354198	0.333814	0.22532
		0.5	0.474448	0.189444	0.196626	0.154356	0.365822	0.343865	0.21523
		0.9	0.48425	0.160554	0.157672	0.150566	0.383066	0.356924	0.21255
	0.7	0.2	0.477015	0.216848	0.233017	0.170035	0.370019	0.341923	0.217845
		0.5	0.488394	0.18788	0.193188	0.155925	0.382363	0.352497	0.20769
		0.9	0.496848	0.159122	0.156225	0.150538	0.397723	0.365312	0.19956

TABLE 4.4: RMSE of Various Estimators when n=10 and T=5 for β

MA(1) RMSE (β) T=5, n=10									
λ	δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.250504	0.263805	0.396458	1.187268	0.218905	0.219924	0.20545
		0.5	0.250419	0.243285	0.40671	4.046063	0.218397	0.221104	0.20541
		0.9	0.249421	0.213854	0.412945	1.684626	0.226237	0.22665	0.20541
	0.5	0.2	0.244276	0.263418	0.365716	0.927186	0.214277	0.20894	0.20325
		0.5	0.244163	0.2429	0.378143	2.619591	0.213576	0.209596	0.20321
		0.9	0.243211	0.213457	0.386378	1.803927	0.220579	0.215022	0.20202
	0.7	0.2	0.236281	0.262838	0.339085	0.98524	0.208637	0.198325	0.19321
		0.5	0.236146	0.242349	0.351579	3.536562	0.207974	0.202823	0.19309
		0.9	0.235255	0.21294	0.360527	1.910593	0.214198	0.20374	0.19201
0.5	0.3	0.2	0.250293	0.243275	0.403526	17.56959	0.218077	0.220607	0.21645
		0.5	0.249643	0.220931	0.413439	2.056303	0.224667	0.219876	0.20546
		0.9	0.248274	0.192265	0.419289	1.192488	0.235427	0.23356	0.20523
	0.5	0.2	0.24396	0.242919	0.374743	16.51176	0.214095	0.209383	0.20345
		0.5	0.2434	0.220416	0.386261	2.278197	0.219444	0.213624	0.20021
		0.9	0.242188	0.191676	0.393326	1.221834	0.228925	0.222047	0.190552
	0.7	0.2	0.23588	0.242376	0.34847	46.67633	0.208922	0.198613	0.19005
		0.5	0.235407	0.219761	0.36023	2.449666	0.213424	0.202511	0.19001
		0.9	0.234353	0.190999	0.367141	1.244405	0.22145	0.213908	0.18525
0.7	0.3	0.2	0.249546	0.221652	0.407445	2.085189	0.22319	0.223804	0.18053
		0.5	0.248602	0.199618	0.417262	1.344188	0.231488	0.230809	0.18051
		0.9	0.247087	0.172888	0.422379	0.992766	0.242558	0.240554	0.18051
	0.5	0.2	0.243182	0.221289	0.380186	2.195701	0.217791	0.212712	0.18051
		0.5	0.242413	0.199042	0.391037	1.388721	0.225521	0.219665	0.18025
		0.9	0.241123	0.172231	0.396885	1.002273	0.236813	0.228996	0.18002
	0.7	0.2	0.235091	0.220753	0.354725	2.234975	0.211856	0.202039	0.18002
		0.5	0.234482	0.198348	0.365306	1.423317	0.219226	0.208746	0.18002
		0.9	0.23341	0.171505	0.370501	1.005782	0.228171	0.217503	0.18001

Table 4.5: Best Estimator using Absolute bias Criterion at various combinations of n and T for δ when the true model is AR (1)

n	δ	p/T	5	10	15	20
10	0.3	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	AH(d)
	0.5	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	AH(d)
	0.7	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	AH(d)
20	0.3	0.2	MM	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	MM	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
50	0.3	0.2	P-est.	AH(d)	AH(d)	AH(d)
		0.5	P-est.	AH(d)	AH(d)	AH(d)
		0.9	P-est.	AH(d)	AH(d)	AH(d)
	0.5	0.2	P-est.	AH(d)	AH(d)	AH(d)
		0.5	P-est.	AH(d)	AH(d)	AH(d)
		0.9	P-est.	AH(d)	AH(d)	AH(d)
	0.7	0.2	P-est.	AH(d)	AH(d)	AH(d)
		0.5	P-est.	AH(d)	AH(d)	AH(d)
		0.9	P-est.	AH(d)	AH(d)	AH(d)
100	0.3	0.2	P-est.	P-est.	P-est.	ABGMM
		0.5	P-est.	P-est.	P-est.	ABGMM
		0.9	P-est.	P-est.	P-est.	ABGMM
	0.5	0.2	P-est.	P-est.	P-est.	ABGMM
		0.5	P-est.	P-est.	P-est.	ABGMM
		0.9	P-est.	P-est.	P-est.	ABGMM
	0.7	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	ABGMM
200	0.3	0.2	ABGMM	ABGMM	ABGMM	ABGMM
		0.5	ABGMM	ABGMM	ABGMM	ABGMM
		0.9	SYS1	ABGMM	ABGMM	ABGMM
	0.5	0.2	ABGMM	ABGMM	ABGMM	ABGMM
		0.5	ABGMM	ABGMM	ABGMM	ABGMM
		0.9	SYS1	ABGMM	ABGMM	ABGMM
	0.7	0.2	ABGMM	ABGMM	ABGMM	ABGMM
		0.5	ABGMM	ABGMM	ABGMM	ABGMM
		0.9	SYS1	ABGMM	ABGMM	ABGMM

From table 4.5, the result showed that the proposed modified estimator performed better than all other estimators when the time periods is 5, 10 and 15 while AH (d) estimator also performed relatively well for $T=20$ when the cross sectional dimension is 10. The proposed modified estimator performed better than other estimators for all the time periods, except when $T=5$, while MM estimator also performed better when the degree of autocorrelation is low and high. When $n=50$, the proposed modified estimator outperformed all other estimators when the time period is 5 while AH (d) estimator performed best as the time period increases. But as n increases to 100, the proposed modified estimator exhibit the best performance at all the time periods, except when $T=20$, where ABGMM performed relatively well. ABGMM performance is more pronounced when $n=200$ for all time periods while SYS1 performs better than all other estimators when the degree of autocorrelation is high when $T=5$.

Table 4.6: Best Estimator using Absolute bias Criterion at various combinations of n and T for β when the true model is AR (1)

n	δ	ρ/T	5	10	15	20
10	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
20	0.3	0.2	P-est.	ABGMM	ABGMM	P-est.
		0.5	P-est.	ABGMM	ABGMM	P-est.
		0.9	M	ABGMM	ABGMM	P-est.
	0.5	0.2	P-est.	ABGMM	ABGMM	P-est.
		0.5	P-est.	ABGMM	ABGMM	P-est.
		0.9	P-est.	ABGMM	ABGMM	P-est.
	0.7	0.2	P-est.	ABGMM	ABGMM	P-est.
		0.5	P-est.	ABGMM	ABGMM	P-est.
		0.9	P-est.	ABGMM	ABGMM	P-est.
50	0.3	0.2	P-est.	M	P-est.	P-est.
		0.5	P-est.	M	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	M	P-est.	P-est.
		0.5	P-est.	M	P-est.	P-est.

		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	MM	P-est.	P-est.
		0.5	P-est.	M	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
100	0.3	0.2	M	P-est.	P-est.	P-est.
		0.5	ABGMM	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	M	P-est.	P-est.	P-est.
		0.5	MM/ABGMM	P-est.	P-est.	P-est.
		0.9	ABGMM	P-est.	P-est.	P-est.
	0.7	0.2	M	P-est.	P-est.	P-est.
		0.5	MM	P-est.	P-est.	P-est.
		0.9	ABGMM	P-est.	P-est.	P-est.
200	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	MM	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	ABGMM	P-est.	P-est.	P-est.
		0.5	ABGMM	P-est.	P-est.	P-est.
		0.9	ABGMM	P-est.	P-est.	P-est.
	0.7	0.2	MM	P-est.	P-est.	P-est.
		0.5	ABGMM	P-est.	P-est.	P-est.
		0.9	ABGMM	P-est.	P-est.	P-est.

From table 4.6, the results showed that when n is small, the proposed modified estimator outperformed all other estimators, except when $n=20$, ABGMM estimator performed better than all other estimators when the time periods is 10 and 15. But n increased to 50, the proposed modified estimator outperformed all other estimators when the time periods is 5, 10

and 20 while the robust estimators (M and MM) performed relatively well when the degree of autocorrelation is low and moderate for $T=10$. As n increases, the proposed modified estimator performed better than all other estimators when the time period is greater than 5 for all the degrees of autocorrelation while ABGMM estimator and the robust estimators (M and MM) performed better when the time period is 5.

Table 4.7: Best Estimator using RMSE Criterion at various combinations of n and T for δ when the true model is AR (1)

n	Δ	ρ/T	5	10	15	20
10	0.3	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.5	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.7	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
20	0.3	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.7	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
50	0.3	0.2	AH(d)	AH(d)	AH(d)	ABGMM
		0.5	AH(d)	AH(d)	AH(d)	ABGMM
		0.9	P-est.	AH(d)	AH(d)	ABGMM
	0.5	0.2	AH(d)	AH(d)	AH(d)	ABGMM
		0.5	AH(d)	AH(d)	AH(d)	ABGMM
		0.9	AH(d)	AH(d)	AH(d)	ABGMM
	0.7	0.2	AH(d)	AH(d)	AH(d)	ABGMM
		0.5	AH(d)	AH(d)	AH(d)	ABGMM
		0.9	AH(d)	AH(d)	AH(d)	ABGMM

100	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.

	0.7	0.9	P-est.	P-est.	P-est.	P-est.
		0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
200	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.

From table 4.7, the simulation result showed that when n is small (10 and 20), AH (d) estimator outperformed all other estimators when the time period is small, but as T increases, the proposed modified estimator performed better than all other estimators. But when $n=50$, AH (d) outperformed all other estimators when the time periods is 5, 10 and 15 while ABGMM also performed best at $T=20$. When n is large, the proposed modified estimator performed quite well than all other estimators for all time periods and all the degrees of autocorrelation.

Table 4.8: Best Estimator using RMSE Criterion at various combinations of n and T for β when the true model is AR (1)

n	δ	ρ/T	5	10	15	20
10	0.3	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
	0.5	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
	0.7	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
20	0.3	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
	0.5	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
	0.7	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
50	0.3	0.2	MM	P-est.	P-est.	P-est.
		0.5	MM	P-est.	P-est.	P-est.
		0.9	MM	P-est.	P-est.	P-est.
	0.5	0.2	MM	P-est.	P-est.	P-est.
		0.5	MM	P-est.	P-est.	P-est.
		0.9	MM	P-est.	P-est.	P-est.
		0.2	MM	P-est.	P-est.	P-est.
		0.5	MM	P-est.	P-est.	P-est.
		0.9	MM	P-est.	P-est.	P-est.
100	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
200	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.9	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.

From table 4.8, the simulation result revealed that when n is small, the proposed modified estimator outperformed all other estimators when the time periods is 5, 15 and 20 while

ABGMM estimator also performed better than all other estimators when $T=10$ for the degrees of autocorrelation. When $n=50$, the MM estimator performed better than all other estimators when $T=5$ while the proposed modified estimator outperformed all other estimators when T increases for all the degrees of autocorrelation. When n is large, the proposed modified estimator performed better than all other estimators for all the time periods and all the degrees of autocorrelation.

Table 4.9: Best Estimator using Absolute bias Criterion at various combinations of n and T for δ when the true model is MA (1)

n	δ	Θ/T	5	10	15	20
10	0.3	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	AH(d)
	0.5	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	AH(d)
	0.7	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	AH(d)
20	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
50	0.3	0.2	P-est.	AH(d)	AH(d)	AH(d)
		0.5	P-est.	AH(d)	AH(d)	AH(d)
		0.9	P-est.	AH(d)	AH(d)	AH(d)
	0.5	0.2	P-est.	AH(d)	AH(d)	AH(d)
		0.5	P-est.	AH(d)	AH(d)	AH(d)
		0.9	P-est.	AH(d)	AH(d)	AH(d)
	0.7	0.2	P-est.	AH(d)	AH(d)	AH(d)
		0.5	P-est.	AH(d)	AH(d)	AH(d)
		0.9	P-est.	AH(d)	AH(d)	AH(d)
100	0.3	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	ABGMM
		0.9	P-est.	P-est.	P-est.	ABGMM
	0.5	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	ABGMM
		0.9	P-est.	P-est.	P-est.	ABGMM
	0.7	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	ABGMM
		0.9	P-est.	P-est.	P-est.	ABGMM
200	0.3	0.2	ABGMM	ABGMM	ABGMM	ABGMM
		0.5	ABGMM	ABGMM	ABGMM	ABGMM/M
		0.9	ABGMM	ABGMM	ABGMM	ABGMM
	0.5	0.2	ABGMM	ABGMM	ABGMM	ABGMM
		0.5	ABGMM	ABGMM	ABGMM	MM
		0.9	ABGMM	ABGMM	ABGMM	ABGMM
	0.7	0.2	ABGMM	ABGMM	ABGMM	ABGMM
		0.5	ABGMM	ABGMM	ABGMM	M
		0.9	ABGMM	ABGMM	ABGMM	M

From table 4.9, the result showed that when n is small, the proposed modified estimator outperformed all other estimators except when $n=20$ and $T=10$, AH (d) estimator performed better than all other estimators for all degrees of autocorrelation. When $n=50$, the proposed modified estimator outperformed all other estimators when T is small while the AH (d) estimator performed better than all other estimator when the time period is 10, 15 and 20. But n increases to 100, the proposed modified estimator outperformed all other estimators for the periods is 5, 10 and 15 while ABGMM and AH (d) estimator performed better than all other estimators for all the degrees of autocorrelation. When $n=200$, ABGMM outperformed all other estimators for time periods while the M and MM performed better in some cases when $T=20$.

Table 4.10: Best Estimator using Absolute bias Criterion at various combinations of n and T for β when the true model is MA (1)

n	δ	θ/T	5	10	15	20
10	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
20	0.3	0.2	P-est.	ABGMM	MM	P-est.
		0.5	P-est.	ABGMM	MM	P-est.
		0.9	P-est.	MM	MM	P-est.
	0.5	0.2	P-est.	ABGMM	MM	P-est.
		0.5	P-est.	ABGMM	MM	P-est.
		0.9	P-est.	ABGMM	MM	P-est.
	0.7	0.2	P-est.	ABGMM	MM	P-est.
		0.5	P-est.	ABGMM	MM	P-est.
		0.9	P-est.	ABGMM	MM	P-est.
50	0.3	0.2	P-est.	MM	MM	P-est.
		0.5	P-est.	MM	MM	P-est.
		0.9	P-est.	ABGMM/SYS1	ABGMM	P-est.
	0.5	0.2	P-est.	MM	MM	P-est.
		0.5	P-est.	ABGMM	ABGMM	P-est.

	0.7	0.9	P-est.	ABGMM	ABGMM	P-est.
		0.2	P-est.	MM	MM	P-est.
		0.5	P-est.	ABGMM	ABGMM	P-est.
		0.9	P-est.	ABGMM	ABGMM	P-est.
100	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
200	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.

From table 4.10, the simulation result showed that the proposed modified estimator outperformed all other estimators for all the sample sizes, except when $n=20$ and 50 and $T=10$ and 15 , the ABGMM and MM estimators performed better than all other estimators.

Table 4.11: Best Estimator using RMSE Criterion at various combinations of n and T for δ when the true model is AR (1)

n	δ	θ/T	5	10	15	20
10	0.3	0.2	ABGMM	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.5	0.2	ABGMM	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.7	0.2	ABGMM	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
20	0.3	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.5	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.7	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
50	0.3	0.2	AH(d)	AH(d)	AH(d)	ABGMM
		0.5	AH(d)	AH(d)	AH(d)	ABGMM
		0.9	AH(d)	AH(d)	AH(d)	ABGMM
	0.5	0.2	AH(d)	AH(d)	AH(d)	ABGMM
		0.5	AH(d)	AH(d)	AH(d)	ABGMM
		0.9	AH(d)	AH(d)	AH(d)	ABGMM
	0.7	0.2	AH(d)	AH(d)	AH(d)	ABGMM
		0.5	AH(d)	AH(d)	AH(d)	ABGMM
		0.9	AH(d)	AH(d)	AH(d)	ABGMM
100	0.3	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.5	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.7	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
200	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.

From table 4.11, the result showed that when $n=10$, ABGMM estimator outperformed all other estimators when the degree of autocorrelation is low while AH (d) estimator performed when the degree of autocorrelation is moderate and high when $T=5$. As T increases, the proposed modified estimator outperformed all other estimators. When $n=20$ and 100 , AH (d) estimator performed better than all other estimators when $T=5$. But as T increases, the proposed modified estimator outperformed all other estimators. When $n=50$, AH (d) estimator performed better than all other estimators when time periods is 5, 10 and 15 while ABGMM estimator performed better than all other estimators when the time period is 20 for all the degrees of autocorrelation. For sample size 200, the proposed modified estimator outperformed all other estimators for all degrees of autocorrelation.

Table 4.12: Best Estimator using RMSE Criterion at various combinations of n and T for β when the true model is AR (1)

n	δ	θ/T	5	10	15	20
10	0.3	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
	0.5	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
	0.7	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
20	0.3	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
	0.5	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
	0.7	0.2	P-est.	ABGMM	P-est.	P-est.
		0.5	P-est.	ABGMM	P-est.	P-est.
		0.9	P-est.	ABGMM	P-est.	P-est.
50	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
100	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
200	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.

From table 4.12, the simulation result revealed that the proposed modified estimator outperformed all other estimators for all the sample sizes, except when n is small and $T=10$, that ABGMM estimator performed better than all other estimators for all the degrees of autocorrelation.

Table 4.13: Best Estimator using Absolute bias Criterion at various combinations of n and T for δ when the true model is ARMA (1, 1)

n	δ	θ/T	5	10	15	20
10	0.3	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	AH(d)
	0.5	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	AH(d)
	0.7	0.2	P-est.	P-est.	P-est.	AH(d)
		0.5	P-est.	P-est.	P-est.	AH(d)
		0.9	P-est.	P-est.	P-est.	AH(d)
20	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
50	0.3	0.2	P-est.	AH(d)	AH(d)	AH(d)
		0.5	P-est.	AH(d)	AH(d)	AH(d)
		0.9	P-est.	AH(d)	AH(d)	AH(d)
	0.5	0.2	P-est.	AH(d)	AH(d)	AH(d)
		0.5	P-est.	AH(d)	AH(d)	AH(d)
		0.9	P-est.	AH(d)	AH(d)	AH(d)
	0.7	0.2	P-est.	AH(d)	AH(d)	AH(d)
		0.5	P-est.	AH(d)	AH(d)	AH(d)
		0.9	P-est.	AH(d)	AH(d)	AH(d)
100	0.3	0.2	P-est.	P-est.	P-est.	ABGMM
		0.5	P-est.	P-est.	P-est.	ABGMM
		0.9	P-est.	P-est.	P-est.	ABGMM
	0.5	0.2	P-est.	P-est.	P-est.	ABGMM
		0.5	P-est.	P-est.	P-est.	ABGMM
		0.9	P-est.	P-est.	P-est.	ABGMM
	0.7	0.2	P-est.	P-est.	P-est.	ABGMM
		0.5	P-est.	P-est.	P-est.	ABGMM
		0.9	P-est.	P-est.	P-est.	ABGMM
200	0.3	0.2	ABGMM	ABGMM	ABGMM	ABGMM

		0.5	ABGMM	ABGMM	ABGMM	ABGMM
		0.9	ABGMM	ABGMM	ABGMM	ABGMM
	0.5	0.2	ABGMM	ABGMM	ABGMM	ABGMM
		0.5	ABGMM	ABGMM	ABGMM	ABGMM
		0.9	ABGMM	ABGMM	ABGMM	ABGMM
	0.7	0.2	ABGMM	ABGMM	ABGMM	ABGMM
		0.5	ABGMM	ABGMM	ABGMM	ABGMM
		0.9	ABGMM	ABGMM	ABGMM	ABGMM

From table 4.13, the simulation result revealed that when n is small, the proposed modified estimator outperformed all other estimators for all the time periods except when $n=10$ and $T=20$, AH (d) estimator performed better than all other estimator for all the degrees of autocorrelation. When $n=50$, the proposed modified estimator performed better than all other estimators for $T=5$, but as T increases, the AH (d) estimator performed better than all other estimator for all the degrees of autocorrelation. When $n=100$, the proposed modified estimator outperformed all other estimators for time periods 5, 10 and 15 while ABGMM performed better than all other estimators as T increases to 20. But as n increases to 200, ABGMM outperformed all other estimators for all time periods and for all the degrees of autocorrelation.

Table 4.14: Best Estimator using Absolute bias Criterion at various combinations of n and T for β when the true model is ARMA (1, 1)

n	δ	θ/T	5	10	15	20
10	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
20	0.3	0.2	P-est.	ABGMM	ABGMM	P-est.
		0.5	P-est.	ABGMM	ABGMM	P-est.
		0.9	P-est.	ABGMM	ABGMM	P-est.
	0.5	0.2	P-est.	ABGMM	ABGMM	P-est.
		0.5	P-est.	ABGMM	ABGMM	P-est.
		0.9	P-est.	ABGMM	ABGMM	P-est.
	0.7	0.2	P-est.	ABGMM	ABGMM	P-est.
		0.5	P-est.	ABGMM	ABGMM	P-est.
		0.9	P-est.	ABGMM	ABGMM	P-est.
50	0.3	0.2	P-est.	M	AH(d)	P-est.
		0.5	P-est.	MM	AH(d)	P-est.
		0.9	P-est.	MM	AH(d)	P-est.
	0.5	0.2	P-est.	M	AH(d)	P-est.
		0.5	P-est.	M	AH(d)	P-est.
		0.9	P-est.	MM	AH(d)	P-est.
	0.7	0.2	P-est.	SYS1	AH(d)	P-est.
		0.5	P-est.	M	AH(d)	P-est.
		0.9	P-est.	M	AH(d)	P-est.
100	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
200	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.

From table 4.14, the simulation result showed that when n is small, the proposed modified estimator outperformed all other estimators for all the time periods except when $n=20$ and $T=10$ and 15 , ABGMM performed better than all other estimators for all the degrees of autocorrelation. When $n=50$, the proposed modified estimator performed better than all other estimators when $T=5$ and 20 while the robust estimators (M and MM) performed better at $T=10$, and AH (d) estimator outperformed better than all other estimators when $T=15$. As n increases to 200 , the proposed modified estimator performed better than all other estimators for all the time periods and for all the degrees of autocorrelation.

Table 4.15: Best Estimator using RMSE Criterion at various combinations of n and T for δ when the true model is ARMA (1, 1)

n	δ	θ/T	5	10	15	20
10	0.3	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)/ABGMM	P-est.	P-est.	P-est.
	0.5	0.2	ABGMM	P-est.	P-est.	P-est.
		0.5	ABGMM	P-est.	P-est.	P-est.
		0.9	ABGMM	P-est.	P-est.	P-est.
	0.7	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	ABGMM	P-est.	P-est.	P-est.
20	0.3	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.5	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.7	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)/SYS1	P-est.	P-est.	P-est.
50	0.3	0.2	AH(d)	AH(d)	AH(d)	ABGMM
		0.5	AH(d)	AH(d)	AH(d)	ABGMM
		0.9	AH(d)	AH(d)	AH(d)	ABGMM
	0.5	0.2	AH(d)	AH(d)	AH(d)	ABGMM
		0.5	AH(d)	AH(d)	AH(d)	ABGMM
		0.9	AH(d)	AH(d)	AH(d)	ABGMM
	0.7	0.2	AH(d)	AH(d)	AH(d)	ABGMM
		0.5	AH(d)	AH(d)	AH(d)	ABGMM
		0.9	AH(d)	AH(d)	AH(d)	ABGMM
100	0.3	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.5	0.2	AH(d0)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
	0.7	0.2	AH(d)	P-est.	P-est.	P-est.
		0.5	AH(d)	P-est.	P-est.	P-est.
		0.9	AH(d)	P-est.	P-est.	P-est.
200	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.

From table 4.15 the simulation result revealed that when $n=10$, AH (d) and ABGMM estimators performs better than all other estimator when T is 5. But as T increases, the proposed modified estimator performed better than all other estimators. When $n=20$, AH (d) estimator performed better than all other estimators when $T=5$ while the proposed modified estimator outperformed all other estimator when time periods is 10, 15 and 20. As n increases to 50, AH (d) estimator outperformed all other estimators at $T=5, 10$ and 15 when ABGMM performed better than all other estimators when the time period is 20. When n is large, the proposed modified estimator outperformed all other estimators for all time periods except when $n=100$ and $T=5$ that AH (d) estimator performed better than all other estimators for all degrees of autocorrelation.

Table 4.16: Best Estimator using RMSE Criterion at various combinations of n and T for β when the true model is ARMA (1, 1)

n	δ	θ/T	5	10	15	20
10	0.3	0.2	MM	ABGMM	P-est.	P-est.
		0.5	ABGMM	ABGMM	P-est.	P-est.
		0.9	ABGMM	ABGMM	P-est.	P-est.
	0.5	0.2	ABGMM	ABGMM	P-est.	P-est.
		0.5	ABGMM	ABGMM	P-est.	P-est.
		0.9	ABGMM	ABGMM	P-est.	P-est.
	0.7	0.2	ABGMM	ABGMM	P-est.	P-est.
		0.5	ABGMM	ABGMM	P-est.	P-est.
		0.9	ABGMM	ABGMM	P-est.	P-est.
20	0.3	0.2	ABGMM	ABGMM	P-est.	P-est.
		0.5	ABGMM	ABGMM	P-est.	P-est.
		0.9	ABGMM	ABGMM	P-est.	P-est.
	0.5	0.2	ABGMM	ABGMM	P-est.	P-est.
		0.5	ABGMM	ABGMM	P-est.	P-est.
		0.9	ABGMM	ABGMM	P-est.	P-est.
	0.7	0.2	ABGMM	ABGMM	P-est.	P-est.
		0.5	ABGMM	ABGMM	P-est.	P-est.
		0.9	ABGMM	ABGMM	P-est.	P-est.
50	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
100	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
200	0.3	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.5	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.
	0.7	0.2	P-est.	P-est.	P-est.	P-est.
		0.5	P-est.	P-est.	P-est.	P-est.
		0.9	P-est.	P-est.	P-est.	P-est.

From table 4.16, the simulation result revealed that when n is small, ABGMM outperformed all other estimators for time periods 5 and 10, while the proposed modified estimator performed better than all other estimators for time periods 15 and 20 for all the degrees of autocorrelation. As n increases, the proposed modified estimator outperformed all other estimators for all the time periods and for all the degrees of autocorrelation.

Table 4.17: Comparison of the Performances of Estimators {M, MM} With Absolute bias Criterion for δ

N	Absolute bias Criterion							
	T=5		T=10		T=15		T=20	
	M	MM	M	MM	M	MM	M	MM
10	27(100%)	0(0%)	6(22%)	21(78%)	6(22%)	21(78%)	21(78%)	6(22%)
20	27(100%)	0(0%)	24(89%)	3(11%)	9(33%)	18(67%)	0(0%)	27(100%)
50	6(22%)	21(78)	27(100%)	0(0%)	21(78%)	6(22%)	21(78%)	6(22%)
100	3(11%)	24(89%)	15(56%)	12(44%)	0(100%)	27(100%)	9(33%)	18(67%)
200	9(33%)	18(67%)	9(33%)	18(67%)	6(22%)	21(78%)	12(44%)	15(56%)

From table 4.17, the bias for δ at various sample sizes showed that M estimator outperformed MM estimator at sample size 10 and 20, the MM estimator outperformed M estimator at sample sizes 50, 100 and 200 when T is small with minimum biases. When T is 10, M estimator outperformed MM estimator at sample sizes 20, 50 and 100 while MM estimator outperformed M estimator at sample sizes 10 and 200. As T increased to fifteen, MM estimator outperformed M estimator at sample sizes 10, 20, 100 and 200, the MM estimator performed better at 50. When T=20, MM estimator outperformed M estimator at sample size 10 and 50 while M estimator outperformed MM estimator at sample sizes 20, 100 and 200.

Table 4.18: Comparison of the Performances of Estimators {M, MM} With Absolute bias Criterion for β

	Bias Criterion							
	T=5		T=10		T=15		T=20	
N	M	MM	M	MM	M	MM	M	MM
10	6(22%)	21(78%)	9(33%)	18(67%)	0(0%)	27(100%)	0(0%)	9(100%)
20	27(100%)	0(0%)	18(67%)	9(33%)	9(33%)	18(67%)	3(11%)	24(89%)
50	3(11%)	24(89%)	27(100%)	0(0%)	6(22%)	21(78%)	24(89%)	3(11%)
100	9(33%)	18(67%)	18(67%)	9(33%)	0(0%)	27(100%)	9(33%)	18(67%)
200	6(22%)	21(78%)	3(11%)	24(89%)	15(56%)	12(44%)	21(78%)	6(22%)

From table 4.18, the bias for β at various sample sizes showed that MM estimator outperformed M estimator at sample sizes 10, 50, 100 and 200, the M estimator outperformed MM estimator at sample size 20 when T is small with minimum biases. When T =10, M estimator outperformed MM estimator at sample sizes 20, 50 and 100 while the MM estimator outperformed M at sample sizes 10 and 200. MM estimator outperformed M estimator at sample sizes 10, 20, 50 and 100, the M estimator outperformed MM at sample size 200 when T is 15. When T=20, MM estimator outperformed M at sample sizes 10, 20 and 100, M estimator outperformed MM at sample sizes 50 and 200.

Table 4.19: Comparison of the Performances of Estimators {M, MM} With Root Mean Squares Error Criterion for δ

	Root Mean Squares Error Criterion							
	T=5		T=10		T=15		T=20	
N	M	MM	M	MM	M	MM	M	MM
10	0(0%)	27(100%)	0(0%)	27(100%)	12(44%)	15(56%)	27(100%)	0(0%)
20	27(100%)	0(0%)	3(11%)	24(89%)	27(100%)	0(0%)	27(100%)	0(0%)
50	0(0%)	27(100%)	18(67%)	9(33%)	27(100%)	0(0%)	27(100%)	0(0%)
100	27(100%)	0(0%)	27(100%)	0(0%)	27(100%)	0(0%)	27(100%)	0(0%)
200	9(33%)	18(67%)	21(78%)	6(22%)	0(0%)	27(100%)	27(100%)	0(0%)

From table 4.19, the RMSE for δ at various sample sizes showed that MM estimator outperformed M estimator at sample sizes 10, 50 and 200, M estimator outperformed at sample sizes 20 and 200 when T is small. When T is 10, MM estimator outperformed M estimator at sample sizes 10 and 20, M estimator outperformed MM estimator at sample sizes 50, 100 and 200. M estimator outperformed MM estimator at sample sizes 20, 50 and 100, MM estimator outperformed M estimator at sample sizes 10 and 200 when T is moderate. But when T is large, M estimator outperformed MM estimator at all sample sizes.

Table 4.20: Comparison of the Performances of Estimators {M, MM} With Root Mean Squares Error Criterion for β

	Root Mean Squares Error Criterion							
	T=5		T=10		T=15		T=20	
n	M	MM	M	MM	M	MM	M	MM
10	0(0%)	27(100%)	27(100%)	0(0%)	27(100%)	0(0%)	6(22%)	21(78%)
20	27(100%)	0(0%)	21(78%)	6(22%)	27(100%)	0(0%)	27(100%)	0(0%)
50	6(22%)	21(67%)	0(0%)	27(100%)	27(100%)	0(0%)	27(100%)	0(0%)
100	27(100%)	0(0%)	15(56%)	12(44%)	27(100%)	0(0%)	27(100%)	0(0%)
200	27(100%)	0(0%)	3(11%)	24(89%)	0(0%)	27(100%)	6(22%)	21(78%)

The RMSE of β at various sample sizes showed that M estimator outperformed MM estimator at sample sizes 20, 100 and 200, MM estimator outperformed M estimator at sample sizes 10 and 50 when T is small. When T is 10, M estimator outperformed MM estimator at sample sizes 10, 20 and 100 while MM estimator outperformed M estimator at sample sizes 50 and 200. The M estimator outperformed MM estimator at sample sizes 10, 20, 50 and 100, MM estimator outperformed M estimator at sample size 200 when T is moderate. M estimator outperformed MM estimator at sample sizes 20, 50 and 100 while MM estimator outperformed M estimator at sample size 10 and 200. Thus performance at bias and RMSE indicates that M estimator has overall best performance compare with MM estimator.

4.2 Application to Real Life Data

In this study, the discussed estimators using macroeconomic panel data from Penn World Table (PWT) were applied to investigate the relationship between the real GDP per capita and exchange rate to estimate the important of technique selection.

4.3 An Empirical Application: Real GDP and Exchange Rate in OPEC Countries.

Considering a simple dynamic real GDP equation of this form:

$$GDP_{it} = GDP_{i,t-1} + R_{it}\beta + \mu_i + \nu_{it}$$

Which is similar to the baseline dynamic model in equation (3.12), where $i = 1, 2, \dots, n$ and $t = 1, 2, \dots, T$. GDP_{it} is the log of real GDP per capita, R_{it} is the exchange rate, μ_i is the country specific-effects that contains various unobserved factors and ν_{it} is the random disturbance term. The dependent variable is the log of real GDP per capita. The explanatory variable includes the lagged dependent variable and the exchange rate. The time period covered by our sample is from 1986 to 2005 over 12 OPEC countries. The data used for our empirical study is a computed data downloaded from PWT 2012.

4.4 Empirical Result

The results from empirical data for 12 OPEC countries over the periods of 20 years were presented. The level of autocorrelation is 0.6706 (moderate level of autocorrelation). The estimates of the parameters and the corresponding standard errors are provided in the table 4.21 below:

Table 4.21: Empirical Results of estimation of CGDP on Exchange Rate of OPEC Countries

Estimation Method	OLS	ABGMM	SYS1	AH(d)	M	MM	P. EST
Real GDP(δ)	2.01534	1.08356	1.06959	1.13965	2.31568	2.23566	1.09453
Standard Error	0.25843	0.00197	0.01125	0.00348	0.01791	0.01525	0.00154
Exchange Rate(β)	2.59708	0.14074	0.07841	0.15463	0.88307	0.89403	1.15324
Standard Error	0.02443	0.02382	0.23175	0.02423	0.22751	0.03522	0.01824

From table 4.21, for the estimate of δ and β , the proposed modified estimator outperforms all other estimators because it has smaller standard error. Therefore, the above results corroborate our findings from Monte Carlo simulation of datasets for modeling panel data.

CHAPTER FIVE

SUMMARY AND CONCLUSION

5.1 Introduction

One of the problems with Dynamic Panel Data Models is autocorrelation due to the presence of lagged dependent variable among the regressors. Hence, there is need to estimate the model in the presence of autocorrelation.

In this study, the performance of Dynamic Panel Model estimators in the presence of autocorrelation was investigated. Our study focused specifically on the following seven estimation procedures: OLS, ABGMM, SYS1, AH (d), M, MM and the Proposed modified estimator. Both simulated and Real Life data were used.

5.2 Summary

Comparing the estimation method listed above with ranked their performance using the finite sampling properties of estimators in the presence of autocorrelation. The results of performance of the estimators considered at various level of autocorrelation. These estimators were ranked using the ranks 1, 2, 3, 4, 5, 6 and 7 at different values of T and n. The study also specifies when the error components follow AR (1), MA (1) and ARMA (1, 1). In most cases, the performances follow similar pattern when the autoregressive parameter (δ and λ) and parameters of error components (ρ and θ) are varied. 1000 replications (R) of the experiments are performed on cross-sections, $n = 10, 20, 50, 100$ and 200 and time periods, $T = 5, 10, 15$ and 20, for different degrees of autocorrelation and coefficient values ρ and θ : 0.2, 0.5 and 0.9. The estimators of the parameters of Dynamic Panel Data models with respect to their performances in terms of absolute bias and RMSE are as follow:

For estimate of δ in terms of absolute bias, when n is small, the proposed modified estimator outperformed all other estimators for all the time periods, except when $T=20$, AH (d) estimator performed better than all other estimators for all different generation scheme. However, when $n=20$, MM estimator also performed when the degree of autocorrelation is low and high under the generation scheme of AR (1). As n increases to 50, the proposed modified estimator performed better than all other estimators when $T=5$ while AH (d) estimator performed better than all other estimators as T increases for all different generation scheme. When $n=100$, the proposed modified estimator performed relatively well for the time periods is 5, 10 and 15 respectively while ABGMM and AH (d) performed better in some

cases under generating scheme of AR (1) and MA (1), expect under generating scheme of ARMA (1,1) while ABGMM performed better than all other estimators when $T=20$. When $n=200$, ABGMM performed better than all other estimators for all the time periods, expect in some cases while SYS1, M and MM performed better than all other estimators.

For the estimate of β in terms of absolute bias, when n is small, the proposed modified estimator outperformed all other estimators, expect when $n=20$ and $T=10$ and 20 while ABGMM performed better than all other estimators under generation scheme of AR (1) and ARMA (1, 1). In addition, under generation scheme of MA (1), when $n=20$ and $T=15$, MM estimator performed relatively well than other estimators.

When n is large, the proposed modified estimator outperformed all other estimators for all time periods under generating scheme of MA (1) and ARMA (1, 1) while ABGMM, M and MM performed better in some cases when $T=5$ for AR (1). However, when $n=50$, M and MM performed better when $T=10$ under generation scheme of AR (1), ABGMM, M, SYS1 and MM performed better when $T=10$ and 15 for MA (1) and M, MM, SYS1 performed better when $T=10$ while AH (d) performed better when $T=15$ under generating scheme of ARMA (1, 1).

For estimate of δ in terms of RMSE, AH (d) estimator outperformed all other estimators for the sample sizes, expect when $n=200$ while the proposed modified estimator performed better than all the estimators under the generating scheme of MA (1) and ARMA (1, 1) when $T = 5$. As T increases, the proposed modified estimator outperformed all other estimators for the sample sizes, expect when $n=50$, AH (d) estimator performed relatively well when $T=5$ and 10 while ABGMM performed best at $T=20$.

For the estimate of β in terms of RMSE, when n is small, the proposed modified estimator outperformed all other estimators when $T=5, 15$ and 20 while ABGMM performed better than all other estimators when $T=10$ under generating scheme of AR (1) and MA (1). Then ABGMM performed better than all other estimators when $T=5$ and 10 . When n is large, the proposed modified estimator outperformed all other estimators for all time periods and for all different generating scheme, expect when $n=50$ and $T=5$ that MM estimator performed quite well under generating scheme of AR (1).

The absolute bias of the OLS estimator for the estimate of β is more severe than the estimate of δ for all sample sizes irrespective of the time periods.

The Simulation results on various generating mechanism showed that based on a root mean squares error criterion, the proposed modified estimator performed well against the existing estimators.

Furthermore, the study also observed that when the value of the autoregressive parameter of the explanatory variable λ is varies, the absolute bias and RMSE of the estimators improves as the values of λ increases. The result of our findings showed that, as to be expected all estimators (with the exception Blundell-Bond System GMM and OLS) generally performed better with small T and large T. However, the proposed modified estimator seems to show the largest improvement as n and T increases.

Turning to the empirical comparison, the results showed that the proposed modified estimator outperformed all other estimators. Therefore, the real- life data results corroborate our findings from the Monte Carlo simulation results.

5.3 Conclusion

This study concluded that in estimating the parameters of dynamic panel models in the presence of autocorrelation of the error term, the proposed modified estimator is more preferable. It is recommended to use the proposed modified estimator when dealing with panel data models in the presence of serial correlation.

5.4 Contribution to Knowledge

The research contributes to the existing literature in the following ways

- (i) The performances of dynamic panel data model estimators in the presence of serial correlation were evaluated.
- (ii) A new estimator was modified and compare with existing estimators at different sample sizes and levels of autocorrelation.
- (iii) Two robust estimators (M and MM) were also applied under dynamic panel data models.

5.5 Suggestion for Future Studies

The following suggestions may be considered for future research

- 1) A study of the behaviour of some dynamic panel data estimators when heterogeneity or both autocorrelation and heterogeneity is present in the disturbance term.
- 2) Study of two-way error component dynamic panel data models when autocorrelation

is present.

- 3) Study of estimation and test procedures for unbalanced panel data models.

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APPENDIX 1

TABLE 1: Absolute Bias of various Estimators when n=10 and T=5 for δ

AR(1) ABSOLUTE BIAS (δ) T=5, n=10									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.047656	0.017434	0.001584	0.072468	0.006718	0.013961	0.001503
		0.5	0.042466	0.023116	0.018672	0.008905	0.020523	0.041127	0.001443
		0.9	0.040698	0.032956	0.028485	0.050407	0.050594	0.053034	0.001317
	0.5	0.2	0.050202	0.017431	0.007572	0.067371	0.010845	0.020835	0.001535
		0.5	0.045067	0.022726	0.013322	0.000595	0.016723	0.042929	0.001435
		0.9	0.04247	0.031386	0.022473	0.050799	0.063871	0.063938	0.001357
	0.7	0.2	0.052018	0.017399	0.00957	0.060782	0.01665	0.025838	0.001501
		0.5	0.046916	0.022316	0.009754	0.01766	0.013303	0.047996	0.001502
		0.9	0.043508	0.029894	0.017484	0.051467	0.071098	0.074004	0.001501
0.5	0.3	0.2	0.050078	0.012988	0.000761	0.060368	0.013503	0.01712	0.001494
		0.5	0.043932	0.018433	0.009896	0.144753	0.047487	0.05667	0.001454
		0.9	0.039629	0.026352	0.015269	0.079186	0.04715	0.059122	0.001323
	0.5	0.2	0.052684	0.013016	0.004014	0.056369	0.016512	0.020307	0.001206
		0.5	0.046685	0.018258	0.00559	0.192809	0.029867	0.059176	0.001205
		0.9	0.041528	0.025083	0.011653	0.080581	0.044989	0.06828	0.001193
	0.7	0.2	0.054565	0.013023	0.006084	0.050205	0.027939	0.022407	0.001485
		0.5	0.048692	0.018034	0.003325	0.388648	0.011655	0.057165	0.001376
		0.9	0.042679	0.023895	0.009024	0.082089	0.05405	0.077815	0.001321
0.7	0.3	0.2	0.051772	0.009641	5.71E-05	0.037673	0.031735	0.017574	0.001453
		0.5	0.045072	0.014547	0.004092	0.589482	0.058581	0.063209	0.001404
		0.9	0.03859	0.020231	0.00524	0.100813	0.044386	0.055346	0.001357
	0.5	0.2	0.054379	0.009665	0.00162	0.030468	0.031175	0.020318	0.001563
		0.5	0.047901	0.014535	0.001703	0.569011	0.042687	0.062122	0.001419
		0.9	0.040763	0.019607	0.004202	0.103169	0.040584	0.064908	0.001402
	0.7	0.2	0.056254	0.009679	0.002321	0.017604	0.009257	0.020753	0.001398
		0.5	0.049979	0.014465	0.000632	0.455577	0.031401	0.060111	0.001216
		0.9	0.042485	0.019289	0.003944	0.042485	0.056823	0.072579	0.001195

TABLE 2: Absolute Bias of various Estimators when n=10 and T=5 for β

AR(1) ABSOLUTE BIAS (β) T=5, n=10									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.028835	0.025626	0.035443	3.995608	0.013552	0.004279	0.005702
		0.5	0.030481	0.025641	0.017633	0.186976	0.022089	0.020042	0.005123
		0.9	0.031068	0.016196	0.00164	1.528404	0.042899	0.026898	0.005073
	0.5	0.2	0.027252	0.025229	0.045244	3.633275	0.015797	0.009308	0.004515
		0.5	0.028744	0.024255	0.031675	0.201849	0.020405	0.02181	0.004417
		0.9	0.029543	0.016354	0.014987	1.541584	0.049381	0.032166	0.004325
	0.7	0.2	0.025344	0.024794	0.048182	3.101834	0.017697	0.012671	0.004494
		0.5	0.026688	0.022916	0.042362	1.037939	0.018013	0.026651	0.004312
		0.9	0.027625	0.016465	0.026888	1.543895	0.051326	0.036389	0.004219
0.5	0.3	0.2	0.028373	0.023307	0.030421	1.971206	0.003865	0.007263	0.004441
		0.5	0.030111	0.02284	0.022846	4.537994	0.038947	0.02729	0.004425
		0.9	0.031576	0.017259	0.007477	1.787163	0.043003	0.02832	0.004391
	0.5	0.2	0.026831	0.02285	0.041887	1.789134	0.001775	0.009113	0.004121
		0.5	0.028403	0.021498	0.035681	5.951168	0.02574	0.028685	0.003913
		0.9	0.030004	0.017184	0.020293	1.804242	0.04061	0.033522	0.004009
	0.7	0.2	0.024967	0.022366	0.048658	1.513816	0.00506	0.00991	0.004459
		0.5	0.026387	0.020207	0.045116	11.59548	0.01697	0.026735	0.004251
		0.9	0.028045	0.017064	0.031083	1.806838	0.044348	0.037461	0.004225
0.7	0.3	0.2	0.028158	0.020761	0.028166	0.832433	0.006097	0.008097	0.004314
		0.5	0.029891	0.020079	0.024086	13.11666	0.052249	0.029892	0.004221
		0.9	0.031988	0.0173	0.021829	1.797492	0.042014	0.02449	0.004198
	0.5	0.2	0.026653	0.020266	0.039359	0.633822	0.003723	0.009178	0.004519
		0.5	0.028221	0.018839	0.035687	12.50096	0.038836	0.028757	0.004438
		0.9	0.03031	0.016829	0.030529	0.103169	0.03914	0.030156	0.004282
	0.7	0.2	0.024825	0.019753	0.047195	0.303799	0.007045	0.008761	0.004321
		0.5	0.026249	0.017646	0.044358	9.765125	0.029845	0.026643	0.004213
		0.9	0.028164	0.016007	0.036938	1.819815	0.046063	0.033547	0.004182

TABLE 3: RMSE of various Estimators when n=10 and T=5 for δ

AR(1) RMSE (δ) T=5, n=10									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.390578	0.274066	0.548435	0.148394	0.33103	0.316731	0.248948
		0.5	0.37267	0.253897	0.349828	0.127307	0.343493	0.317478	0.23307
		0.9	0.363491	0.256259	0.240481	0.135672	0.351536	0.310558	0.186527
	0.5	0.2	0.409019	0.271727	0.486112	0.145714	0.354622	0.328157	0.245461
		0.5	0.390378	0.246969	0.317507	0.126903	0.365856	0.330681	0.22963
		0.9	0.379828	0.245384	0.226372	0.13585	0.378163	0.324974	0.23585
	0.7	0.2	0.425503	0.269078	0.429279	0.142545	0.374764	0.337593	0.192543
		0.5	0.406193	0.240098	0.286414	0.128045	0.387059	0.350776	0.174508
		0.9	0.394255	0.235287	0.211096	0.136124	0.39911	0.338526	0.182146
0.5	0.3	0.2	0.401874	0.251233	0.379942	0.14223	0.333238	0.319855	0.24332
		0.5	0.381131	0.238433	0.298317	0.192852	0.342782	0.320757	0.298532
		0.9	0.364277	0.233751	0.216235	0.149117	0.349595	0.31335	0.248171
	0.5	0.2	0.420505	0.249849	0.353878	0.140497	0.353468	0.330791	0.198027
		0.5	0.398836	0.231872	0.272574	0.23107	0.363116	0.334843	0.287013
		0.9	0.380878	0.223744	0.200572	0.149894	0.371768	0.327938	0.195123
	0.7	0.2	0.437357	0.248175	0.326793	0.138059	0.37282	0.340971	0.181312
		0.5	0.414892	0.22533	0.247496	0.408955	0.381355	0.348633	0.45595
		0.9	0.395724	0.214646	0.18419	0.150732	0.392655	0.346069	0.201713
0.7	0.3	0.2	0.410819	0.226003	0.28226	0.133645	0.333772	0.326292	0.187161
		0.5	0.388604	0.218302	0.242219	0.603111	0.345284	0.321813	0.623228
		0.9	0.365072	0.204307	0.177084	0.161817	0.349669	0.315022	0.181719
	0.5	0.2	0.429685	0.225089	0.268036	0.13177	0.354097	0.337235	0.181021
		0.5	0.406367	0.212426	0.224058	0.583106	0.364941	0.338065	0.241062
		0.9	0.382484	0.198101	0.165656	0.163321	0.374072	0.328403	0.216621
	0.7	0.2	0.446871	0.223944	0.253067	0.129377	0.371455	0.346949	0.177923
		0.5	0.422641	0.206521	0.206074	0.473047	0.382946	0.352597	0.274723
		0.9	0.398956	0.19485	0.156683	0.165154	0.395347	0.342564	0.201782

TABLE 4: RMSE of various Estimators when n=10 and T=5 for β

AR(1) RMSE (β) T=5, n=10									
λ	Δ	ρ	OLS	ABGMM1	SYS 2	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.251886	0.249623	0.421206	4.149194	0.219226	0.219369	0.20915
		0.5	0.255228	0.223488	0.431521	0.49339	0.228394	0.21902	0.20712
		0.9	0.256895	0.208138	0.423727	1.544177	0.240795	0.220541	0.20215
	0.5	0.2	0.245645	0.248565	0.396717	3.784562	0.216845	0.208885	0.20882
		0.5	0.248344	0.219436	0.417166	0.486164	0.224934	0.209122	0.20632
		0.9	0.250902	0.202864	0.414729	1.556929	0.239642	0.210867	0.20112
	0.7	0.2	0.2376	0.247443	0.367433	3.251641	0.213266	0.198195	0.19215
		0.5	0.239547	0.215181	0.395535	1.120904	0.220461	0.210102	0.190051
		0.9	0.242691	0.197547	0.401303	1.558765	0.23478	0.201792	0.182181
0.5	0.3	0.2	0.252037	0.229587	0.414019	2.05628	0.219939	0.219587	0.20138
		0.5	0.255931	0.203936	0.42021	4.551375	0.227428	0.219226	0.20009
		0.9	0.258292	0.180886	0.423767	1.797379	0.236971	0.21908	0.185121
	0.5	0.2	0.245538	0.228802	0.39076	1.878211	0.215033	0.208869	0.182172
		0.5	0.248553	0.200124	0.402674	5.960887	0.222246	0.209638	0.180121
		0.9	0.251527	0.176188	0.410834	1.814091	0.232457	0.20976	0.172151
	0.7	0.2	0.237276	0.227929	0.36672	1.610533	0.210147	0.198928	0.19938
		0.5	0.239342	0.196118	0.382876	11.60013	0.216743	0.200593	0.192151
		0.9	0.24263	0.171512	0.390264	1.81631	0.228027	0.201359	0.188182
0.7	0.3	0.2	0.251448	0.208995	0.408992	0.912411	0.21865	0.221804	0.187132
		0.5	0.255599	0.184761	0.413054	13.11938	0.231232	0.219294	0.179824
		0.9	0.258174	0.156404	0.414433	1.804607	0.236613	0.217547	0.189213
	0.5	0.2	0.244796	0.208415	0.386051	0.733455	0.214502	0.211199	0.182317
		0.5	0.247904	0.181363	0.392326	12.50371	0.225162	0.210405	0.171398
		0.9	0.250913	0.153351	0.394581	1.820006	0.233824	0.207786	0.189858
	0.7	0.2	0.236423	0.207737	0.363544	0.472067	0.210147	0.201177	0.179521
		0.5	0.238461	0.177772	0.372078	9.768458	0.218984	0.202041	0.175312
		0.9	0.241783	0.151653	0.370422	1.826587	0.229545	0.200074	0.174321

TABLE 5: Absolute Bias of various Estimators when n=200 and T=5 for δ

AR(1) ABSOLUTE BIAS (δ) T=5, n=200									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.014701	0.000345	0.016002	0.001887	0.015518	0.003193	0.00191
		0.5	0.014915	0.002016	0.003968	0.001857	0.007062	0.002279	0.00218
		0.9	0.015252	0.000945	0.000842	0.001834	0.001771	0.010394	0.000713
	0.5	0.2	0.016398	0.000686	0.015012	0.001865	0.017046	0.003543	0.00512
		0.5	0.016644	0.002299	0.003514	0.001847	0.008509	0.002126	0.003151
		0.9	0.01688	0.001267	0.000617	0.00183	0.002693	0.012303	0.00185
	0.7	0.2	0.017908	0.00099	0.014115	0.001853	0.019826	0.004345	0.00512
		0.5	0.01824	0.002449	0.003497	0.001844	0.013166	0.002193	0.00423
		0.9	0.018331	0.001581	0.00061	0.00183	0.003222	0.014291	0.00138
0.5	0.3	0.2	0.016862	0.000431	0.012617	0.001837	0.012426	0.002096	0.00451
		0.5	0.017606	0.001341	0.004092	0.001852	0.005966	0.002937	0.001515
		0.9	0.017595	0.000948	0.000925	0.001845	0.006	0.012791	0.001312
	0.5	0.2	0.018643	9.08E-05	0.012152	0.001825	0.014109	0.002467	0.006191
		0.5	0.019449	0.001437	0.003943	0.001845	0.007479	0.002696	0.004121
		0.9	0.019234	0.001284	0.000784	0.001841	0.004515	0.013747	0.002162
	0.7	0.2	0.020247	0.000229	0.011625	0.001815	0.015719	0.002011	0.00251
		0.5	0.02114	0.001526	0.004006	0.001842	0.010402	0.001759	0.001812
		0.9	0.020722	0.00159	0.000756	0.001837	0.00498	0.014816	0.001513
0.7	0.3	0.2	0.018459	0.001023	0.009846	0.001798	0.009316	0.000955	0.003152
		0.5	0.019763	0.000484	0.004149	0.001838	0.004471	0.004403	0.000321
		0.9	0.019538	0.000882	0.001141	0.00185	0.008589	0.014407	0.005652
	0.5	0.2	0.020318	0.000684	0.00951	0.00179	0.010957	0.000637	0.006832
		0.5	0.0217	0.000606	0.003951	0.001833	0.006282	0.003784	0.005212
		0.9	0.021227	0.00106	0.000901	0.001847	0.005072	0.014975	0.005005
	0.7	0.2	0.021997	0.00036	0.00914	0.001782	0.011962	0.000417	0.001261
		0.5	0.023439	0.000728	0.003853	0.00183	0.009588	0.002815	0.001071
		0.9	0.022741	0.001254	0.000803	0.001844	0.003021	0.01531	0.001524

TABLE 6: Absolute Bias of various Estimators when n=200 and T=5 for β

AR(1) ABSOLUTE BIAS(β) T=5, n=200									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.008121	0.001377	0.041478	0.865827	0.013296	0.001909	0.001213
		0.5	0.008091	0.002418	0.042135	0.35496	0.00749	0.001199	0.001093
		0.9	0.008024	0.003625	0.033368	0.120407	0.005266	0.00278	0.001023
	0.5	0.2	0.008682	0.001193	0.04088	0.645449	0.013689	0.002005	0.002952
		0.5	-0.01664	0.002299	0.003514	0.314693	0.008083	0.001307	0.002312
		0.9	0.008594	0.003725	0.038876	0.116431	0.005124	0.003985	0.001912
	0.7	0.2	0.009037	0.001018	0.033562	0.437103	0.014829	0.002497	0.004213
		0.5	0.009016	0.002556	0.048757	0.267034	0.010869	0.001498	0.004103
		0.9	0.00896	0.00379	0.040535	0.110919	0.004815	0.005088	0.003512
0.5	0.3	0.2	0.008114	0.002006	0.027229	0.240524	0.011179	0.001373	0.002593
		0.5	0.008077	0.001334	0.044858	0.185426	0.006621	0.002097	0.001813
		0.9	0.007993	0.002429	0.040822	0.094924	0.002316	0.004867	0.001512
	0.5	0.2	0.008678	0.001806	0.027179	0.187455	0.01179	0.00056	0.002513
		0.5	0.00864	0.001376	0.04927	0.168246	0.007633	0.002034	0.002171
		0.9	0.008567	0.002612	0.044922	0.089798	0.00314	0.005454	0.001991
	0.7	0.2	0.009034	0.001612	0.023007	0.132431	0.012158	0.001053	0.001592
		0.5	0.008999	0.0014	0.048149	0.14785	0.00901	0.001624	0.001482
		0.9	0.008937	0.002769	0.045438	0.082738	0.003067	0.006003	0.002817
0.7	0.3	0.2	0.008114	0.002379	0.018373	0.067266	0.009668	0.001134	0.002195
		0.5	0.008064	0.000344	0.040176	0.104059	0.004835	0.002832	0.002031
		0.9	0.007973	0.001711	0.04431	0.073448	0.000357	0.006351	0.001811
	0.5	0.2	0.008678	0.002176	0.019556	0.047419	0.010263	0.000724	0.003152
		0.5	0.008626	0.000427	0.043807	0.095743	0.005984	0.002488	0.001915
		0.9	0.008544	0.001826	0.047325	0.070823	0.002428	0.006662	0.001892
	0.7	0.2	0.009034	0.001971	0.017299	0.026208	0.010343	0.000485	0.001512
		0.5	0.008985	0.000503	0.042776	0.085585	0.007657	0.002008	0.001322
		0.9	0.008912	0.001934	0.046155	0.066687	0.003309	0.006791	0.001553

TABLE 7: RMSE of various Estimators when n=200 and T=5 for δ

AR(1) RMSE (δ) T=5, n=200									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.083299	0.079507	0.13533	0.027668	0.072882	0.070146	0.025615
		0.5	0.082231	0.071815	0.0821	0.027665	0.070989	0.070805	0.024615
		0.9	0.081118	0.062086	0.051388	0.027662	0.070669	0.072198	0.023153
	0.5	0.2	0.086403	0.078411	0.130301	0.027667	0.076391	0.073521	0.025515
		0.5	0.085095	0.070586	0.079803	0.027664	0.074452	0.074022	0.02352
		0.9	0.083858	0.061115	0.050218	0.027662	0.074303	0.075779	0.022716
	0.7	0.2	0.089034	0.077302	0.123791	0.027666	0.079613	0.076309	0.026172
		0.5	0.087548	0.069364	0.077025	0.027664	0.07782	0.076607	0.025161
		0.9	0.086178	0.060242	0.048786	0.027662	0.077342	0.078749	0.025101
0.5	0.3	0.2	0.085723	0.072856	0.092398	0.027665	0.072225	0.070185	0.024152
		0.5	0.083916	0.066009	0.066967	0.027664	0.071514	0.070838	0.024002
		0.9	0.081219	0.057136	0.047057	0.027662	0.071219	0.072393	0.023195
	0.5	0.2	0.088812	0.071721	0.091356	0.027665	0.075729	0.058573	0.024562
		0.5	0.086762	0.064747	0.065966	0.027664	0.074888	0.07408	0.023512
		0.9	0.083955	0.055956	0.045969	0.027661	0.074692	0.075862	0.022151
	0.7	0.2	0.091454	0.070578	0.089485	0.027664	0.078632	0.076486	0.023152
		0.5	0.089189	0.063522	0.064418	0.027664	0.077796	0.076734	0.023152
		0.9	0.086294	0.055042	0.044822	0.027661	0.077677	0.078733	0.023151
0.7	0.3	0.2	0.087605	0.065864	0.069262	0.027663	0.071715	0.070422	0.023616
		0.5	0.085202	0.059983	0.054829	0.027663	0.072014	0.071005	0.021651
		0.9	0.081216	0.052294	0.041451	0.027661	0.072336	0.072924	0.021421
	0.5	0.2	0.090703	0.064789	0.068935	0.027663	0.075291	0.073953	0.020198
		0.5	0.088053	0.05872	0.05433	0.027663	0.075313	0.074375	0.020901
		0.9	0.083983	0.05111	0.040695	0.027661	0.075662	0.076323	0.020907
	0.7	0.2	0.093373	0.063704	0.06817	0.027662	0.078063	0.076923	0.020021
		0.5	0.090485	0.057547	0.05346	0.027663	0.078102	0.077258	0.01905
		0.9	0.086394	0.050025	0.039797	0.027661	0.078479	0.079106	0.019002

TABLE 8: RMSE of various Estimators when n=20 and T=5 for β

AR(1) RMSE (β) T=5, n=20									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.05019	0.075001	0.496731	0.889241	0.054185	0.057241	0.045152
		0.5	0.050205	0.071292	0.429309	0.365711	0.05281	0.057372	0.042152
		0.9	0.050218	0.062317	0.402008	0.128654	0.05272	0.057838	0.042002
	0.5	0.2	0.049103	0.074416	0.466208	0.67199	0.052845	0.055495	0.044142
		0.5	0.049117	0.0704	0.414074	0.32609	0.051544	0.055562	0.042312
		0.9	0.049139	0.061574	0.382304	0.124798	0.051565	0.056129	0.042013
	0.7	0.2	0.047669	0.073782	0.425114	0.468517	0.051424	0.053473	0.044951
		0.5	0.047682	0.069447	0.389906	0.279383	0.050469	0.053433	0.043712
		0.9	0.047709	0.060765	0.358314	0.119445	0.050037	0.054131	0.042119
0.5	0.3	0.2	0.050192	0.071017	0.481363	0.26379	0.053725	0.057224	0.045513
		0.5	0.050208	0.06812	0.461532	0.196528	0.053222	0.057334	0.044717
		0.9	0.050241	0.059297	0.428059	0.102391	0.05297	0.057718	0.046173
	0.5	0.2	0.049103	0.070439	0.448137	0.214577	0.052354	0.039677	0.043165
		0.5	0.049122	0.067231	0.438888	0.179933	0.051855	0.055482	0.042715
		0.9	0.049159	0.058469	0.404823	0.097481	0.05171	0.055989	0.041315
	0.7	0.2	0.047669	0.069811	0.409455	0.165481	0.050706	0.053518	0.045613
		0.5	0.047689	0.066339	0.408005	0.160297	0.050326	0.053356	0.043715
		0.9	0.047724	0.057761	0.376837	0.090799	0.050072	0.053988	0.042913
0.7	0.3	0.2	0.050191	0.065332	0.468318	0.096339	0.05361	0.057284	0.047731
		0.5	0.050212	0.063259	0.466975	0.114951	0.053593	0.05729	0.046713
		0.9	0.050253	0.05473	0.444455	0.08007	0.053484	0.057908	0.045516
	0.5	0.2	0.049102	0.064784	0.43602	0.082622	0.052266	0.05563	0.045217
		0.5	0.049127	0.062423	0.442893	0.107153	0.052065	0.055493	0.044515
		0.9	0.049172	0.054084	0.421358	0.077528	0.052157	0.056186	0.043215
	0.7	0.2	0.047667	0.064201	0.399551	0.070803	0.050449	0.053691	0.042152
		0.5	0.047694	0.061605	0.411742	0.097702	0.050356	0.053479	0.042013
		0.9	0.047739	0.053369	0.392504	0.073591	0.050536	0.054176	0.040132

TABLE 9: Absolute Bias of various Estimators when n=100 and T=10 for δ

AR(1) ABSOLUTE BIAS (δ) T=10 n=100									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.010162	0.002366	0.000856	0.001943	0.006294	0.003315	0.000659
		0.5	0.014119	0.002366	0.000856	0.001439	0.005037	0.012855	0.000656
		0.9	0.017738	0.002366	0.000856	0.001375	0.011992	0.027358	0.000646
	0.5	0.2	0.011296	0.002366	0.000856	0.001778	0.009465	0.002811	0.000617
		0.5	0.015175	0.002366	0.000856	0.001407	0.001025	0.011307	0.000612
		0.9	0.018785	0.002366	0.000856	0.00135	0.01385	0.026603	0.000615
	0.7	0.2	0.012288	0.002366	0.000856	0.001704	0.013032	0.001193	0.000573
		0.5	0.016157	0.002366	0.000856	0.001403	0.002276	0.009166	0.000552
		0.9	0.019613	0.002366	0.000856	0.001339	0.014626	0.025221	0.000542
0.5	0.3	0.2	0.010752	0.001971	0.000917	0.002038	0.00663	0.003568	0.000493
		0.5	0.014645	0.001971	0.000917	0.001482	0.0008	0.010549	0.000489
		0.9	0.018336	0.001971	0.000917	0.001278	0.017411	0.027317	0.000452
	0.5	0.2	0.011918	0.001971	0.000917	0.001973	0.008596	0.003715	0.000359
		0.5	0.015792	0.001971	0.000917	0.001467	0.001243	0.010266	0.000345
		0.9	0.019141	0.001971	0.000917	0.001255	0.019169	0.026781	0.000303
	0.7	0.2	0.012915	0.001871	0.000917	0.001933	0.010972	0.00333	0.000303
		0.5	0.016797	0.001971	0.000917	0.001465	0.003106	0.009465	0.000301
		0.9	0.019852	0.001971	0.000917	0.001236	0.018329	0.025473	0.000305
0.7	0.3	0.2	0.011103	0.001583	0.00093	0.002018	0.005509	0.003682	0.000512
		0.5	0.014684	0.001583	0.00093	0.001522	0.002739	0.009607	0.000588
		0.9	0.018185	0.001583	0.00093	0.001179	0.011333	0.026221	0.000513
	0.5	0.2	0.012212	0.001583	0.00093	0.002001	0.007298	0.004016	0.000442
		0.5	0.015806	0.001583	0.00093	0.001523	0.004163	0.009485	0.000453
		0.9	0.018978	0.001583	0.00093	0.001156	0.014307	0.025271	0.000492
	0.7	0.2	0.013202	0.001583	0.00093	0.001994	0.00935	0.004129	0.000435
		0.5	0.016819	0.001583	0.00093	0.001528	0.005553	0.009283	0.000392
		0.9	0.019634	0.001583	0.00093	0.001138	0.011675	0.023964	0.000323

TABLE 10: Absolute Bias of various Estimators when n=100 and T=10 for β

AR(1) ABSOLUTE BIAS (β) T=10 n=100									
Λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.005226	0.001686	0.008901	0.13796	0.004651	0.002985	0.000712
		0.5	0.005268	0.001686	0.008901	0.278886	0.000242	0.00788	0.000712
		0.9	0.005302	0.001686	0.008901	0.21008	0.002152	0.013556	0.000712
	0.5	0.2	0.005507	0.001686	0.008901	0.050582	0.006209	0.002681	0.000712
		0.5	0.005518	0.001686	0.008901	0.301403	0.002319	0.006576	0.000712
		0.9	0.00552	0.001686	0.008901	0.22266	0.002509	0.012058	0.000712
	0.7	0.2	0.005665	0.001686	0.008901	0.132346	0.007969	0.001594	0.000712
		0.5	0.005649	0.001686	0.008901	0.299475	0.004041	0.005153	0.000712
		0.9	0.005619	0.001686	0.008901	0.226287	0.002664	0.010371	0.000712
0.5	0.3	0.2	0.005213	0.001686	0.006797	0.165622	0.005364	0.003479	0.000828
		0.5	0.005241	0.001686	0.006797	0.18566	0.002435	0.007709	0.000828
		0.9	0.005261	0.001686	0.006797	0.209055	0.005155	0.014884	0.000828
	0.5	0.2	0.005493	0.001686	0.006797	0.113463	0.0061	0.003629	0.000828
		0.5	0.005493	0.001686	0.006797	0.192262	0.002482	0.007204	0.000828
		0.9	0.005474	0.001686	0.006797	0.216684	0.005792	0.01356	0.000828
	0.7	0.2	0.00565	0.001686	0.006797	0.080844	0.00667	0.003372	0.000828
		0.5	0.005625	0.001686	0.006797	0.188972	0.003303	0.006471	0.000828
		0.9	0.005571	0.001686	0.006797	0.221133	0.004789	0.011986	0.000828
0.7	0.3	0.2	0.005202	0.0017	0.00567	0.112689	0.005917	0.003373	0.000593
		0.5	0.005219	0.0017	0.00567	0.131154	0.002782	0.007516	0.000593
		0.9	0.005231	0.0017	0.00567	0.207022	0.00259	0.015083	0.000593
	0.5	0.2	0.005481	0.0017	0.00567	0.100365	0.006125	0.003616	0.000593
		0.5	0.005474	0.0017	0.00567	0.128354	0.00319	0.007198	0.000593
		0.9	0.005432	0.0017	0.00567	0.212186	0.004009	0.013602	0.000593
	0.7	0.2	0.005637	0.0017	0.00567	0.093513	0.006711	0.003658	0.000593
		0.5	0.005606	0.0017	0.00567	0.123548	0.003787	0.006798	0.000593
		0.9	0.005527	0.0017	0.00567	0.214625	0.002217	0.012088	0.000593

TABLE 11: RMSE of various Estimators when n=100 and T=10 for δ

AR(1) RMSE (δ) T=10, n=100									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.086834	0.053971	0.029848	0.033363	0.062293	0.068802	0.004555
		0.5	0.082579	0.053971	0.029848	0.033334	0.063398	0.069382	0.004555
		0.9	0.080142	0.053971	0.029848	0.033327	0.065864	0.073357	0.004555
	0.5	0.2	0.089539	0.053971	0.029848	0.033351	0.065622	0.071572	0.004555
		0.5	0.084923	0.053971	0.029848	0.033332	0.066261	0.071813	0.004555
		0.9	0.082066	0.053971	0.029848	0.033326	0.069298	0.075855	0.004555
	0.7	0.2	0.092067	0.053971	0.029848	0.033345	0.068842	0.073691	0.004555
		0.5	0.087143	0.053971	0.029848	0.033331	0.069034	0.073631	0.004555
		0.9	0.083844	0.053971	0.029848	0.033326	0.072183	0.077537	0.004555
0.5	0.3	0.2	0.089208	0.052921	0.029535	0.033367	0.063216	0.068647	0.004315
		0.5	0.08406	0.052921	0.029535	0.033339	0.064258	0.068579	0.004315
		0.9	0.080999	0.052921	0.029535	0.033326	0.068586	0.073075	0.004315
	0.5	0.2	0.091845	0.052921	0.029535	0.033362	0.066369	0.071419	0.004315
		0.5	0.086474	0.052921	0.029535	0.033338	0.067456	0.07121	0.004315
		0.9	0.083051	0.052921	0.029535	0.033326	0.0723	0.075407	0.004315
	0.7	0.2	0.094332	0.052921	0.029535	0.03336	0.069387	0.073528	0.004315
		0.5	0.088774	0.052921	0.029535	0.033337	0.070283	0.0732	0.004315
		0.9	0.084917	0.052921	0.029535	0.033326	0.074783	0.076992	0.004315
0.7	0.3	0.2	0.090358	0.051753	0.029279	0.033364	0.063555	0.0686	0.004106
		0.5	0.084886	0.051753	0.029279	0.03334	0.065002	0.06845	0.004106
		0.9	0.081564	0.051753	0.029279	0.033324	0.06805	0.072314	0.004106
	0.5	0.2	0.092944	0.051753	0.029279	0.033363	0.066673	0.071369	0.004106
		0.5	0.087263	0.051753	0.029279	0.033341	0.068304	0.071093	0.004106
		0.9	0.083583	0.051753	0.029279	0.033324	0.071821	0.074565	0.004106
	0.7	0.2	0.095402	0.051753	0.029279	0.033363	0.069433	0.073477	0.004106
		0.5	0.089578	0.051753	0.029279	0.033341	0.071092	0.073178	0.004106
		0.9	0.085489	0.051753	0.029279	0.033323	0.074089	0.076245	0.004106

TABLE 12: RMSE of various Estimators when n=100 and T=10 for β

AR(1) RMSE (β) T=10, n=100									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.046725	0.054374	0.142975	0.219334	0.048309	0.051785	0.004375
		0.5	0.046703	0.054374	0.142975	0.289698	0.049394	0.052106	0.004375
		0.9	0.046706	0.054374	0.142975	0.214528	0.051118	0.053303	0.004375
	0.5	0.2	0.046148	0.054374	0.142975	0.162587	0.048083	0.050066	0.004375
		0.5	0.046137	0.054374	0.142975	0.310673	0.048935	0.050225	0.004375
		0.9	0.046139	0.054374	0.142975	0.226679	0.050537	0.051483	0.004375
	0.7	0.2	0.045279	0.054374	0.142975	0.1911	0.047672	0.048018	0.004375
		0.5	0.04528	0.054374	0.142975	0.307942	0.04826	0.048083	0.004375
		0.9	0.045282	0.054374	0.142975	0.230055	0.049624	0.049317	0.004375
0.5	0.3	0.2	0.046743	0.053667	0.093351	0.18981	0.04951	0.051402	0.004015
		0.5	0.04672	0.053667	0.093351	0.194616	0.050692	0.051383	0.004015
		0.9	0.046719	0.053667	0.093351	0.2123	0.052634	0.052929	0.004015
	0.5	0.2	0.046171	0.053667	0.093351	0.144044	0.049109	0.049682	0.004015
		0.5	0.046158	0.053667	0.093351	0.200492	0.050271	0.049608	0.004015
		0.9	0.046161	0.053667	0.093351	0.219716	0.052236	0.051028	0.004015
	0.7	0.2	0.045307	0.053667	0.093351	0.116598	0.048446	0.047659	0.004015
		0.5	0.045303	0.053667	0.093351	0.196844	0.049478	0.047565	0.004015
		0.9	0.04531	0.053667	0.093351	0.224	0.051136	0.048835	0.004015
0.7	0.3	0.2	0.046762	0.052801	0.071473	0.127854	0.050332	0.051082	0.004007
		0.5	0.046736	0.052801	0.071473	0.138391	0.0516	0.051033	0.004007
		0.9	0.046732	0.052801	0.071473	0.209321	0.053378	0.052375	0.004007
	0.5	0.2	0.046193	0.052801	0.071473	0.116425	0.049905	0.049337	0.004007
		0.5	0.046176	0.052801	0.071473	0.135503	0.051157	0.049267	0.004007
		0.9	0.046183	0.052801	0.071473	0.214374	0.052746	0.050445	0.004007
	0.7	0.2	0.045333	0.052801	0.071473	0.109656	0.049139	0.047325	0.004007
		0.5	0.045326	0.052801	0.071473	0.130671	0.050253	0.047272	0.004007
		0.9	0.045337	0.052801	0.071473	0.216727	0.051596	0.048317	0.004007

TABLE 13: Absolute Bias of various Estimators when n=100 and T=15 for δ

AR(1) ABSOLUTE BIAS (δ) T=15 n=100									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.00209	0.000178	0.004742	0.000329	0.003792	0.005353	0.000129
		0.5	0.000543	0.000178	0.036184	0.00066	0.012017	0.001983	0.000129
		0.9	0.001301	0.000178	0.243007	0.000749	0.024622	0.016524	0.000129
	0.5	0.2	0.002009	0.000178	0.379443	0.000448	0.003006	0.006178	0.000129
		0.5	0.000729	0.000178	0.432619	0.000667	0.009952	0.001161	0.000129
		0.9	0.001498	0.000178	0.445991	0.00075	0.024691	0.016615	0.000129
	0.7	0.2	0.001897	0.008114	0.279132	0.000533	0.000683	0.007487	0.000129
		0.5	0.000818	0.000178	0.154346	0.000674	0.00754	0.000195	0.000129
		0.9	0.001707	0.000178	0.004742	0.00075	0.024817	0.016	0.000129
0.5	0.3	0.2	0.00182	0.000258	0.423611	0.000493	0.00384	0.004783	0.000253
		0.5	0.000744	0.000258	0.361835	0.000645	0.015716	0.00326	0.000253
		0.9	0.001909	0.000258	0.004619	0.000759	0.029172	0.015999	0.000253
	0.5	0.2	0.001755	0.000258	0.423611	0.000532	0.002009	0.004997	0.000253
		0.5	0.00094	0.000258	0.004619	0.00065	0.01239	0.001964	0.000253
		0.9	0.00205	0.000258	0.004619	0.000756	0.028856	0.015471	0.000253
	0.7	0.2	0.001655	0.000258	0.004619	0.000563	0.00094	0.005247	0.000253
		0.5	0.001028	0.000258	0.009232	0.000655	0.009989	0.000658	0.000253
		0.9	0.002242	0.000258	0.431144	0.000749	0.029693	0.014519	0.000253
0.7	0.3	0.2	0.001375	0.000343	0.429176	0.000596	0.003228	0.004733	0.000154
		0.5	0.000983	0.000343	0.004476	0.000659	0.015998	0.003053	0.000154
		0.9	0.002307	0.006547	0.411029	0.000756	0.026156	0.015713	0.000154
	0.5	0.2	0.001335	0.000343	0.395602	0.000609	0.001654	0.004556	0.000154
		0.5	0.001153	0.000343	0.105353	0.00066	0.012393	0.001802	0.000154
		0.9	0.002399	0.000343	0.106363	0.000748	0.026699	0.01523	0.000154
	0.7	0.2	0.00127	0.000343	0.066881	0.000618	0.000231	0.004339	0.000154
		0.5	0.001224	0.007018	0.003564	0.00066	0.010981	0.000795	0.000154
		0.9	0.002488	0.000343	0.042802	0.00074	0.026044	0.014079	0.000154

TABLE 14: Absolute Bias of various Estimators when n=100 and T=15 for β

AR(1) ABSOLUTE BIAS (β) T=15 n=100									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.000388	0.003738	0.003051	1.465972	0.003682	0.004229	0.000249
		0.5	0.000543	0.003738	0.032688	0.381204	0.011654	0.004228	0.000249
		0.9	0.000382	0.003738	0.234419	0.141197	0.017417	0.011176	0.000249
	0.5	0.2	0.00055	0.003738	0.378165	1.069381	0.00221	0.005093	0.000249
		0.5	0.000544	0.003738	0.421913	0.35528	0.008757	0.002914	0.000249
		0.9	0.000545	0.003738	0.430263	0.136807	0.015852	0.010508	0.000249
	0.7	0.2	0.000688	0.000638	0.284458	0.783225	0.000365	0.006243	0.000249
		0.5	0.000683	0.003738	0.148621	0.324399	0.006248	0.00161	0.000249
		0.9	0.000684	0.003738	0.003051	0.13317	0.014364	0.009385	0.000249
0.5	0.3	0.2	0.000377	0.003719	0.398948	0.672611	0.003027	0.004432	0.000212
		0.5	0.00038	0.003719	0.338243	0.298621	0.013922	0.0049	0.000212
		0.9	0.000386	0.003719	0.003039	0.101853	0.021108	0.011902	0.000212
	0.5	0.2	0.00054	0.003719	0.398948	0.570788	0.001058	0.004577	0.000212
		0.5	0.000543	0.003719	0.003039	0.282627	0.010114	0.00327	0.000212
		0.9	0.000549	0.003719	0.003039	0.102836	0.018834	0.010708	0.000212
	0.7	0.2	0.000679	0.003719	0.003039	0.485556	0.000182	0.004846	0.000212
		0.5	0.000682	0.003719	0.008	0.263268	0.007566	0.001766	0.000212
		0.9	0.000688	0.003719	0.364134	0.106403	0.017793	0.009323	0.000212
0.7	0.3	0.2	0.00037	0.003669	0.40921	0.361006	0.001888	0.004846	0.000189
		0.5	0.00038	0.003669	0.002731	0.219366	0.012824	0.003792	0.000189
		0.9	0.000391	0.0011	0.385559	0.086114	0.018503	0.011592	0.000189
	0.5	0.2	0.000534	0.003669	0.377479	0.328734	0.00056	0.004545	0.000189
		0.5	0.000543	0.003669	0.099331	0.211222	0.009219	0.002421	0.000189
		0.9	0.000553	0.003669	0.099401	0.090543	0.017149	0.010398	0.000189
	0.7	0.2	0.000674	0.003669	0.061482	0.299328	0.000757	0.004344	0.000189
		0.5	0.000682	0.000894	0.005258	0.202735	0.007463	0.001286	0.000189
		0.9	0.000691	0.003669	0.03757	0.094781	0.015339	0.008892	0.000189

TABLE 15: RMSE of various Estimators when n=100 and T=15 for δ

AR(1) RMSE (δ) T=15, N=100									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.069447	0.02978	0.019782	0.021935	0.048466	0.05733	0.003187
		0.5	0.064965	0.02978	0.121943	0.021944	0.050036	0.057425	0.003187
		0.9	0.061559	0.02978	0.331623	0.021948	0.054835	0.059697	0.003187
	0.5	0.2	0.07249	0.02978	0.413811	0.021938	0.051153	0.060798	0.003187
		0.5	0.067713	0.02978	0.443385	0.021944	0.052327	0.060764	0.003187
		0.9	0.063997	0.02978	0.452553	0.021948	0.057356	0.063003	0.003187
	0.7	0.2	0.075193	0.029967	0.355687	0.02194	0.053358	0.063691	0.003187
		0.5	0.070084	0.02978	0.261882	0.021945	0.054151	0.063476	0.003187
		0.9	0.066103	0.02978	0.019782	0.021948	0.059442	0.065416	0.003187
0.5	0.3	0.2	0.071342	0.029375	0.443561	0.021937	0.049141	0.057552	0.003151
		0.5	0.066471	0.029375	0.411172	0.021943	0.051582	0.057903	0.003151
		0.9	0.062436	0.029375	0.01959	0.021947	0.057	0.059915	0.003151
	0.5	0.2	0.074411	0.029375	0.443561	0.021938	0.051908	0.060948	0.003151
		0.5	0.069222	0.029375	0.01959	0.021943	0.053404	0.061243	0.003151
		0.9	0.064865	0.029375	0.01959	0.021947	0.059225	0.062956	0.003151
	0.7	0.2	0.07715	0.029375	0.01959	0.021939	0.054146	0.063706	0.003151
		0.5	0.071636	0.029375	0.050775	0.021943	0.055124	0.063961	0.003151
		0.9	0.066969	-0.39303	0.029375	0.021947	0.061647	0.065238	0.003151
0.7	0.3	0.2	0.072132	0.028888	0.451939	0.021939	0.049278	0.057585	0.002835
		0.5	0.067157	0.028888	0.01949	0.021942	0.052068	0.058065	0.002835
		0.9	0.06278	0.028355	0.447028	0.021947	0.055294	0.059494	0.002835
	0.5	0.2	0.075182	0.028888	0.434048	0.021939	0.051883	0.060947	0.002835
		0.5	0.069845	0.028888	0.220696	0.021942	0.053827	0.061467	0.002835
		0.9	0.065157	0.028888	0.222892	0.021946	0.058049	0.062542	0.002835
	0.7	0.2	0.077909	0.028888	0.172776	0.021939	0.054077	0.063671	0.002835
		0.5	0.072292	0.028777	0.020735	0.021942	0.055736	0.064248	0.002835
		0.9	0.067227	0.028888	0.138468	0.021946	0.059756	0.064831	0.002835

TABLE 16: RMSE of various Estimators when n=100 and T=15 for β

AR(1) RMSE (β) T=15, n=100									
λ	Δ	ρ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.036255	0.02918	0.09436	1.470408	0.03812	0.044872	0.001315
		0.5	0.036227	0.02918	0.150096	0.38439	0.039884	0.045656	0.001315
		0.9	0.03621	0.02918	0.327164	0.143905	0.042497	0.047307	0.001315
	0.5	0.2	0.035439	0.02918	0.410037	1.074441	0.03746	0.044381	0.001315
		0.5	0.035419	0.02918	0.433838	0.358462	0.038568	0.044913	0.001315
		0.9	0.035395	0.02918	0.437759	0.139495	0.041159	0.046514	0.001315
	0.7	0.2	0.034436	0.027609	0.356879	0.788746	0.036507	0.043515	0.001315
		0.5	0.034421	0.02918	0.267297	0.327587	0.037122	0.043758	0.001315
		0.9	0.034396	0.02918	0.09436	0.135809	0.039597	0.045118	0.001315
0.5	0.3	0.2	0.036228	0.029003	0.414937	0.675517	0.038531	0.044936	0.001293
		0.5	0.03621	0.029003	0.382508	0.30082	0.040737	0.045914	0.001293
		0.9	0.036172	0.029003	0.063073	0.104424	0.043983	0.047604	0.001293
	0.5	0.2	0.035413	0.029003	0.414937	0.573935	0.037939	0.044339	0.001293
		0.5	0.035402	0.029003	0.063073	0.284851	0.039099	0.045135	0.001293
		0.9	0.035368	0.029003	0.063073	0.105305	0.042288	0.046628	0.001293
	0.7	0.2	0.034411	0.029003	0.063073	0.488871	0.037033	0.043331	0.001293
		0.5	0.034405	0.029003	0.076345	0.265533	0.037621	0.043967	0.001293
		0.9	0.034377	0.029003	0.39679	0.108709	0.040927	0.045153	0.001293
0.7	0.3	0.2	0.03621	0.028716	0.428699	0.363315	0.03858	0.044925	0.001088
		0.5	0.036198	0.028716	0.048747	0.221093	0.040598	0.045903	0.001088
		0.9	0.036163	0.026714	0.417306	0.088172	0.042426	0.047316	0.001088
	0.5	0.2	0.035395	0.028716	0.411896	0.331166	0.037964	0.044278	0.001088
		0.5	0.035389	0.028716	0.211732	0.212968	0.039103	0.045194	0.001088
		0.9	0.035362	0.028716	0.211883	0.092456	0.041323	0.046362	0.001088
	0.7	0.2	0.034393	0.028716	0.1694	0.301846	0.037075	0.043218	0.001088
		0.5	0.034391	0.026808	0.049411	0.204497	0.037816	0.044082	0.001088
		0.9	0.034374	0.028716	0.136616	0.096558	0.039737	0.044905	0.001088

APPENDIX 2

TABLE 17: Absolute Bias of various Estimators when n=50 and T=5 for δ

MA(1) ABSOLUTE BIAS (δ) T=5 n=50									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.01042	0.006816	0.035096	0.006102	0.004706	0.005862	0.002762
		0.5	0.010462	0.002796	0.026624	0.005924	0.006509	0.007941	0.002612
		0.9	0.009866	0.001092	0.019535	0.005887	0.011238	0.009211	0.001039
	0.5	0.2	0.009355	0.005537	0.02971	0.00634	0.007327	0.009407	0.002753
		0.5	0.00936	0.001683	0.02445	0.00605	0.005769	0.010902	0.001512
		0.9	0.008703	0.001954	0.018614	0.005948	0.009507	0.011959	0.001413
	0.7	0.2	0.008188	0.004289	0.024808	0.006467	0.01004	0.013344	0.002918
		0.5	0.008152	0.000599	0.022197	0.006154	0.007753	0.014181	0.000281
		0.9	0.007427	0.002792	0.017606	0.006012	0.008319	0.017388	0.002132
0.5	0.3	0.2	0.010463	0.004436	0.025744	0.006001	0.004972	0.007532	0.002817
		0.5	0.010431	0.000342	0.021031	0.005856	0.01044	0.008829	0.002782
		0.9	0.009918	0.004326	0.016351	0.005771	0.024333	0.007544	0.002528
	0.5	0.2	0.009487	0.003444	0.023277	0.0061	0.006439	0.010647	0.002888
		0.5	0.009392	0.00114	0.019879	0.005915	0.008332	0.01159	0.001112
		0.9	0.0088	0.004891	0.01582	0.0058	0.019333	0.013987	0.002762
	0.7	0.2	0.008406	0.002476	0.020823	0.006172	0.007808	0.016118	0.002777
		0.5	0.008243	0.001912	0.018602	0.005974	0.011227	0.007636	0.001121
		0.9	0.007565	0.005435	0.015188	0.00584	0.01144	0.018384	0.002673
0.7	0.3	0.2	0.0101	0.00247	0.020139	0.005997	0.006575	0.008165	0.002322
		0.5	0.010128	0.002464	0.017216	0.005833	0.017447	0.0079	0.002213
		0.9	0.00978	0.006234	0.013868	0.005702	0.021248	0.00935	0.002131
	0.5	0.2	0.009179	0.001743	0.018706	0.006038	0.007606	0.014131	0.001555
		0.5	0.009133	0.003009	0.016481	0.005854	0.010092	0.015331	0.001215
		0.9	0.008696	0.006583	0.013514	0.005704	0.026831	0.016442	0.001817
	0.7	0.2	0.008152	0.001033	0.017232	0.006072	0.00798	0.014297	0.001021
		0.5	0.008023	0.003535	0.015638	0.005881	0.010958	0.019034	0.001422
		0.9	0.007492	0.006916	0.01307	0.005722	0.022204	0.015534	0.002315

TABLE 18: Absolute Bias of various Estimators when n=50 and T=5 for β

MA(1) ABSOLUTE BIAS (β) T=5 n=50									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.001947	0.00263	0.062203	0.909726	0.002952	0.00315	0.001728
		0.5	0.001982	0.002892	0.066081	0.435783	0.003523	0.005071	0.001656
		0.9	0.002018	0.002837	0.067868	0.234005	0.007434	0.006384	0.001512
	0.5	0.2	0.000472	0.00309	0.051496	0.885611	0.004514	0.004807	0.000285
		0.5	0.000527	0.003246	0.057217	0.472503	0.003269	0.006356	0.000262
		0.9	0.000585	0.003078	0.059972	0.2635	0.005612	0.007685	0.000172
	0.7	0.2	0.000895	0.003513	0.040162	0.809787	0.005554	0.006674	0.000178
		0.5	0.000821	0.003573	0.04713	0.486376	0.004551	0.007646	0.000152
		0.9	0.000742	0.003304	0.050552	0.284738	0.004685	0.011767	0.000137
0.5	0.3	0.2	0.001988	0.002218	0.064475	0.402985	0.002497	0.004438	0.001618
		0.5	0.002003	0.002836	0.067202	0.276044	0.006843	0.005957	0.001519
		0.9	0.002019	0.002974	0.068225	0.190528	0.016215	0.00718	0.001505
	0.5	0.2	0.000529	0.002537	0.055334	0.428779	0.003472	0.005749	0.000492
		0.5	0.00056	0.003039	0.059021	0.312937	0.004884	0.007164	0.000427
		0.9	0.000593	0.003085	0.060695	0.220134	0.012511	0.010684	0.000392
	0.7	0.2	0.000824	0.002831	0.045127	0.434825	0.004503	0.009818	0.000496
		0.5	0.000776	0.003225	0.049344	0.339721	0.006255	0.00587	0.000487
		0.9	0.000726	0.003188	0.051388	0.245367	0.007441	0.012677	0.000348
0.7	0.3	0.2	0.002021	0.001819	0.065377	0.229369	0.003437	0.005052	0.001821
		0.5	0.002019	0.002621	0.067449	0.197288	0.011612	0.007139	0.001808
		0.9	0.00202	0.002857	0.067695	0.157283	0.014455	0.006858	0.001717
	0.5	0.2	0.000577	0.002025	0.056856	0.257877	0.00427	0.009677	0.000159
		0.5	0.000588	0.002724	0.059538	0.230157	0.006751	0.011073	0.000151
		0.9	0.000601	0.002893	0.060365	0.185198	0.017525	0.012282	0.000132
	0.7	0.2	0.000762	0.002216	0.047077	0.277473	0.004516	0.007515	0.000127
		0.5	0.000738	0.002818	0.049993	0.257808	0.006789	0.012589	0.000127
		0.9	0.000712	0.002925	0.051125	0.210684	0.013905	0.010264	0.000115

TABLE 19: RMSE of various Estimators when n=50 and T=5 for δ

MA(1) RMSE (δ) T=5, n=50									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.228964	0.153571	0.233558	0.067777	0.136686	0.133808	0.084188
		0.5	0.241613	0.143262	0.156332	0.067761	0.136659	0.134374	0.084182
		0.9	0.250708	0.128607	0.107377	0.067759	0.136213	0.135123	0.084179
	0.5	0.2	0.236288	0.152822	0.232289	0.067811	0.145421	0.142072	0.084181
		0.5	0.248244	0.142401	0.159189	0.067784	0.145429	0.142346	0.084187
		0.9	0.256631	0.127764	0.109839	0.067775	0.144708	0.142717	0.084182
	0.7	0.2	0.242638	0.152012	0.224109	0.067833	0.154282	0.149128	0.082172
		0.5	0.253973	0.141478	0.159274	0.067803	0.153439	0.149148	0.082101
		0.9	0.261739	0.126856	0.111323	0.06779	0.152349	0.152499	0.081003
0.5	0.3	0.2	0.239772	0.143128	0.158719	0.067768	0.136563	0.134314	0.085818
		0.5	0.250296	0.132035	0.116772	0.067756	0.136232	0.134916	0.085008
		0.9	0.257173	0.116967	0.085417	0.06775	0.137709	0.13772	0.084213
	0.5	0.2	0.247073	0.142301	0.16103	0.067788	0.145562	0.142287	0.084001
		0.5	0.256745	0.131047	0.119873	0.067772	0.144583	0.142579	0.083733
		0.9	0.262804	0.115996	0.087858	0.067762	0.144844	0.143615	0.082128
	0.7	0.2	0.253429	0.141399	0.160447	0.067804	0.153175	0.151571	0.082115
		0.5	0.262338	0.129994	0.121736	0.067786	0.15262	0.161221	0.082007
		0.9	0.267683	0.11497	0.089862	0.067775	0.151264	0.151985	0.081097
0.7	0.3	0.2	0.245921	0.132366	0.119786	0.067767	0.135577	0.134723	0.083328
		0.5	0.254963	0.121042	0.092887	0.067754	0.136282	0.136075	0.082023
		0.9	0.260304	0.106012	0.070611	0.067745	0.136957	0.136378	0.082009
	0.5	0.2	0.253217	0.131557	0.122155	0.067782	0.144355	0.147057	0.081772
		0.5	0.261315	0.120064	0.095525	0.067766	0.143767	0.145896	0.080078
		0.9	0.265784	0.10505	0.072732	0.067755	0.145517	0.150576	0.080017
	0.7	0.2	0.259587	0.130656	0.123103	0.067795	0.152442	0.148937	0.080019
		0.5	0.266846	0.119017	0.097541	0.067778	0.151547	0.153618	0.079282
		0.9	0.270558	0.10404	0.074662	0.067765	0.151914	0.151019	0.076021

TABLE 20: RMSE of various Estimators when n=50 and T=5 for β

MA(1) RMSE (β) T=5, n=50									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.10643	0.116697	0.363142	0.952899	0.102783	0.102808	0.036214
		0.5	0.106475	0.106656	0.380534	0.464541	0.103419	0.102636	0.036003
		0.9	0.106515	0.092443	0.393002	0.253823	0.104602	0.102487	0.036413
	0.5	0.2	0.105835	0.116085	0.339367	0.926208	0.103167	0.101755	0.035281
		0.5	0.105895	0.105975	0.359424	0.499245	0.103604	0.101405	0.034137
		0.9	0.105944	0.091822	0.375828	0.281829	0.104418	0.101058	0.033125
	0.7	0.2	0.104415	0.115352	0.319733	0.848097	0.103128	0.099783	0.032173
		0.5	0.104481	0.105191	0.338098	0.511573	0.103029	0.099296	0.032065
		0.9	0.104529	0.091121	0.356224	0.302012	0.103461	0.103642	0.031068
0.5	0.3	0.2	0.106474	0.106831	0.374752	0.433557	0.103422	0.102564	0.037582
		0.5	0.106501	0.095521	0.392852	0.296849	0.104301	0.10248	0.035217
		0.9	0.106524	0.081447	0.407798	0.204885	0.105526	0.102513	0.034101
	0.5	0.2	0.105883	0.10604	0.353666	0.45726	0.103712	0.101332	0.033252
		0.5	0.10593	0.094727	0.373942	0.332076	0.104018	0.101074	0.033102
		0.9	0.105966	0.080776	0.392135	0.233323	0.104519	0.103415	0.032751
	0.7	0.2	0.104459	0.10514	0.332805	0.461632	0.102909	0.103484	0.031292
		0.5	0.104516	0.093853	0.352639	0.357631	0.103148	0.101179	0.031251
		0.9	0.104554	0.080049	0.372442	0.257731	0.10296	0.103273	0.030288
0.7	0.3	0.2	0.10651	0.09643	0.381164	0.253949	0.103816	0.102344	0.035178
		0.5	0.106519	0.085116	0.400609	0.213694	0.104713	0.102557	0.034008
		0.9	0.106532	0.071937	0.418106	0.168627	0.104943	0.102744	0.034005
	0.5	0.2	0.10592	0.095573	0.362341	0.280227	0.103739	0.107146	0.035888
		0.5	0.105952	0.084313	0.3835	0.245007	0.103917	0.105883	0.032881
		0.9	0.105981	0.071286	0.403856	0.195496	0.104767	0.106802	0.031872
	0.7	0.2	0.10449	0.09462	0.341966	0.298137	0.103025	0.098726	0.034186
		0.5	0.104535	0.083444	0.362811	0.271539	0.102972	0.104793	0.032525
		0.9	0.104569	0.070595	0.384546	0.220252	0.10308	0.099503	0.031293

TABLE 21: Absolute Bias of various Estimators when n=100 and T=10 for δ

MA(1) ABSOLUTE BIAS (δ) T=10 n=100									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.006317	0.002366	0.000856	0.002911	0.004795	0.001026	0.000818
		0.5	0.006866	0.002366	0.000856	0.002701	0.004857	0.001538	0.000818
		0.9	0.007416	0.002366	0.000856	0.002562	0.003704	0.002087	0.000818
	0.5	0.2	0.007284	0.002366	0.000856	0.002662	0.006846	0.00102	0.000818
		0.5	0.00783	0.002366	0.000856	0.002636	0.004667	0.001864	0.000818
		0.9	0.008368	0.002366	0.000856	0.002552	0.005313	0.002494	0.000818
	0.7	0.2	0.008142	0.002366	0.000856	0.002475	0.010174	0.000478	0.000818
		0.5	0.008683	0.002366	0.000856	0.002575	0.006206	0.002022	0.000818
		0.9	0.009209	0.002366	0.000856	0.002544	0.005036	0.002826	0.000818
0.5	0.3	0.2	0.006799	0.001971	0.000917	0.002579	0.004196	0.001674	0.000724
		0.5	0.007342	0.001971	0.000917	0.002615	0.003959	0.001985	0.000724
		0.9	0.007855	0.001971	0.000917	0.002621	0.002099	0.002249	0.000724
	0.5	0.2	0.007775	0.001971	0.000917	0.002531	0.004604	0.002016	0.000724
		0.5	0.008309	0.001971	0.000917	0.002614	0.005338	0.002425	0.000724
		0.9	0.008806	0.001971	0.000917	0.002639	0.004359	0.002682	0.000724
	0.7	0.2	0.00864	0.001971	0.000917	0.002489	0.006507	0.002136	0.000724
		0.5	0.009164	0.001971	0.000917	0.002612	0.004523	0.002794	0.000724
		0.9	0.009644	0.001971	0.000917	0.002656	0.0058	0.003073	0.000724
0.7	0.3	0.2	0.007175	0.001583	0.00093	0.002441	0.004315	0.002262	0.000569
		0.5	0.007697	0.001583	0.00093	0.002571	0.003381	0.002323	0.000569
		0.9	0.008172	0.001583	0.00093	0.002663	0.001201	0.00234	0.000569
	0.5	0.2	0.008155	0.001583	0.00093	0.002443	0.003615	0.002717	0.000569
		0.5	0.008664	0.001583	0.00093	0.002593	0.004981	0.00279	0.000569
		0.9	0.00912	0.001583	0.00093	0.002696	0.002773	0.002782	0.000569
	0.7	0.2	0.009023	0.001583	0.00093	0.00245	0.004503	0.003044	0.000569
		0.5	0.009518	0.001583	0.00093	0.002616	0.006117	0.003211	0.000569
		0.9	0.009957	0.001583	0.00093	0.002729	0.005163	0.003186	0.000569

TABLE 22: Absolute Bias of various Estimators when n=100 and T=10 for β

MA(1) ABSOLUTE BIAS (β) T=10 n=100									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.005215	0.001686	0.008901	1.316431	0.005597	3.90E-05	0.000002
		0.5	0.005213	0.001686	0.008901	0.703366	0.005966	0.000262	0.000016
		0.9	0.005209	0.001686	0.008901	0.407741	0.005185	0.000535	0.000016
	0.5	0.2	0.005508	0.001686	0.008901	1.015553	0.006366	0.000179	0.000016
		0.5	0.005505	0.001686	0.008901	0.656104	0.005452	0.000644	0.000016
		0.9	0.005501	0.001686	0.008901	0.407374	0.006141	0.000877	0.000016
	0.7	0.2	0.005676	0.001686	0.008901	0.772075	0.008021	5.39E-05	0.000016
		0.5	0.005673	0.001686	0.008901	0.60163	0.005891	0.000863	0.000016
		0.9	0.005669	0.001686	0.008901	0.403103	0.005635	0.001136	0.000016
0.5	0.3	0.2	0.005212	0.001686	0.006797	0.603573	0.005452	0.00043	0.000075
		0.5	0.00521	0.001686	0.006797	0.465017	0.005309	0.000475	0.000075
		0.9	0.005208	0.001686	0.006797	0.346572	0.003703	0.000497	0.000075
	0.5	0.2	0.005505	0.001686	0.006797	0.558526	0.005304	0.000823	0.000075
		0.5	0.005502	0.001686	0.006797	0.4647	0.006057	0.000864	0.000075
		0.9	0.005499	0.001686	0.006797	0.357511	0.005237	0.000814	0.000075
	0.7	0.2	0.005672	0.001686	0.006797	0.512399	0.00601	0.001003	0.000075
		0.5	0.005669	0.001686	0.006797	0.458576	0.005215	0.001168	0.000075
		0.9	0.005667	0.001686	0.006797	0.365327	0.006014	0.001072	0.000075
0.7	0.3	0.2	0.005209	0.0017	0.00567	0.367716	0.005691	0.000789	0.000081
		0.5	0.005207	0.0017	0.00567	0.346758	0.004829	0.00061	0.000081
		0.9	0.005206	0.0017	0.00567	0.300869	0.002641	0.000451	0.000081
	0.5	0.2	0.005501	0.0017	0.00567	0.363316	0.004908	0.001185	0.000081
		0.5	0.005499	0.0017	0.00567	0.356084	0.00572	0.000965	0.000081
		0.9	0.005498	0.0017	0.00567	0.314874	0.003651	0.000754	0.000081
	0.7	0.2	0.005668	0.0017	0.00567	0.357249	0.005101	0.001445	0.000081
		0.5	0.005666	0.0017	0.00567	0.362186	0.006207	0.001253	0.000081
		0.9	0.005665	0.0017	0.00567	0.326752	0.005241	0.000997	0.000081

TABLE 23: RMSE of various Estimators when n=100 and T=10 for δ

MA(1) RMSE (δ) T=10, n=100									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.092393	0.053971	0.029848	0.033426	0.062089	0.068904	0.005371
		0.5	0.09517	0.053971	0.029848	0.033409	0.062298	0.068699	0.005371
		0.9	0.096541	0.053971	0.029848	0.033398	0.062218	0.068475	0.005371
	0.5	0.2	0.094823	0.053971	0.029848	0.033405	0.065206	0.071744	0.005371
		0.5	0.097252	0.053971	0.029848	0.033404	0.065105	0.07155	0.005371
		0.9	0.098291	0.053971	0.029848	0.033398	0.065107	0.071332	0.005371
	0.7	0.2	0.097092	0.053971	0.029848	0.03339	0.068165	0.073959	0.005371
		0.5	0.099202	0.053971	0.029848	0.033399	0.067721	0.073796	0.005371
		0.9	0.09994	0.053971	0.029848	0.033397	0.067575	0.073604	0.005371
0.5	0.3	0.2	0.094677	0.052921	0.029535	0.033399	0.062315	0.06872	0.005188
		0.5	0.096844	0.052921	0.029535	0.033403	0.062228	0.068489	0.005188
		0.9	0.097636	0.052921	0.029535	0.033403	0.06178	0.068286	0.005188
	0.5	0.2	0.097022	0.052921	0.029535	0.033395	0.065209	0.071537	0.005188
		0.5	0.098853	0.052921	0.029535	0.033403	0.065203	0.071316	0.005188
		0.9	0.099349	0.052921	0.029535	0.033405	0.064557	0.071116	0.005188
	0.7	0.2	0.099225	0.052921	0.029535	0.033392	0.067948	0.07374	0.005188
		0.5	0.100746	0.052921	0.029535	0.033403	0.067605	0.073554	0.005188
		0.9	0.100971	0.052921	0.029535	0.033407	0.067011	0.073368	0.005188
0.7	0.3	0.2	0.095693	0.051753	0.029279	0.033388	0.062498	0.068562	0.005007
		0.5	0.097429	0.051753	0.029279	0.033399	0.061998	0.068324	0.005007
		0.9	0.097844	0.051753	0.029279	0.033407	0.061451	0.068191	0.005007
	0.5	0.2	0.09799	0.051753	0.029279	0.033389	0.065315	0.07136	0.005007
		0.5	0.09941	0.051753	0.029279	0.033402	0.064814	0.071132	0.005007
		0.9	0.099561	0.051753	0.029279	0.03341	0.06401	0.071	0.005007
	0.7	0.2	0.100155	0.051753	0.029279	0.03339	0.067836	0.073557	0.005007
		0.5	0.101284	0.051753	0.029279	0.033404	0.067383	0.073358	0.005007
		0.9	0.101193	0.051753	0.029279	0.033414	0.066408	0.073227	0.005007

TABLE 24: RMSE of various Estimators when n=100 and T=10 for β

MA(1) RMSE (β) T=10, n=100									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.046746	0.054374	0.142975	1.324323	0.048461	0.051608	0.003243
		0.5	0.046757	0.054374	0.142975	0.70804	0.048928	0.051362	0.003243
		0.9	0.04677	0.054374	0.142975	0.410742	0.048975	0.051142	0.003243
	0.5	0.2	0.046165	0.054374	0.142975	1.024833	0.048078	0.049947	0.003243
		0.5	0.046177	0.054374	0.142975	0.6611	0.048403	0.049697	0.003243
		0.9	0.046191	0.054374	0.142975	0.410459	0.048597	0.049451	0.003243
	0.7	0.2	0.045291	0.054374	0.142975	0.782644	0.047536	0.047997	0.003243
		0.5	0.045305	0.054374	0.142975	0.606899	0.047649	0.04777	0.003243
		0.9	0.045318	0.054374	0.142975	0.406254	0.047718	0.047527	0.003243
0.5	0.3	0.2	0.046762	0.053667	0.093351	0.608919	0.049002	0.051323	0.003192
		0.5	0.046768	0.053667	0.093351	0.468247	0.048971	0.051161	0.003192
		0.9	0.046773	0.053667	0.093351	0.348645	0.048613	0.051107	0.003192
	0.5	0.2	0.046183	0.053667	0.093351	0.564157	0.048527	0.049633	0.003192
		0.5	0.046189	0.053667	0.093351	0.468006	0.048593	0.049455	0.003192
		0.9	0.046194	0.053667	0.093351	0.359603	0.048232	0.049362	0.003192
	0.7	0.2	0.045312	0.053667	0.093351	0.518218	0.047831	0.047677	0.003192
		0.5	0.045317	0.053667	0.093351	0.461936	0.047674	0.04751	0.003192
		0.9	0.04532	0.053667	0.093351	0.367435	0.047441	0.047395	0.003192
0.7	0.3	0.2	0.046777	0.052801	0.071473	0.371747	0.049282	0.051086	0.003113
		0.5	0.046777	0.052801	0.071473	0.349198	0.048882	0.051039	0.003113
		0.9	0.046775	0.052801	0.071473	0.302428	0.04824	0.051157	0.003113
	0.5	0.2	0.0462	0.052801	0.071473	0.367377	0.048701	0.049365	0.003113
		0.5	0.046198	0.052801	0.071473	0.35853	0.048481	0.049292	0.003113
		0.9	0.046195	0.052801	0.071473	0.31643	0.047718	0.049364	0.003113
	0.7	0.2	0.04533	0.052801	0.071473	0.361289	0.04789	0.047399	0.003113
		0.5	0.045327	0.052801	0.071473	0.364628	0.047695	0.047319	0.003113
		0.9	0.045321	0.052801	0.071473	0.328306	0.046986	0.047349	0.003113

TABLE 25: Absolute Bias of various Estimators when n=10 and T=20 for δ

			MA(1) ABSOLUTE BIAS (δ)			T=20	n=10		
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.023932	0.023425	0.224845	0.007995	0.025311	0.017465	0.008418
		0.5	0.02852	0.023425	0.224845	0.007374	0.02463	0.015633	0.008418
		0.9	0.033908	0.023425	0.224845	0.006776	0.025882	0.014502	0.008418
	0.5	0.2	0.027514	0.023425	0.224845	0.006896	0.026528	0.022294	0.008212
		0.5	0.032156	0.023425	0.224845	0.006862	0.026845	0.014905	0.008172
		0.9	0.037507	0.023425	0.224845	0.006495	0.027336	0.013379	0.008092
	0.7	0.2	0.030667	0.023425	0.224845	0.006095	0.027667	0.015987	0.008082
		0.5	0.035341	0.023425	0.224845	0.006403	0.029071	0.013527	0.008054
		0.9	0.040634	0.023425	0.224845	0.00622	0.027559	0.011795	0.008003
0.5	0.3	0.2	0.027575	0.023425	0.224845	0.007134	0.023822	0.016452	0.007594
		0.5	0.033103	0.023425	0.224845	0.006927	0.025276	0.014541	0.007594
		0.9	0.039025	0.023425	0.224845	0.006692	0.028718	0.013675	0.007594
	0.5	0.2	0.031211	0.023425	0.224845	0.00666	0.027879	0.015518	0.007594
		0.5	0.036737	0.023425	0.224845	0.006638	0.027632	0.013373	0.007594
		0.9	0.042566	0.023425	0.224845	0.0065	0.029696	0.415883	0.007594
	0.7	0.2	0.034407	0.023425	0.224845	0.006253	0.029567	0.013815	0.007594
		0.5	0.03991	0.023425	0.224845	0.00635	0.029934	0.209369	0.007594
		0.9	0.045632	0.023425	0.224845	0.00629	0.029003	0.010538	0.007594
0.7	0.3	0.2	0.030876	0.023425	0.224845	0.006612	0.024483	0.016018	0.007485
		0.5	0.036892	0.023425	0.224845	0.006645	0.027284	0.014373	0.007372
		0.9	0.043036	0.023425	0.224845	0.006633	0.028894	0.013633	0.007218
	0.5	0.2	0.03452	0.023425	0.224845	0.006323	0.028233	0.014754	0.007382
		0.5	0.040494	0.023425	0.224845	0.006437	0.028008	0.358175	0.007288
		0.9	0.046511	0.023425	0.224845	0.006484	0.032001	0.012193	0.007123
	0.7	0.2	0.037715	0.023425	0.224845	0.006055	0.031282	0.177503	0.007008
		0.5	0.043629	0.023425	0.224845	0.006215	0.030777	0.011111	0.007005
		0.9	0.049509	0.023425	0.224845	0.006307	0.030202	0.010404	0.007001

TABLE 26: Absolute Bias of various Estimators when n=10 and T=20 for β

			MA(1) ABSOLUTE BIAS (β)			T=20	n=10		
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.3	0.2	0.01976	0.022738	0.375945	1.484793	0.01876	0.013523	0.002524
		0.5	0.019747	0.022738	0.375945	0.623359	0.018454	0.011687	0.002524
		0.9	0.019784	0.022738	0.375945	0.291712	0.0192	0.009839	0.002524
	0.5	0.2	0.020206	0.022738	0.375945	1.343132	0.019294	0.014244	0.002524
		0.5	0.02018	0.022738	0.375945	0.596842	0.019242	0.010001	0.002524
		0.9	0.020206	0.022738	0.375945	0.282856	0.020053	0.008184	0.002524
	0.7	0.2	0.020242	0.022738	0.375945	1.131852	0.019874	0.009871	0.002524
		0.5	0.020205	0.022738	0.375945	0.543283	0.019706	0.007951	0.002524
		0.9	0.020225	0.022738	0.375945	0.264174	0.019263	0.006332	0.002524
0.5	0.3	0.2	0.019749	0.022738	0.375945	0.605546	0.01818	0.012259	0.002315
		0.5	0.019764	0.022738	0.375945	0.336256	0.019335	0.010002	0.002315
		0.9	0.019795	0.022738	0.375945	0.19665	0.020934	0.008165	0.002315
	0.5	0.2	0.020186	0.022738	0.375945	0.562943	0.019756	0.010261	0.002315
		0.5	0.020186	0.022738	0.375945	0.325419	0.020352	0.008246	0.002315
		0.9	0.020208	0.022738	0.375945	0.193648	0.021072	0.285292	0.002315
	0.7	0.2	0.020216	0.022738	0.375945	0.496328	0.020273	0.007889	0.002315
		0.5	0.020204	0.022738	0.375945	0.301646	0.020327	0.048674	0.002315
		0.9	0.02022	0.022738	0.375945	0.184169	0.020326	0.004805	0.002315
0.7	0.3	0.2	0.019774	0.022738	0.375945	0.325628	0.019582	0.01125	0.002218
		0.5	0.019793	0.022738	0.375945	0.216536	0.02052	0.009009	0.002218
		0.9	0.019797	0.022738	0.375945	0.146974	0.020417	0.007269	0.002218
	0.5	0.2	0.020206	0.022738	0.375945	0.301254	0.020401	0.009171	0.002218
		0.5	0.02021	0.022738	0.375945	0.208643	0.020938	0.23359	0.002218
		0.9	0.020207	0.022738	0.375945	0.145544	0.02158	0.005702	0.002218
	0.7	0.2	0.020233	0.022738	0.375945	0.265697	0.021111	0.028135	0.002218
		0.5	0.020226	0.022738	0.375945	0.192947	0.021237	0.005263	0.002218
		0.9	0.020217	0.022738	0.375945	0.139366	0.020542	0.004056	0.002218

TABLE 27: RMSE of various Estimators when n=10 and T=20 for δ

MA(1) RMSE (δ) T=20, n=10									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.260556	0.000549	0.050555	0.072109	0.165311	0.173129	0.000382
		0.5	0.270726	0.000549	0.050555	0.07205	0.16445	0.173869	0.000382
		0.9	0.276066	0.000549	0.050555	0.072004	0.163234	0.174847	0.000382
	0.5	0.2	0.268265	0.000549	0.050555	0.072018	0.174474	0.189163	0.000382
		0.5	0.278138	0.000549	0.050555	0.072011	0.173169	0.179445	0.000382
		0.9	0.283258	0.000549	0.050555	0.071986	0.171965	0.180329	0.000382
	0.7	0.2	0.274794	0.000549	0.050555	0.071971	0.181495	0.184874	0.000382
		0.5	0.28446	0.000549	0.050555	0.071984	0.180818	0.183528	0.000382
		0.9	0.289452	0.000549	0.050555	0.071972	0.179242	0.184276	0.000382
0.5	0.3	0.2	0.26967	0.000549	0.050555	0.072024	0.164406	0.173921	0.000335
		0.5	0.276237	0.000549	0.050555	0.072013	0.163091	0.174503	0.000335
		0.9	0.278531	0.000549	0.050555	0.072005	0.164282	0.175222	0.000335
	0.5	0.2	0.277771	0.000549	0.050555	0.07199	0.173487	0.179425	0.000335
		0.5	0.283933	0.000549	0.050555	0.071993	0.172002	0.179905	0.000335
		0.9	0.285907	0.000549	0.050555	0.071992	0.173154	0.45325	0.000335
	0.7	0.2	0.284741	0.000549	0.050555	0.071968	0.181452	0.183436	0.000335
		0.5	0.290597	0.000549	0.050555	0.071977	0.179651	0.278352	0.000335
		0.9	0.292351	0.000549	0.050555	0.071982	0.179944	0.184455	0.000335
0.7	0.3	0.2	0.273635	0.000549	0.050555	0.071981	0.16345	0.17469	0.000288
		0.5	0.277918	0.000549	0.050555	0.071994	0.163859	0.175074	0.000288
		0.9	0.278492	0.000549	0.050555	0.072007	0.163477	0.175579	0.000288
	0.5	0.2	0.282108	0.000549	0.050555	0.071962	0.172836	0.180041	0.000288
		0.5	0.285906	0.000549	0.050555	0.07198	0.172478	0.400828	0.000288
		0.9	0.286089	0.000549	0.050555	0.071998	0.172862	0.180974	0.000288
	0.7	0.2	0.289483	0.000549	0.050555	0.07195	0.180442	0.255267	0.000288
		0.5	0.292903	0.000549	0.050555	0.071969	0.179751	0.184162	0.000288
		0.9	0.292797	0.000549	0.050555	0.07199	0.179934	0.184743	0.000288

TABLE 28: RMSE of various Estimators when n=10 and T=20 for β

MA(1) RMSE (β) T=20, n=10									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.127961	0.000517	0.141335	1.531582	0.131322	0.130607	0.000419
		0.5	0.128022	0.000517	0.141335	0.654163	0.131362	0.130873	0.000419
		0.9	0.128049	0.000517	0.141335	0.313836	0.130761	0.131221	0.000419
	0.5	0.2	0.124714	0.000517	0.141335	1.391642	0.12851	0.132129	0.000419
		0.5	0.124786	0.000517	0.141335	0.629579	0.128223	0.126224	0.000419
		0.9	0.124829	0.000517	0.141335	0.306575	0.127563	0.12639	0.000419
	0.7	0.2	0.120669	0.000517	0.141335	1.182043	0.124594	0.121202	0.000419
		0.5	0.120744	0.000517	0.141335	0.578321	0.124626	0.120992	0.000419
		0.9	0.120796	0.000517	0.141335	0.289934	0.123588	0.121008	0.000419
0.5	0.3	0.2	0.128032	0.000517	0.141335	0.637194	0.131308	0.131031	0.000402
		0.5	0.128038	0.000517	0.141335	0.360408	0.130525	0.130986	0.000402
		0.9	0.12803	0.000517	0.141335	0.214988	0.131877	0.131027	0.000402
	0.5	0.2	0.124794	0.000517	0.141335	0.596876	0.128478	0.126295	0.000402
		0.5	0.124816	0.000517	0.141335	0.351414	0.127724	0.126136	0.000402
		0.9	0.124824	0.000517	0.141335	0.213317	0.12877	0.311847	0.000402
	0.7	0.2	0.12075	0.000517	0.141335	0.533082	0.125009	0.120986	0.000402
		0.5	0.120781	0.000517	0.141335	0.330028	0.124038	0.130039	0.000402
		0.9	0.1208	0.000517	0.141335	0.205629	0.124364	0.120611	0.000402
0.7	0.3	0.2	0.128063	0.000517	0.141335	0.350713	0.130969	0.13141	0.000377
		0.5	0.128037	0.000517	0.141335	0.236625	0.13151	0.131124	0.000377
		0.9	0.12802	0.000517	0.141335	0.16245	0.131749	0.13091	0.000377
	0.5	0.2	0.124836	0.000517	0.141335	0.32869	0.128232	0.126461	0.000377
		0.5	0.124826	0.000517	0.141335	0.230452	0.128224	0.265367	0.000377
		0.9	0.124821	0.000517	0.141335	0.162122	0.128622	0.125912	0.000377
	0.7	0.2	0.120797	0.000517	0.141335	0.296239	0.124331	0.12403	0.000377
		0.5	0.120799	0.000517	0.141335	0.217089	0.123992	0.120602	0.000377
		0.9	0.120803	0.000517	0.141335	0.15749	0.124664	0.120366	0.000377

TABLE 29: Absolute Bias of various Estimators when $n=20$ and $T=20$ for δ

			MA(1) ABSOLUTE BIAS (δ)			T=20	n=20		
λ	Δ	Θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.001404	0.009577	0.224845	0.007228	0.010404	0.004955	0.001203
		0.5	0.000468	0.375945	0.224845	0.006826	0.008733	0.005489	0.000351
		0.9	0.001438	0.009577	0.224845	0.006823	0.007961	0.00551	0.001282
	0.5	0.2	0.00072	0.009577	0.224845	0.007531	0.01101	0.005855	0.000542
		0.5	0.000323	0.375945	0.224845	0.006995	0.010704	0.006332	0.000254
		0.9	0.002349	0.009577	0.224845	0.006912	0.008755	0.006595	0.001253
	0.7	0.2	0.000207	0.009577	0.224845	0.007568	0.013119	0.007182	0.000153
		0.5	0.000938	0.375945	0.224845	0.007096	0.011835	0.007099	0.000438
		0.9	0.003078	0.009577	0.224845	0.006979	0.009868	0.007433	0.002578
0.5	0.3	0.2	0.000218	0.00941	0.224845	0.006857	0.009119	0.004983	0.000157
		0.5	0.00091	0.00941	0.224845	0.00679	0.007534	0.005356	0.000123
		0.9	0.002199	0.00941	0.224845	0.006869	0.008458	0.005188	0.001881
	0.5	0.2	0.001031	0.00941	0.224845	0.006997	0.011297	0.005574	0.001542
		0.5	0.001835	0.00941	0.224845	0.006897	0.008273	0.006226	0.001245
		0.9	0.003232	0.00941	0.224845	0.006943	0.007202	0.006428	0.002352
	0.7	0.2	0.001672	0.00941	0.224845	0.007073	0.012287	0.00611	0.001178
		0.5	0.002584	0.00941	0.224845	0.00698	0.010265	0.006795	0.001055
		0.9	0.004084	0.00941	0.224845	0.007008	0.007037	0.007334	0.000983
0.7	0.3	0.2	0.002183	0.009326	0.224845	0.006795	0.007916	0.004795	0.001583
		0.5	0.002408	0.009326	0.224845	0.006827	0.007385	0.005012	0.001552
		0.9	0.00302	0.009326	0.224845	0.006945	0.009602	0.004776	0.001473
	0.5	0.2	0.003137	0.009326	0.224845	0.00687	0.010207	0.005378	0.001408
		0.5	0.003454	0.009326	0.224845	0.006899	0.006754	0.005998	0.001369
		0.9	0.004153	0.009326	0.224845	0.007005	0.00786	0.00614	0.001342
	0.7	0.2	0.003916	0.009326	0.224845	0.006921	0.011637	0.005717	0.001295
		0.5	0.004325	0.009326	0.224845	0.00696	0.007002	0.006624	0.001248
		0.9	0.005108	0.009326	0.224845	0.007063	0.00557	0.007172	0.001128

TABLE 30: Absolute Bias of various Estimators when n=20 and T=20 for β

			MA(1) ABSOLUTE BIAS (β)			T=20	n=20		
λ	δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.008709	0.012435	0.375945	0.937231	0.007428	0.004343	0.002412
		0.5	0.008755	0.012435	0.375945	0.379813	0.006835	0.005114	0.002559
		0.9	0.008798	0.012435	0.375945	0.242713	0.006425	0.00563	0.002612
	0.5	0.2	0.009052	0.012435	0.375945	1.367257	0.007074	0.004951	0.002419
		0.5	0.009093	0.012435	0.375945	0.499321	0.00741	0.005488	0.002411
		0.9	0.009134	0.012435	0.375945	0.286538	0.006502	0.006087	0.002359
	0.7	0.2	0.009201	0.012435	0.375945	1.63793	0.007819	0.005856	0.002315
		0.5	0.009236	0.012435	0.375945	0.599441	0.007116	0.005812	0.002309
		0.9	0.009276	0.012435	0.375945	0.326905	0.006517	0.006335	0.002232
0.5	0.3	0.2	0.008746	0.012626	0.375945	0.40865	0.007004	0.004783	0.002589
		0.5	0.008789	0.012626	0.375945	0.25046	0.006182	0.005453	0.002525
		0.9	0.008828	0.012626	0.375945	0.186987	0.007211	0.005718	0.002503
	0.5	0.2	0.009083	0.012626	0.375945	0.51069	0.007481	0.005042	0.002419
		0.5	0.009123	0.012626	0.375945	0.299632	0.006313	0.005786	0.002613
		0.9	0.009161	0.012626	0.375945	0.212484	0.005235	0.006272	0.002595
	0.7	0.2	0.009225	0.012626	0.375945	0.585734	0.007261	0.005275	0.002518
		0.5	0.009263	0.012626	0.375945	0.343519	0.006872	0.005889	0.002498
		0.9	0.0093	0.012626	0.375945	0.23728	0.004906	0.006528	0.002343
0.7	0.3	0.2	0.008775	0.012678	0.375945	0.274027	0.00643	0.005051	0.002889
		0.5	0.008813	0.012678	0.375945	0.198197	0.005962	0.005534	0.002712
		0.9	0.008849	0.012678	0.375945	0.159163	0.00837	0.005608	0.002693
	0.5	0.2	0.00911	0.012678	0.375945	0.312609	0.007212	0.005257	0.002652
		0.5	0.009146	0.012678	0.375945	0.223722	0.005175	0.005944	0.002519
		0.9	0.00918	0.012678	0.375945	0.175764	0.00608	0.006267	0.002445
	0.7	0.2	0.009252	0.012678	0.375945	0.341897	0.007091	0.00529	0.002419
		0.5	0.009285	0.012678	0.375945	0.246923	0.004953	0.006059	0.002312
		0.9	0.009318	0.012678	0.375945	0.192306	0.003686	0.006607	0.002218

TABLE 31: RMSE of various Estimators when n=20 and T=20 for δ

MA(1) RMSE (δ) T=20, n=20									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.174095	0.054283	0.224845	0.042857	0.109464	0.131423	0.035219
		0.5	0.179553	0.054283	0.224845	0.042797	0.109087	0.131569	0.035225
		0.9	0.182002	0.054283	0.224845	0.042802	0.1084	0.131793	0.034922
	0.5	0.2	0.178597	0.054283	0.224845	0.042911	0.115636	0.140817	0.034728
		0.5	0.183791	0.054283	0.224845	0.042825	0.115426	0.140843	0.034519
		0.9	0.186014	0.054283	0.224845	0.042816	0.114865	0.140924	0.034443
	0.7	0.2	0.182355	0.054283	0.224845	0.042916	0.121135	0.148832	0.034214
		0.5	0.187352	0.054283	0.224845	0.042841	0.120999	0.14872	0.034101
		0.9	0.189419	0.054283	0.224845	0.042826	0.120456	0.148641	0.033822
0.5	0.3	0.2	0.178847	0.053934	0.224845	0.042805	0.109022	0.131458	0.036982
		0.5	0.18258	0.053934	0.224845	0.042796	0.108509	0.131843	0.035822
		0.9	0.183994	0.053934	0.224845	0.04281	0.108137	0.13218	0.036832
	0.5	0.2	0.183508	0.053934	0.224845	0.042828	0.115406	0.140742	0.036593
		0.5	0.186928	0.053934	0.224845	0.042814	0.114812	0.14102	0.035239
		0.9	0.188072	0.053934	0.224845	0.042822	0.114052	0.141237	0.035028
	0.7	0.2	0.187452	0.053934	0.224845	0.04284	0.120941	0.148636	0.034818
		0.5	0.190629	0.053934	0.224845	0.042827	0.120608	0.14878	0.033542
		0.9	0.191571	0.053934	0.224845	0.042833	0.119698	0.148848	0.033409
0.7	0.3	0.2	0.18043	0.053708	0.224845	0.0428	0.108326	0.13151	0.035832
		0.5	0.183395	0.053708	0.224845	0.042805	0.10814	0.132017	0.035813
		0.9	0.184542	0.053708	0.224845	0.042822	0.108376	0.132383	0.035972
	0.5	0.2	0.185218	0.053708	0.224845	0.042813	0.115005	0.140705	0.035773
		0.5	0.18784	0.053708	0.224845	0.042817	0.114372	0.141125	0.035814
		0.9	0.188691	0.053708	0.224845	0.042833	0.114245	0.141391	0.035712
	0.7	0.2	0.18931	0.053708	0.224845	0.042821	0.120389	0.148511	0.035525
		0.5	0.19166	0.053708	0.224845	0.042827	0.119883	0.148806	0.035414
		0.9	0.192282	0.053708	0.224845	0.042843	0.119102	0.148935	0.034999

TABLE 32: RMSE of various Estimators when n=20 and T=20 for β

MA(1) RMSE (β) T=20, n=20									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.082956	0.05414	0.375945	0.969725	0.08469	0.098764	0.04496
		0.5	0.08292	0.05414	0.375945	0.403665	0.085002	0.099283	0.04343
		0.9	0.082893	0.05414	0.375945	0.25609	0.084932	0.100002	0.04341
	0.5	0.2	0.08145	0.05414	0.375945	1.388718	0.083404	0.098716	0.04242
		0.5	0.081422	0.05414	0.375945	0.518118	0.083773	0.099123	0.04242
		0.9	0.081399	0.05414	0.375945	0.298447	0.083948	0.099702	0.04196
	0.7	0.2	0.079328	0.05414	0.375945	1.654074	0.081783	0.097533	0.04154
		0.5	0.079304	0.05414	0.375945	0.614978	0.082153	0.097817	0.04124
		0.9	0.079284	0.05414	0.375945	0.337627	0.082389	0.098257	0.04025
0.5	0.3	0.2	0.082927	0.054106	0.375945	0.430919	0.084891	0.099144	0.04569
		0.5	0.082897	0.054106	0.375945	0.2665	0.084881	0.099911	0.04569
		0.9	0.082881	0.054106	0.375945	0.196922	0.085012	0.100743	0.04569
	0.5	0.2	0.081426	0.054106	0.375945	0.528707	0.083809	0.099003	0.04569
		0.5	0.081403	0.054106	0.375945	0.313731	0.083865	0.099659	0.04569
		0.9	0.081389	0.054106	0.375945	0.221777	0.083673	0.100365	0.04569
	0.7	0.2	0.079305	0.054106	0.375945	0.600994	0.082236	0.097725	0.04569
		0.5	0.079286	0.054106	0.375945	0.3561	0.082446	0.098258	0.04569
		0.9	0.079275	0.054106	0.375945	0.245983	0.082137	0.098829	0.04569
0.7	0.3	0.2	0.082906	0.054048	0.375945	0.288906	0.08482	0.099547	0.04598
		0.5	0.082885	0.054048	0.375945	0.209416	0.084995	0.100436	0.04598
		0.9	0.082879	0.054048	0.375945	0.166665	0.085445	0.101261	0.04598
	0.5	0.2	0.081406	0.054048	0.375945	0.325921	0.084051	0.099329	0.04598
		0.5	0.081391	0.054048	0.375945	0.234178	0.08396	0.100117	0.04598
		0.9	0.081386	0.054048	0.375945	0.182999	0.08401	0.10083	0.04598
	0.7	0.2	0.079284	0.054048	0.375945	0.354009	0.082318	0.097986	0.04598
		0.5	0.079273	0.054048	0.375945	0.256705	0.082351	0.09865	0.04598
		0.9	0.079271	0.054048	0.375945	0.199267	0.081937	0.099236	0.04598

TABLE 33: Absolute Bias of various Estimators when n=50 and T=20 for δ

			MA(1) ABSOLUTE BIAS (δ)			T=20	n=50		
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.016403	0.010059	0.445694	0.000201	0.000333	0.003947	0.002913
		0.5	0.014482	0.014987	0.352266	0.000217	0.000779	0.0038	0.002913
		0.9	0.011567	0.00992	0.053138	0.000228	0.000205	0.004002	0.002913
	0.5	0.2	0.016544	0.019353	0.148945	0.00019	0.00033	0.004531	0.002913
		0.5	0.014527	0.017468	0.218937	0.000207	2.71E-06	0.00423	0.002913
		0.9	0.011551	0.016294	0.442758	0.000221	0.002146	0.004544	0.002913
	0.7	0.2	0.016542	0.018349	0.342915	0.000183	0.002298	0.005552	0.002913
		0.5	0.01444	0.016477	0.423254	0.000199	0.000138	0.004568	0.002913
		0.9	0.011413	0.01037	0.344037	0.000214	0.002237	0.004854	0.002913
0.5	0.3	0.2	0.014603	0.017468	0.401125	0.000213	0.000616	0.003736	0.002839
		0.5	0.012322	0.017394	0.433883	0.000228	8.06E-05	0.003883	0.002839
		0.9	0.009391	0.015303	0.387089	0.00024	0.001021	0.004223	0.002839
	0.5	0.2	0.014707	0.018208	0.301762	0.000205	0.000465	0.004104	0.002839
		0.5	0.012337	0.017394	0.438271	0.000221	0.001861	0.004358	0.002839
		0.9	0.009359	0.014702	0.356069	0.000236	0.000928	0.004895	0.002839
	0.7	0.2	0.014672	0.018208	0.246992	0.000197	0.000279	0.004445	0.002839
		0.5	0.012225	0.019238	0.274664	0.000215	0.000474	0.004591	0.002839
		0.9	0.009209	0.015371	0.419202	0.000231	0.003365	0.005303	0.002839
0.7	0.3	0.2	0.012558	0.010844	0.147232	0.00022	0.001367	0.003868	0.002745
		0.5	0.010235	0.018216	0.409147	0.000235	9.75E-05	0.004152	0.002745
		0.9	0.007476	0.006703	0.160441	0.000249	0.001716	0.004464	0.002745
	0.5	0.2	0.012632	0.017974	0.367918	0.000214	5.92E-05	0.004251	0.002745
		0.5	0.010234	0.015317	0.391017	0.000231	0.001925	0.004724	0.002745
		0.9	0.00744	0.015475	0.296547	0.000247	0.000267	0.005249	0.002745
	0.7	0.2	0.012572	0.016292	0.344492	0.000208	0.000619	0.004452	0.002745
		0.5	0.01011	0.015317	0.345507	0.000227	0.003736	0.005026	0.002745
		0.9	0.00729	0.008421	0.41825	0.000244	0.002498	0.005775	0.002745

TABLE 34: Absolute Bias of various Estimators when n=50 and T=20 for β

λ	Δ	θ	MA(1) ABSOLUTE BIAS (β)			T=20	n=50		
			OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.004061	0.014793	0.466512	0.562397	0.001809	0.00654	0.001552
		0.5	0.004111	0.034587	0.358149	0.477685	0.002219	0.006606	0.001552
		0.9	0.00414	0.028947	0.049809	0.362366	0.001789	0.006902	0.001552
	0.5	0.2	0.003786	0.04026	0.154154	0.463333	0.001488	0.006215	0.001552
		0.5	0.003833	0.038928	0.226632	0.454913	0.001653	0.006233	0.001552
		0.9	0.003862	0.038463	0.457094	0.365051	0.003306	0.006672	0.001552
	0.7	0.2	0.00347	0.039276	0.354679	0.38918	0.002517	0.006259	0.001552
		0.5	0.003514	0.037946	0.440547	0.425685	0.001492	0.005816	0.001552
		0.9	0.003542	0.022159	0.355361	0.361428	0.003249	0.006275	0.001552
0.5	0.3	0.2	0.004113	0.03986	0.45362	0.476971	0.002101	0.006471	0.001453
		0.5	0.004137	0.040693	0.490052	0.393115	0.001686	0.006812	0.001453
		0.9	0.004144	0.038823	0.43037	0.313947	0.000297	0.007233	0.001453
	0.5	0.2	0.003837	0.04093	0.333523	0.459066	0.001397	0.006056	0.001453
		0.5	0.00386	0.040693	0.495225	0.398282	0.003016	0.006531	0.001453
		0.9	0.003867	0.037922	0.394667	0.326455	0.002122	0.007137	0.001453
	0.7	0.2	0.00352	0.04093	0.279701	0.43604	0.001319	0.005657	0.001453
		0.5	0.003541	0.041955	0.31079	0.39642	0.002116	0.006083	0.001453
		0.9	0.003548	0.039378	0.466654	0.334333	0.003945	0.006834	0.001453
0.7	0.3	0.2	0.00414	0.022952	0.154542	0.386665	0.002866	0.00665	0.001105
		0.5	0.004139	0.04326	0.465075	0.277589	0.000729	0.007514	0.001105
		0.9	0.004146	0.008073	0.167385	0.331849	0.001527	0.00711	0.001105
	0.5	0.2	0.003864	0.04235	0.416471	0.390927	0.001922	0.006294	0.001105
		0.5	0.003869	0.03754	0.446522	0.34527	0.00304	0.006934	0.001105
		0.9	0.003863	0.040921	0.331862	0.293389	0.000636	0.007521	0.001105
	0.7	0.2	0.003547	0.039823	0.390666	0.389407	0.001576	0.005829	0.001105
		0.5	0.003551	0.03754	0.390838	0.353735	0.004316	0.006557	0.001105
		0.9	0.003546	0.017851	0.472172	0.306104	0.00291	0.007312	0.001105

TABLE 35: RMSE of various Estimators when n=50 and T=20 for δ

MA(1) RMSE (δ) T=20, n=50									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.081899	0.036076	0.452857	0.032915	0.062392	0.075449	0.025765
		0.5	0.085049	0.021849	0.403882	0.032925	0.062729	0.075422	0.024156
		0.9	0.087315	0.026717	0.154464	0.032931	0.062314	0.075609	0.025152
	0.5	0.2	0.085061	0.019399	0.260836	0.032904	0.06658	0.080036	0.020953
		0.5	0.087859	0.020281	0.319264	0.032916	0.06706	0.079918	0.020717
		0.9	0.089777	0.01922	0.451105	0.032925	0.066853	0.080012	0.020512
	0.7	0.2	0.087933	0.021087	0.398415	0.032896	0.070331	0.084031	0.027712
		0.5	0.090421	0.021222	0.441261	0.032909	0.070796	0.083797	0.025613
		0.9	0.092029	0.031093	0.398902	0.032919	0.070878	0.083795	0.026131
0.5	0.3	0.2	0.084523	0.022007	0.426094	0.032922	0.062744	0.075433	0.019316
		0.5	0.087359	0.020271	0.442744	0.032931	0.062362	0.075545	0.019316
		0.9	0.089322	0.020301	0.419567	0.032939	0.062157	0.075867	0.019291
	0.5	0.2	0.087596	0.021012	0.367135	0.032914	0.06701	0.079924	0.018127
		0.5	0.090053	0.020271	0.445041	0.032926	0.066848	0.079965	0.018002
		0.9	0.091657	0.020874	0.402193	0.032935	0.066603	0.080216	0.019817
	0.7	0.2	0.090388	0.021012	0.33536	0.032908	0.070686	0.083795	0.019026
		0.5	0.092511	0.019295	0.353283	0.03292	0.070797	0.083765	0.019015
		0.9	0.093798	0.020347	0.436223	0.032931	0.070663	0.083939	0.019012
0.7	0.3	0.2	0.086016	0.032334	0.252096	0.032925	0.06246	0.075541	0.022413
		0.5	0.088657	0.018287	0.426477	0.032935	0.062127	0.075754	0.022221
		0.9	0.090452	0.037285	0.264018	0.032945	0.062205	0.076172	0.022165
	0.5	0.2	0.089026	0.02084	0.404901	0.03292	0.066975	0.079958	0.021077
		0.5	0.091279	0.021787	0.417166	0.032932	0.06668	0.080116	0.021065
		0.9	0.092713	0.020392	0.364663	0.032943	0.066664	0.080492	0.021062
	0.7	0.2	0.091757	0.022276	0.392593	0.032915	0.0709	0.08375	0.023165
		0.5	0.093669	0.021787	0.392908	0.032928	0.070923	0.083855	0.022172
		0.9	0.094787	0.03432	0.43154	0.03294	0.070586	0.084174	0.020165

TABLE 36: RMSE of various Estimators when n=50 and T=20 for β

MA(1) RMSE (β) T=20, n=50									
λ	Δ	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.3	0.2	0.050816	0.040749	0.47406	0.592911	0.052019	0.055271	0.034275
		0.5	0.050773	0.040542	0.425053	0.488012	0.052214	0.055174	0.032172
		0.9	0.050723	0.039066	0.210872	0.366976	0.052281	0.055211	0.032152
	0.5	0.2	0.050313	0.041669	0.302	0.496836	0.051782	0.054954	0.034065
		0.5	0.050271	0.040911	0.350307	0.465799	0.05198	0.054804	0.034002
		0.9	0.050222	0.040442	0.463859	0.36979	0.052088	0.054784	0.033017
	0.7	0.2	0.049372	0.041264	0.426928	0.423304	0.05107	0.054177	0.032192
		0.5	0.049332	0.040994	0.460869	0.436946	0.051214	0.053939	0.031926
		0.9	0.049286	0.041145	0.41565	0.366288	0.051399	0.053862	0.030562
0.5	0.3	0.2	0.05077	0.042883	0.482522	0.487199	0.052266	0.055181	0.031762
		0.5	0.050732	0.04264	0.499504	0.39845	0.052199	0.055177	0.031516
		0.9	0.050691	0.042201	0.47114	0.316923	0.052483	0.055253	0.031217
	0.5	0.2	0.050265	0.042879	0.419446	0.469501	0.051991	0.054799	0.030198
		0.5	0.050229	0.04264	0.50209	0.403728	0.051958	0.054759	0.030209
		0.9	0.050189	0.041908	0.453382	0.329472	0.052205	0.05479	0.030107
	0.7	0.2	0.049323	0.042879	0.385874	0.446474	0.051175	0.053926	0.031982
		0.5	0.04929	0.043299	0.403776	0.401946	0.051195	0.053844	0.031872
		0.9	0.049253	0.042399	0.489072	0.337395	0.051492	0.053838	0.031612
0.7	0.3	0.2	0.050731	0.043109	0.293003	0.392061	0.052272	0.055214	0.035565
		0.5	0.050698	0.04455	0.485549	0.335229	0.052388	0.055238	0.034162
		0.9	0.050665	0.040156	0.30379	0.279711	0.052531	0.055325	0.034012
	0.5	0.2	0.050224	0.044255	0.46388	0.396293	0.052014	0.054777	0.033106
		0.5	0.050194	0.043294	0.474969	0.348657	0.052129	0.054775	0.033017
		0.9	0.050163	0.043897	0.414983	0.295517	0.052314	0.054834	0.033006
	0.7	0.2	0.049281	0.043736	0.449166	0.394707	0.051306	0.053845	0.032962
		0.5	0.049254	0.043294	0.448691	0.357126	0.051579	0.053822	0.031976
		0.9	0.049226	0.041858	0.488698	0.308244	0.051586	0.053856	0.030673

APPENDIX 3

TABLE 37: Absolute Bias of various Estimators when n=200 and T=5 for δ

			ARMA(1,1) ABSOLUTE BIAS			($\delta=0.3$)	T=5	n=200	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.015801	5.02E-05	0.014611	0.001861	0.015128	0.002409	0.002182
		0.5	0.017019	0.000279	0.011736	0.001817	0.01047	0.000637	0.002181
		0.9	0.017945	0.000482	0.00875	0.001777	0.008288	0.000813	0.002173
	0.5	0.2	0.015974	0.001781	0.005127	0.001849	0.009747	0.001274	0.002172
		0.5	0.017494	0.001196	0.006019	0.001834	0.016137	9.08E-05	0.002172
		0.9	0.018704	0.00076	0.005938	0.001802	0.01434	0.00043	0.002159
	0.9	0.2	0.015837	0.001672	0.001639	0.001837	0.001312	0.011649	0.002152
		0.5	0.016567	0.002278	0.002274	0.001832	0.000591	0.010666	0.00215
		0.9	0.017673	0.001804	0.00307	0.001817	0.006822	0.007171	0.002131
0.5	0.2	0.2	0.017473	0.000348	0.014272	0.001847	0.016357	0.002342	0.002155
		0.5	0.018663	5.92E-06	0.011739	0.001808	0.011259	0.000105	0.002151
		0.9	0.019556	0.00026	0.008868	0.001773	0.008668	0.001384	0.002154
	0.5	0.2	0.017777	0.001883	0.004896	0.001841	0.012106	0.000917	0.002154
		0.5	0.019228	0.001384	0.005783	0.001827	0.018003	0.000257	0.002154
		0.9	0.02036	0.000906	0.005834	0.001797	0.018	0.000741	0.002154
	0.9	0.2	0.017447	0.002012	0.001418	0.001833	1.39E-06	0.012694	0.002154
		0.5	0.018159	0.002339	0.002072	0.001827	0.002208	0.010843	0.002154
		0.9	0.019363	0.001853	0.002915	0.001812	0.011283	0.006899	0.002154
0.7	0.2	0.2	0.018963	0.000659	0.013823	0.001834	0.017816	0.001975	0.002093
		0.5	0.020129	0.000273	0.011628	0.001799	0.012908	0.000472	0.002091
		0.9	0.020992	5.11E-05	0.008912	0.001767	0.010374	0.001991	0.002094
	0.5	0.2	0.019419	0.002018	0.004885	0.001838	0.014769	0.000501	0.002085
		0.5	0.020799	0.00155	0.005723	0.001823	0.017922	0.00086	0.002072
		0.9	0.021843	0.001041	0.0058	0.001792	0.020269	0.001146	0.002066
	0.9	0.2	0.018865	0.002297	0.001317	0.001832	0.000859	0.013801	0.002061
		0.5	0.019634	0.002387	0.002051	0.001823	0.002309	0.011185	0.002053
		0.9	0.020881	0.001889	0.002864	0.001808	0.014318	0.006603	0.002048

TABLE 38: Absolute Bias of various Estimators when n=200 and T=5 for δ

			ARMA(1,1) ABSOLUTE BIAS			($\delta=0.5$)	T=5	n=200	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.017705	0.000506	0.011017	0.001821	0.010309	0.001079	0.002154
		0.5	0.018487	0.000467	0.008825	0.001793	0.008829	0.000442	0.002154
		0.9	0.018904	0.000374	0.006746	0.001766	0.008687	0.001484	0.002154
	0.5	0.2	0.018596	0.000944	0.0047	0.001846	0.011266	0.00104	0.002143
		0.5	0.019696	0.000652	0.004892	0.001825	0.013895	0.00024	0.002143
		0.9	0.020273	0.000456	0.004684	0.001791	0.01093	0.000651	0.002143
	0.9	0.2	0.018101	0.001575	0.001413	0.001843	0.002942	0.012111	0.002129
		0.5	0.01892	0.001583	0.001955	0.001835	0.002532	0.00978	0.002128
		0.9	0.019836	0.001268	0.002521	0.001812	0.005707	0.005517	0.002122
0.5	0.2	0.2	0.019448	0.00018	0.010838	0.001811	0.011234	0.00059	0.002123
		0.5	0.020178	0.000203	0.008858	0.001788	0.009347	0.000996	0.002123
		0.9	0.02054	0.000181	0.006857	0.001764	0.008876	0.002047	0.002123
	0.5	0.2	0.020468	0.001089	0.004501	0.00184	0.014076	0.000539	0.002136
		0.5	0.021436	0.000821	0.004743	0.00182	0.016815	1.13E-05	0.002132
		0.9	0.021951	0.000579	0.004658	0.001788	0.013608	0.000995	0.002118
	0.9	0.2	0.019752	0.001789	0.001166	0.001839	0.002234	0.012507	0.002118
		0.5	0.020617	0.001636	0.00178	0.001831	0.000539	0.009395	0.002118
		0.9	0.021549	0.001284	0.002382	0.001808	0.009467	0.00537	0.002118
0.7	0.2	0.2	0.021011	0.000131	0.010587	0.001802	0.01234	3.18E-05	0.002088
		0.5	0.02169	4.73E-05	0.008819	0.001782	0.009903	0.001598	0.002072
		0.9	0.022	1.42E-06	0.006918	0.00176	0.009351	0.002614	0.002063
	0.5	0.2	0.022168	0.001235	0.004466	0.001836	0.015278	5.87E-05	0.002042
		0.5	0.022995	0.000973	0.00472	0.001815	0.01918	0.00043	0.002033
		0.9	0.023457	0.000695	0.004665	0.001784	0.015735	0.001328	0.002025
	0.9	0.2	0.021254	0.001977	0.001085	0.001835	0.000802	0.012758	0.002027
		0.5	0.022169	0.001648	0.001721	0.001827	0.002968	0.009207	0.002026
		0.9	0.023095	0.001302	0.002311	0.001805	0.011734	0.005292	0.002017

TABLE 39: Absolute Bias of various Estimators when n=200 and T=5 for δ

			ARMA(1,1) ABSOLUTE BIAS			($\delta=0.7$)	T=5	n=200	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.019057	0.000863	0.008583	0.001792	0.008695	0.000187	0.001992
		0.5	0.019472	0.000578	0.006972	0.001775	0.0087	0.000995	0.001992
		0.9	0.019495	0.000294	0.005413	0.001755	0.009684	0.001816	0.001992
	0.5	0.2	0.020568	0.00029	0.004218	0.001834	0.009693	0.001861	0.001963
		0.5	0.021142	0.000201	0.004138	0.001813	0.010124	0.000607	0.001961
		0.9	0.021324	0.000215	0.003829	0.001777	0.009675	0.001168	0.001953
	0.9	0.2	0.020029	0.001133	0.001329	0.001853	0.005463	0.012981	0.001953
		0.5	0.020671	0.000947	0.001778	0.001842	0.003348	0.009539	0.001953
		0.9	0.021317	0.00079	0.002066	0.001808	0.002215	0.005842	0.001953
0.5	0.2	0.2	0.020857	0.000551	0.008449	0.001785	0.009244	0.000377	0.001886
		0.5	0.0212	0.000338	0.006987	0.001772	0.008943	0.00154	0.001882
		0.9	0.021153	0.000126	0.005497	0.001755	0.009911	0.002375	0.001881
	0.5	0.2	0.022402	0.000431	0.004031	0.001831	0.012736	0.001073	0.001877
		0.5	0.022905	0.000348	0.004041	0.001808	0.012035	0.000806	0.001872
		0.9	0.023006	0.000332	0.003814	0.001776	0.011874	0.001449	0.001875
	0.9	0.2	0.021717	0.001175	0.001166	0.001849	0.004199	0.0128	0.001887
		0.5	0.022407	0.000982	0.001606	0.001837	0.001099	0.009513	0.001879
		0.9	0.023007	0.000806	0.001937	0.001806	0.004172	0.005916	0.001874
0.7	0.2	0.2	0.022475	0.000255	0.008266	0.001778	0.010041	0.000974	0.001859
		0.5	0.022748	0.000111	0.006954	0.001767	0.009385	0.002114	0.001855
		0.9	0.022634	3.28E-05	0.005545	0.001753	0.009325	0.002906	0.001853
	0.5	0.2	0.024045	0.000577	0.00396	0.001827	0.015635	0.000755	0.001851
		0.5	0.02449	0.000487	0.003996	0.001803	0.015432	0.001039	0.001848
		0.9	0.024516	0.000457	0.003823	0.001774	0.012737	0.001986	0.001846
	0.9	0.2	0.023275	0.00119	0.001115	0.001844	0.005149	0.012719	0.001844
		0.5	0.023986	0.001015	0.00153	0.001833	0.001964	0.009367	0.001841
		0.9	0.024543	0.000831	0.001881	0.001804	0.007318	0.005977	0.001839

TABLE 40: Absolute Bias of various Estimators when n=200 and T=5 for β

			ARMA(1,1) ABSOLUTE BIAS (β)			$\delta=0.3$	T=5	n=200	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.008123	0.001719	0.031389	0.422375	0.012821	0.00136	0.000196
		0.5	0.008131	0.001975	0.019606	0.140698	0.009967	0.000613	0.000196
		0.9	0.008143	0.001993	0.012097	0.017236	0.009025	0.000422	0.000196
	0.5	0.2	0.008097	0.001739	0.044972	0.265017	0.008749	0.001537	0.000196
		0.5	0.008103	0.000761	0.042519	0.162014	0.01246	0.001029	0.000196
		0.9	0.008117	0.000133	0.034763	0.058532	0.011944	0.000729	0.000196
	0.9	0.2	0.008036	0.003293	0.03387	0.124949	0.003864	0.005111	0.000196
		0.5	0.008073	0.00283	0.03897	0.106667	0.003847	0.006019	0.000196
		0.9	0.008091	0.002081	0.038459	0.069185	0.007625	0.004931	0.000196
0.5	0.2	0.2	0.008686	0.001543	0.034049	0.35141	0.013045	0.001079	0.000185
		0.5	0.008693	0.001775	0.025471	0.116919	0.009989	2.22E-05	0.000185
		0.9	0.008703	0.001836	0.018712	0.00831	0.008854	0.000187	0.000185
	0.5	0.2	0.008662	0.001746	0.05161	0.243127	0.009938	0.001447	0.000185
		0.5	0.008671	0.000864	0.049686	0.150335	0.012967	0.001369	0.000185
		0.9	0.008684	0.000225	0.042241	0.053065	0.013543	0.001099	0.000185
	0.9	0.2	0.008609	0.003447	0.040498	0.121558	0.004633	0.005631	0.000185
		0.5	0.008642	0.002861	0.046343	0.102965	0.005328	0.006074	0.000185
		0.9	0.008658	0.00212	0.04605	0.066	0.00962	0.004741	0.000185
0.7	0.2	0.2	0.009041	0.001323	0.031361	0.254588	0.013115	0.000754	0.000181
		0.5	0.009047	0.001582	0.026127	0.085797	0.0103	0.000492	0.000181
		0.9	0.009056	0.001682	0.020998	0.003165	0.009073	0.000735	0.000181
	0.5	0.2	0.009021	0.001737	0.051862	0.213271	0.011093	0.001296	0.000181
		0.5	0.009031	0.000968	0.051296	0.13195	0.012369	0.001807	0.000181
		0.9	0.009043	0.000315	0.04481	0.044858	0.014095	0.00147	0.000181
	0.9	0.2	0.008977	0.003567	0.043273	0.116083	0.004291	0.006185	0.000181
		0.5	0.009005	0.002881	0.048903	0.095545	0.005116	0.006246	0.000181
		0.9	0.009019	0.002144	0.048991	0.060187	0.010615	0.004597	0.000181

TABLE 41: Absolute Bias of various Estimators when N=200 and T=5 for β

			ARMA(1,1) ABSOLUTE BIAS (β)			$\delta=0.5$	T=5	n=200	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.008124	0.002133	0.020765	0.138389	0.009926	0.00096	0.000188
		0.5	0.008137	0.00205	0.014835	0.047399	0.009398	0.000521	0.000188
		0.9	0.008152	0.001828	0.009889	0.003598	0.009132	0.000559	0.000188
	0.5	0.2	0.008081	0.000691	0.044555	0.147394	0.009448	0.001378	0.000188
		0.5	0.0081	5.54E-05	0.039583	0.087958	0.011505	0.000328	0.000188
		0.9	0.008124	0.000295	0.030129	0.027302	0.010396	0.000116	0.000188
	0.9	0.2	0.008025	0.002315	0.042635	0.089572	0.00336	0.005731	0.000188
		0.5	0.008054	0.001918	0.041233	0.073404	0.001665	0.005636	0.000188
		0.9	0.008076	0.001271	0.037251	0.041757	0.005692	0.003549	0.000188
0.5	0.2	0.2	0.008686	0.001911	0.024475	0.112382	0.010104	0.000416	0.000182
		0.5	0.008698	0.00186	0.020543	0.0379	0.00912	5.41E-05	0.000182
		0.9	0.008711	0.001686	0.016461	0.007827	0.008846	1.57E-05	0.000182
	0.5	0.2	0.008649	0.00073	0.049413	0.138682	0.010759	0.001155	0.000182
		0.5	0.008667	0.000171	0.04537	0.082787	0.012561	0.000626	0.000182
		0.9	0.008691	0.000204	0.036879	0.024721	0.011594	0.000508	0.000182
	0.9	0.2	0.008597	0.002418	0.047503	0.085938	0.003731	0.005918	0.000182
		0.5	0.008621	0.001975	0.047097	0.07217	0.003307	0.005423	0.000182
		0.9	0.008644	0.001282	0.044148	0.040563	0.007592	0.003499	0.000182
0.7	0.2	0.2	0.009042	0.001687	0.023508	0.080087	0.010206	5.29E-05	0.000176
		0.5	0.009052	0.001674	0.021801	0.024888	0.008746	0.000576	0.000176
		0.9	0.009063	0.001546	0.019068	0.013836	0.008696	0.000521	0.000176
	0.5	0.2	0.009012	0.000764	0.048863	0.125895	0.011022	0.000898	0.000176
		0.5	0.009028	0.000283	0.046215	0.074308	0.012973	0.001007	0.000176
		0.9	0.00905	0.000115	0.039397	0.020191	0.012073	0.000842	0.000176
	0.9	0.2	0.008963	0.002508	0.048248	0.080373	0.004432	0.006055	0.000176
		0.5	0.008981	0.001987	0.048595	0.067634	0.004634	0.005249	0.000176
		0.9	0.009005	0.001295	0.046835	0.038437	0.008469	0.003485	0.000176

TABLE 42: Absolute Bias of various Estimators when n=200 and T=5 for β

			ARMA(1,1)ABSOLUTE BIAS (β)			$\delta=0.7$	T=5	n=200	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.008127	0.002306	0.015229	0.042783	0.009007	0.000974	0.000213
		0.5	0.008144	0.002021	0.01205	0.009646	0.009058	0.000742	0.000213
		0.9	0.00816	0.001657	0.008338	0.014722	0.009592	0.000799	0.000213
	0.5	0.2	0.008072	6.11E-05	0.040168	0.087118	0.008098	0.001335	0.000213
		0.5	0.008104	0.000433	0.034471	0.049301	0.008655	0.000148	0.000213
		0.9	0.008136	0.000571	0.025861	0.007923	0.009981	0.000206	0.000213
	0.9	0.2	0.008005	0.001577	0.042891	0.073366	0.00011	0.006525	0.000213
		0.5	0.008041	0.001104	0.039364	0.058896	0.000319	0.005336	0.000213
		0.9	0.008072	0.00064	0.035363	0.027988	0.002552	0.00322	0.000213
0.5	0.2	0.2	0.008689	0.002087	0.019285	0.030159	0.008912	0.000385	0.000211
		0.5	0.008704	0.001846	0.017648	0.004575	0.008879	0.00018	0.000211
		0.9	0.008718	0.001531	0.014854	0.017053	0.009481	0.000235	0.000211
	0.5	0.2	0.008642	2.22E-05	0.044183	0.084337	0.009843	0.000947	0.000211
		0.5	0.008671	0.000322	0.039836	0.046133	0.009632	0.00043	0.000211
		0.9	0.0087	0.000492	0.032381	0.007555	0.011149	0.000136	0.000211
	0.9	0.2	0.008574	0.001617	0.046455	0.070967	0.001056	0.006368	0.000211
		0.5	0.008606	0.001124	0.044115	0.057644	0.000861	0.005236	0.000211
		0.9	0.008637	0.00066	0.041595	0.028699	0.00354	0.003306	0.000211
0.7	0.2	0.2	0.009044	0.001872	0.019184	0.015406	0.009246	0.00012	0.000205
		0.5	0.009056	0.001674	0.019223	0.002427	0.008523	0.000324	0.000205
		0.9	0.009068	0.001405	0.017668	0.020667	0.008611	0.000239	0.000205
	0.5	0.2	0.009006	5.39E-05	0.043415	0.076368	0.010848	0.000869	0.000205
		0.5	0.009032	0.000214	0.040942	0.04101	0.010852	0.000684	0.000205
		0.9	0.009056	0.000396	0.034979	0.004985	0.01091	0.000576	0.000205
	0.9	0.2	0.008939	0.001615	0.046102	0.066015	0.000496	0.006275	0.000205
		0.5	0.008967	0.001143	0.04503	0.054469	0.003304	0.005067	0.000205
		0.9	0.008996	0.000671	0.043689	0.028319	0.00532	0.003345	0.000205

TABLE 43: RMSE of various Estimators when n=200 and T=5 for δ

ARMA(1,1) RMSE ($\delta=0.3$) T=5 n=200									
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.085415	0.074553	0.104588	0.027667	0.07296	0.070097	0.025776
		0.5	0.088066	0.066664	0.075756	0.027665	0.07231	0.070226	0.025771
		0.9	0.090494	0.056941	0.054444	0.027663	0.072146	0.070557	0.025772
	0.5	0.2	0.083772	0.067563	0.073469	0.027665	0.071777	0.070482	0.025746
		0.5	0.085855	0.061305	0.060618	0.027664	0.073216	0.070482	0.025742
		0.9	0.087772	0.053515	0.047281	0.027663	0.073184	0.070813	0.025737
	0.9	0.2	0.08177	0.058624	0.049197	0.027662	0.070983	0.072012	0.025693
		0.5	0.082828	0.054339	0.044529	0.027663	0.071873	0.071644	0.025693
		0.9	0.084758	0.048189	0.037972	0.027663	0.073111	0.071091	0.025693
0.5	0.2	0.2	0.088396	0.073444	0.104073	0.027666	0.076364	0.073546	0.025532
		0.5	0.090896	0.065637	0.076369	0.027664	0.075453	0.073774	0.025521
		0.9	0.09317	0.056106	0.05512	0.027663	0.075237	0.074195	0.025517
	0.5	0.2	0.086604	0.06646	0.072515	0.027664	0.075263	0.0738	0.025333
		0.5	0.088521	0.060488	0.060647	0.027664	0.076808	0.073988	0.025333
		0.9	0.09037	0.052824	0.047691	0.027663	0.07709	0.074416	0.025333
	0.9	0.2	0.084513	0.057757	0.048179	0.027662	0.074638	0.075538	0.025333
		0.5	0.085473	0.053528	0.044005	0.027663	0.075312	0.075156	0.025333
		0.9	0.087343	0.047408	0.038019	0.027663	0.076817	0.074648	0.025333
0.7	0.2	0.2	0.090967	0.07232	0.10192	0.027666	0.079124	0.076385	0.025129
		0.5	0.093322	0.064619	0.076313	0.027664	0.078163	0.076742	0.025129
		0.9	0.095474	0.055284	0.055504	0.027662	0.077868	0.07729	0.025129
	0.5	0.2	0.088995	0.065501	0.070718	0.027664	0.078277	0.076568	0.025129
		0.5	0.090781	0.059678	0.060202	0.027664	0.07921	0.076927	0.025129
		0.9	0.092587	0.052141	0.047852	0.027663	0.080156	0.077482	0.025129
	0.9	0.2	0.086779	0.05701	0.046991	0.027662	0.077465	0.078451	0.025129
		0.5	0.08774	0.052721	0.043214	0.027663	0.078003	0.078078	0.025129
		0.9	0.089523	0.046666	0.037845	0.027662	0.0798	0.077682	0.025129

TABLE 44: RMSE of various Estimators when n=200 and T=5 for δ

ARMA(1,1) RMSE ($\delta=0.5$) T=5 n=200									
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.087754	0.067104	0.076024	0.027665	0.072165	0.070276	0.025679
		0.5	0.090097	0.058921	0.058566	0.027664	0.072333	0.070536	0.025679
		0.9	0.092081	0.049746	0.044212	0.027662	0.072259	0.070905	0.025679
	0.5	0.2	0.085436	0.061161	0.059623	0.027664	0.072434	0.070657	0.025679
		0.5	0.08724	0.054634	0.04968	0.027664	0.073109	0.070948	0.025679
		0.9	0.089049	0.047096	0.039683	0.027662	0.072712	0.071237	0.025679
	0.9	0.2	0.08208	0.053477	0.04397	0.027662	0.071832	0.07226	0.025679
		0.5	0.083972	0.04858	0.03896	0.027662	0.072758	0.071852	0.025679
		0.9	0.085645	0.042405	0.033274	0.027662	0.073153	0.071385	0.025679
0.5	0.2	0.2	0.090712	0.066008	0.076169	0.027664	0.075363	0.073808	0.025619
		0.5	0.092879	0.057966	0.059239	0.027663	0.075445	0.074164	0.025619
		0.9	0.094709	0.049	0.044902	0.027662	0.075222	0.074582	0.025619
	0.5	0.2	0.088215	0.060139	0.059311	0.027664	0.076035	0.07414	0.025619
		0.5	0.089908	0.053875	0.049965	0.027664	0.076826	0.074519	0.025619
		0.9	0.091641	0.046434	0.040208	0.027662	0.076315	0.074914	0.025619
	0.9	0.2	0.084829	0.05259	0.043344	0.027662	0.075202	0.075717	0.025619
		0.5	0.086613	0.047687	0.038764	0.027662	0.075916	0.075267	0.025619
		0.9	0.088182	0.041692	0.033496	0.027662	0.076522	0.074977	0.025619
0.7	0.2	0.2	0.093259	0.064913	0.075642	0.027664	0.078104	0.07677	0.025523
		0.5	0.095278	0.057022	0.059537	0.027663	0.077842	0.07725	0.025523
		0.9	0.096985	0.048267	0.0454	0.027662	0.077666	0.077747	0.025523
	0.5	0.2	0.09058	0.05919	0.00896	0.027664	0.078715	0.077073	0.025523
		0.5	0.092197	0.053114	0.049943	0.027663	0.080036	0.077557	0.025523
		0.9	0.093868	0.045788	0.040554	0.027662	0.079327	0.078077	0.025523
	0.9	0.2	0.087142	0.051725	0.042484	0.027662	0.078031	0.078535	0.025523
		0.5	0.088815	0.04683	0.038416	0.027662	0.078509	0.078167	0.025523
		0.9	0.090303	0.041018	0.033537	0.027662	0.07927	0.078096	0.025523

TABLE 45: RMSE of various Estimators when n=200 and T=5 for δ

ARMA(1,1) RMSE ($\delta=0.7$) T=5 n=200									
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.089399	0.060051	0.059194	0.027663	0.072049	0.070564	0.024592
		0.5	0.091397	0.052248	0.047488	0.027663	0.072087	0.070856	0.024592
		0.9	0.093028	0.043895	0.037027	0.027662	0.072443	0.071234	0.024592
	0.5	0.2	0.086516	0.054927	0.049245	0.027664	0.072231	0.071042	0.024592
		0.5	0.088203	0.048741	0.041678	0.027663	0.072472	0.071398	0.024592
		0.9	0.089832	0.041699	0.033991	0.027661	0.072536	0.071672	0.024592
	0.9	0.2	0.082649	0.048487	0.03839	0.027662	0.072626	0.072689	0.024592
		0.5	0.084429	0.043309	0.033903	0.027663	0.073188	0.072166	0.024592
		0.9	0.085906	0.037517	0.029309	0.027661	0.072439	0.072038	0.024592
0.5	0.2	0.2	0.092365	0.059049	0.059407	0.027663	0.075116	0.074187	0.024484
		0.5	0.094172	0.051402	0.048061	0.027662	0.075148	0.074542	0.024484
		0.9	0.095644	0.043249	0.037652	0.027661	0.075455	0.074923	0.024484
	0.5	0.2	0.089296	0.054092	0.049116	0.027664	0.075839	0.074627	0.024484
		0.5	0.090893	0.04803	0.042004	0.027663	0.076171	0.075056	0.024484
		0.9	0.092391	0.041111	0.0345	0.027661	0.076196	0.075401	0.024484
	0.9	0.2	0.085414	0.047503	0.037926	0.027662	0.07593	0.076087	0.024484
		0.5	0.087025	0.04251	0.033943	0.027662	0.076096	0.075686	0.024484
		0.9	0.088412	0.036887	0.029587	0.027662	0.075907	0.075697	0.024484
0.7	0.2	0.2	0.094926	0.058048	0.059267	0.027662	0.07779	0.077272	0.023492
		0.5	0.096577	0.050565	0.048399	0.027662	0.077681	0.077716	0.023492
		0.9	0.097917	0.042614	0.038146	0.027661	0.077768	0.078116	0.023492
	0.5	0.2	0.091704	0.053267	0.04872	0.027663	0.079151	0.077691	0.023492
		0.5	0.093208	0.047326	0.042122	0.027662	0.079447	0.078192	0.023492
		0.9	0.094623	0.040545	0.03487	0.027661	0.07902	0.078626	0.023492
	0.9	0.2	0.087775	0.046526	0.03726	0.027662	0.078742	0.078907	0.023492
		0.5	0.08923	0.041742	0.0338	0.027662	0.078438	0.078686	0.023492
		0.9	0.090562	0.036356	0.029703	0.027662	0.078795	0.078853	0.023492

TABLE 46: RMSE of various Estimators when n=200 and T=5 for β

			ARMA(1,1) RMSE(β) T=5			$\delta=0.3$	n=200		
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.2	0.2	0.050188	0.071714	0.490614	0.44401	0.054267	0.057251	0.030282
		0.5	0.050181	0.065435	0.483064	0.163789	0.053956	0.05733	0.030185
		0.9	0.050171	0.05652	0.477727	0.055156	0.053842	0.057411	0.030098
	0.5	0.2	0.050199	0.068099	0.44791	0.275745	0.053406	0.05724	0.031293
		0.5	0.050192	0.062676	0.458617	0.172621	0.054478	0.057062	0.031285
		0.9	0.050183	0.055055	0.46986	0.073072	0.054591	0.057075	0.031273
	0.9	0.2	0.05022	0.059374	0.410177	0.132126	0.052907	0.057624	0.031234
		0.5	0.050204	0.055298	0.426286	0.113397	0.053685	0.057547	0.031234
		0.9	0.050193	0.049668	0.438971	0.076509	0.054744	0.057184	0.031228
0.5	0.2	0.2	0.049099	0.07109	0.460229	0.375684	0.052761	0.055555	0.031284
		0.5	0.049093	0.064883	0.453324	0.144074	0.052266	0.055717	0.031272
		0.9	0.049084	0.056034	0.448472	0.053914	0.052239	0.05588	0.031213
	0.5	0.2	0.049111	0.067282	0.427851	0.254486	0.052151	0.05544	0.031202
		0.5	0.049103	0.062066	0.439718	0.161802	0.053133	0.05542	0.031192
		0.9	0.049094	0.054525	0.448304	0.06916	0.053423	0.05553	0.031053
	0.9	0.2	0.049135	0.058621	0.389008	0.128859	0.051839	0.05596	0.031048
		0.5	0.049118	0.054587	0.404506	0.109935	0.052388	0.055904	0.031045
		0.9	0.049107	0.049065	0.417218	0.073744	0.053456	0.055591	0.031041
0.7	0.2	0.2	0.047665	0.070507	0.42096	0.284206	0.050969	0.053561	0.031053
		0.5	0.047659	0.064301	0.416805	0.11954	0.050427	0.053801	0.029533
		0.9	0.047651	0.055532	0.413905	0.05376	0.050394	0.054055	0.029532
	0.5	0.2	0.047676	0.066509	0.400243	0.225567	0.050685	0.053375	0.029528
		0.5	0.047668	0.061458	0.412375	0.144742	0.051195	0.053513	0.029515
		0.9	0.047659	0.053982	0.419303	0.063284	0.051759	0.053714	0.029511
	0.9	0.2	0.047702	0.057967	0.363663	0.123545	0.050212	0.054012	0.029502
		0.5	0.047684	0.053895	0.377303	0.102965	0.050582	0.053973	0.029413
		0.9	0.047673	0.048469	0.390719	0.068626	0.051736	0.053739	0.029398

TABLE 47: RMSE of various Estimators when n=200 and T=5 for β

			ARMA(1,1) RMSE (β) T=5			$\delta=0.5$	n=200		
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.050185	0.066293	0.479147	0.16067	0.053866	0.057282	0.046406
		0.5	0.050175	0.058982	0.475773	0.074248	0.054032	0.057376	0.046406
		0.9	0.050163	0.049968	0.474296	0.039912	0.053787	0.057432	0.046406
	0.5	0.2	0.050205	0.063776	0.460581	0.157727	0.053936	0.057146	0.046406
		0.5	0.050193	0.057193	0.463964	0.098827	0.054431	0.057098	0.046406
		0.9	0.050179	0.04914	0.467428	0.044043	0.054302	0.057095	0.046406
	0.9	0.2	0.050229	0.055927	0.43145	0.096445	0.05335	0.057673	0.046406
		0.5	0.050211	0.05126	0.435526	0.080028	0.053992	0.057525	0.046406
		0.9	0.050199	0.045046	0.440656	0.049815	0.054421	0.057072	0.046406
0.5	0.2	0.2	0.049097	0.065743	0.448073	0.138597	0.052215	0.055648	0.045124
		0.5	0.049088	0.058453	0.446913	0.069282	0.052329	0.055828	0.045124
		0.9	0.049077	0.049515	0.44651	0.041453	0.052054	0.055944	0.045124
	0.5	0.2	0.049116	0.063023	0.439734	0.149532	0.052634	0.055427	0.045124
		0.5	0.049104	0.056638	0.443417	0.094497	0.053119	0.055514	0.045124
		0.9	0.04909	0.04865	0.446091	0.043054	0.052973	0.055616	0.045124
	0.9	0.2	0.049143	0.055217	0.410352	0.093031	0.05197	0.05598	0.045124
		0.5	0.049127	0.050578	0.414364	0.078943	0.052432	0.055832	0.045124
		0.9	0.049114	0.044517	0.420819	0.049008	0.052888	0.055514	0.045124
0.7	0.2	0.2	0.047663	0.065169	0.41077	0.112854	0.050405	0.053721	0.043929
		0.5	0.047655	0.0579	0.412015	0.063333	0.050293	0.053989	0.043929
		0.9	0.047646	0.049051	0.413372	0.043654	0.050182	0.054177	0.043929
	0.5	0.2	0.047679	0.062276	0.058534	0.137482	0.050781	0.053452	0.043929
		0.5	0.047669	0.056067	0.414947	0.087237	0.051424	0.053673	0.043929
		0.9	0.047655	0.04815	0.417906	0.041051	0.051291	0.053869	0.043929
	0.9	0.2	0.047681	0.043985	0.395395	0.047404	0.051062	0.053725	0.043929
		0.5	0.047709	0.054488	0.38348	0.087789	0.050367	0.053993	0.043929
		0.9	0.047696	0.049929	0.388187	0.074821	0.050612	0.053891	0.043929

TABLE 48: RMSE of various Estimators when n=200 and T=5 for β

			ARMA(1,1) RMSE (β) T=5			$\delta=0.7$	n=200		
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.050181	0.0601	0.470631	0.070608	0.053775	0.057329	0.039618
		0.5	0.050169	0.0527	0.471389	0.04386	0.053737	0.057405	0.039618
		0.9	0.050156	0.044209	0.472447	0.035059	0.053627	0.057462	0.039618
	0.5	0.2	0.050208	0.058365	0.464691	0.097112	0.05376	0.057154	0.039618
		0.5	0.05019	0.051537	0.464238	0.060793	0.053906	0.057159	0.039618
		0.9	0.050171	0.043691	0.465746	0.029366	0.054139	0.057185	0.039618
	0.9	0.2	0.050239	0.051524	0.444267	0.079152	0.053737	0.057919	0.039618
		0.5	0.050218	0.046535	0.441243	0.064645	0.054305	0.057635	0.039618
		0.9	0.050203	0.040488	0.441876	0.036184	0.053827	0.057303	0.039618
0.5	0.2	0.2	0.049093	0.059604	0.440085	0.063842	0.052078	0.055762	0.038919
		0.5	0.049083	0.052221	0.443087	0.043794	0.0521	0.055906	0.038919
		0.9	0.049071	0.043808	0.445522	0.036846	0.051987	0.055992	0.038919
	0.5	0.2	0.049118	0.057677	0.442375	0.094627	0.052426	0.055493	0.038919
		0.5	0.049101	0.051017	0.443062	0.058544	0.052646	0.055646	0.038919
		0.9	0.049084	0.043243	0.444572	0.029869	0.052881	0.055748	0.038919
	0.9	0.2	0.049154	0.050873	0.421312	0.076898	0.052247	0.056183	0.038919
		0.5	0.049135	0.045976	0.42023	0.063577	0.052451	0.055973	0.038919
		0.9	0.049119	0.040014	0.422282	0.03695	0.052324	0.05579	0.038919
0.7	0.2	0.2	0.04766	0.059072	0.404408	0.057927	0.050339	0.053909	0.038828
		0.5	0.047651	0.051721	0.409247	0.044063	0.050149	0.054129	0.038828
		0.9	0.047641	0.043397	0.413338	0.039239	0.049947	0.054256	0.038828
	0.5	0.2	0.047682	0.056988	0.412498	0.087468	0.050878	0.05362	0.038828
		0.5	0.047666	0.050479	0.414724	0.054767	0.051064	0.053871	0.038828
		0.9	0.047651	0.042843	0.41659	0.029845	0.051075	0.05405	0.038828
	0.9	0.2	0.04772	0.050213	0.392906	0.072278	0.050511	0.054177	0.038828
		0.5	0.047703	0.045426	0.393619	0.060742	0.050535	0.054072	0.038828
		0.9	0.047687	0.039564	0.396951	0.03683	0.050567	0.054021	0.038828

TABLE 49: Absolute Bias of various Estimators when n=100 and T=20 for δ

ARMA(1,1) ABSOLUTE BIAS (δ)									
T=20,			n=100 $\delta=0.3$						
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.2	0.2	0.001414	0.000178	0.004742	0.000397	0.003522	0.005629	0.000274
		0.5	0.000197	0.000178	0.37719	0.000516	0.002615	0.006318	0.000274
		0.9	0.001405	0.000178	0.408636	0.000603	0.001208	0.006843	0.000274
	0.5	0.2	0.001248	0.000178	0.152399	0.000626	0.011992	1.26E-05	0.000274
		0.5	0.00225	0.000178	0.30759	0.000608	0.009138	0.003097	0.000274
		0.9	0.003446	0.000178	0.376933	0.000612	0.00609	0.005466	0.000274
	0.9	0.2	0.002352	-0.00834	0.313771	0.000747	0.025008	0.013087	0.000274
		0.5	0.003586	0.000178	0.364291	0.000714	0.023539	0.006905	0.000274
		0.9	0.004746	0.000178	0.004742	0.000677	0.021665	0.001557	0.000274
0.5	0.2	0.2	0.001286	0.000178	0.004742	0.000459	0.001971	0.005913	0.000249
		0.5	2.82E-05	0.000178	0.37719	0.000537	0.002146	0.006215	0.000249
		0.9	0.001565	0.000178	0.408636	0.00061	0.00146	0.006789	0.000249
	0.5	0.2	0.001352	0.000178	0.152399	0.000635	0.00973	0.000879	0.000249
		0.5	0.002375	0.000178	0.30759	0.000614	0.008246	0.003523	0.000249
		0.9	0.003561	0.000178	0.376933	0.000618	0.005989	0.005738	0.000249
	0.9	0.2	0.002544	0.008342	0.313771	0.000743	0.024286	0.01257	0.000249
		0.5	0.003726	0.000178	0.364291	0.000709	0.021446	0.006024	0.000249
		0.9	0.004812	0.000178	0.004742	0.000676	0.019308	0.000735	0.000249
0.7	0.2	0.2	0.001125	0.000178	0.004742	0.000512	0.000749	0.006181	0.000215
		0.5	0.000156	0.000178	0.37719	0.000557	0.001257	0.005966	0.000215
		0.9	0.001733	0.000178	0.408636	0.000617	0.001046	0.006491	0.000215
	0.5	0.2	0.001498	0.000178	0.152399	0.000643	0.007883	0.001553	0.000215
		0.5	0.002512	0.000178	0.30759	0.000621	0.007294	0.003868	0.000215
		0.9	0.003688	0.000178	0.376933	0.000622	0.004832	0.005765	0.000215
	0.9	0.2	0.002745	0.008342	0.313771	0.000739	0.022681	0.011547	0.000215
		0.5	0.003787	0.000178	0.364291	0.000703	0.01894	0.005246	0.000215
		0.9	0.004905	0.000178	0.004742	0.000675	0.017043	0.000169	0.000215

TABLE 50: Absolute Bias of various Estimators when n=100 and T=20 for δ

			ARAM(1,1) ABSOLUTE BIAS (δ) T=20, n=100 $\delta=0.5$						
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.00069	0.000258	0.097166	0.000525	0.003285	0.005443	0.000493
		0.5	0.00106	0.000258	0.074446	0.000574	0.002463	0.006223	0.000493
		0.9	0.003095	0.000258	0.421155	0.000614	0.000251	0.006895	0.000493
	0.5	0.2	0.001806	0.029468	0.146397	0.000624	0.011744	0.000438	0.000493
		0.5	0.003196	0.000258	0.197786	0.000609	0.008176	0.00347	0.000493
		0.9	0.004775	0.000258	0.225988	0.00061	0.004812	0.005605	0.000493
	0.9	0.2	0.002966	0.000258	0.004619	0.000733	0.026075	0.011414	0.000493
		0.5	0.00434	0.000258	0.266863	0.000687	0.024845	0.005415	0.000493
		0.9	0.005838	0.000258	0.388188	0.000652	0.016363	0.000179	0.000493
0.5	0.2	0.2	0.000571	0.000258	0.097166	0.000543	0.002722	0.005278	0.000445
		0.5	0.001203	0.000258	0.074446	0.000579	0.002415	0.006088	0.000445
		0.9	0.00322	0.000258	0.421155	0.000614	0.000855	0.006894	0.000445
	0.5	0.2	0.001923	0.029468	0.146397	0.000627	0.010512	0.001142	0.000445
		0.5	0.003289	0.000258	0.197786	0.000611	0.007345	0.003684	0.000445
		0.9	0.004867	0.000258	0.225988	0.000611	0.004544	0.005788	0.000445
	0.9	0.2	0.00313	0.000258	0.004619	0.000725	0.024232	0.010666	0.000445
		0.5	0.004464	0.000258	0.266863	0.000682	0.022314	0.004572	0.000445
		0.9	0.005893	0.000258	0.388188	0.00065	0.015541	0.000657	0.000445
0.7	0.2	0.2	0.000431	0.000258	0.097166	0.00056	0.001054	0.005029	0.000408
		0.5	0.001356	0.000258	0.074446	0.000584	0.001698	0.00572	0.000408
		0.9	0.003348	0.000258	0.421155	0.000614	0.000226	0.00664	0.000408
	0.5	0.2	0.002019	0.029468	0.146397	0.00063	0.008561	0.001544	0.000408
		0.5	0.003389	0.000258	0.197786	0.000614	0.006179	0.003751	0.000408
		0.9	0.004969	0.000258	0.225988	0.00061	0.004161	0.005699	0.000408
	0.9	0.2	0.003301	0.000258	0.004619	0.000718	0.022836	0.009574	0.000408
		0.5	0.004515	0.000258	0.266863	0.000678	0.018776	0.003906	0.000408
		0.9	0.005947	0.000258	0.388188	0.000648	0.014641	0.001108	0.000408

TABLE 51: Absolute Bias of various Estimators when n=100 and T=20 for δ

			ARMA(1,1) ABSOLUTE BIAS (δ)			n=100	T=20	$\delta=0.7$	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	7.68E-05	0.000343	0.004476	0.000596	0.003106	0.005462	0.000488
		0.5	0.00217	0.000343	0.440939	0.000608	0.00172	0.00633	0.000488
		0.9	0.004486	0.000343	0.393022	0.00062	0.000189	0.007006	0.000488
	0.5	0.2	0.002332	0.000343	0.004476	0.000634	0.010355	0.00112	0.000488
		0.5	0.003966	0.000343	0.004476	0.000617	0.006304	0.003923	0.000488
		0.9	0.005855	0.000343	0.037151	0.000607	0.002639	0.005815	0.000488
	0.9	0.2	0.003388	0.000343	0.004476	0.00072	0.025071	0.010135	0.000488
		0.5	0.00496	0.000343	0.45967	0.000677	0.020691	0.003748	0.000488
		0.9	0.006599	0.000343	0.004476	0.000634	0.011727	0.001307	0.000488
0.5	0.2	0.2	0.000164	0.000343	0.004476	0.000601	0.002386	0.005232	0.000472
		0.5	0.002277	0.000343	0.440939	0.000607	0.001761	0.006235	0.000472
		0.9	0.004572	0.000343	0.393022	0.000617	0.000141	0.00705	0.000472
	0.5	0.2	0.002382	0.000343	0.004476	0.000636	0.00957	0.001339	0.000472
		0.5	0.004031	0.000343	0.004476	0.000617	0.006049	0.003977	0.000472
		0.9	0.005923	0.000343	0.037151	0.000605	0.003272	0.005873	0.000472
	0.9	0.2	0.003456	0.000343	0.004476	0.000713	0.023115	0.009508	0.000472
		0.5	0.005013	0.000343	0.45967	0.000673	0.019408	0.003419	0.000472
		0.9	0.006626	0.000343	0.004476	0.000631	0.011861	0.001559	0.000472
0.7	0.2	0.2	0.000267	0.000343	0.004476	0.000606	0.00132	0.004818	0.000425
		0.5	0.00239	0.000343	0.440939	0.000606	0.001051	0.005882	0.000425
		0.9	0.004657	0.000343	0.393022	0.000614	0.000363	0.006853	0.000425
	0.5	0.2	0.002436	0.000343	0.004476	0.000637	0.008265	0.001445	0.000425
		0.5	0.004118	0.000343	0.004476	0.000615	0.005495	0.003747	0.000425
		0.9	0.00599	0.000343	0.037151	0.000601	0.003786	0.005743	0.000425
	0.9	0.2	0.003546	0.000343	0.004476	0.000706	0.02152	0.008779	0.000425
		0.5	0.005039	0.000343	0.45967	0.000668	0.017884	0.003277	0.000425
		0.9	0.006633	0.000343	0.004476	0.00063	0.011453	0.001742	0.000425

TABLE 52: Absolute Bias of various Estimators when n=100 and T=20 for β

			ARMA(1,1) ABSOLUTE BIAS (β)			T=20	n=100	$\delta=0.3$	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.000383	0.003738	0.003051	0.983412	0.003182	0.004787	0.000298
		0.5	0.00038	0.003738	0.380786	0.540557	0.00156	0.005889	0.000298
		0.9	0.000383	0.003738	0.415019	0.299652	0.000479	0.006771	0.000298
	0.5	0.2	0.000384	0.003738	0.14925	0.373588	0.012232	0.003054	0.000298
		0.5	0.000386	0.003738	0.313388	0.318967	0.010281	0.000319	0.000298
		0.9	0.000393	0.003738	0.382672	0.245713	0.006306	0.002892	0.000298
	0.9	0.2	0.000389	0.003738	0.320737	0.129095	0.019329	0.010453	0.000298
		0.5	0.000396	0.003738	0.367333	0.14341	0.01968	0.007415	0.000298
		0.9	0.000402	0.003738	0.003051	0.14981	0.019309	0.003922	0.000298
0.5	0.2	0.2	0.000545	0.003738	0.003051	0.815571	0.001522	0.004915	0.000287
		0.5	0.000542	0.003738	0.380786	0.49609	0.001112	0.005445	0.000287
		0.9	0.000545	0.003738	0.415019	0.28796	0.000154	0.006198	0.000287
	0.5	0.2	0.000545	0.003738	0.14925	0.348473	0.009494	0.001885	0.000287
		0.5	0.000547	0.003738	0.313388	0.305946	0.008668	0.00021	0.000287
		0.9	0.000553	0.003738	0.382672	0.23858	0.006096	0.002914	0.000287
	0.9	0.2	0.000551	0.003738	0.320737	0.131427	0.016962	0.009321	0.000287
		0.5	0.000556	0.003738	0.367333	0.14768	0.016648	0.006381	0.000287
		0.9	0.000561	0.003738	0.003051	0.149827	0.016171	0.003135	0.000287
0.7	0.2	0.2	0.000683	0.003738	0.003051	0.668942	2.32E-05	0.005154	0.000267
		0.5	0.000681	0.003738	0.380786	0.447313	0.000199	0.005048	0.000267
		0.9	0.000684	0.003738	0.415019	0.273666	0.00043	0.005582	0.000267
	0.5	0.2	0.000683	0.003738	0.14925	0.321631	0.006756	0.000891	0.000267
		0.5	0.000685	0.003738	0.313388	0.289305	0.007332	0.000718	0.000267
		0.9	0.000691	0.003738	0.382672	0.229448	0.004851	0.002856	0.000267
	0.9	0.2	0.000689	0.003738	0.320737	0.132616	0.014472	0.008024	0.000267
		0.5	0.000692	0.003738	0.367333	0.149683	0.013649	0.005414	0.000267
		0.9	0.000697	0.003738	0.003051	0.148775	0.013618	0.002493	0.000267

TABLE 53: Absolute Bias of various Estimators when n=100 and T=20 for β

			ARMA(1,1) ABSOLUTE BIAS(β)			T=20	n=100	$\delta=0.5$	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.000378	0.003719	0.092196	0.511449	0.002219	0.00522	0.000312
		0.5	0.000382	0.003719	0.068415	0.34947	0.00071	0.006204	0.000312
		0.9	0.00039	0.003719	0.400172	0.236964	0.002167	0.007024	0.000312
	0.5	0.2	0.000385	0.013772	0.135945	0.287466	0.011523	0.002069	0.000312
		0.5	0.000391	0.003719	0.18215	0.255404	0.008021	0.001069	0.000312
		0.9	0.000399	0.003719	0.212152	0.208455	0.003906	0.004008	0.000312
	0.9	0.2	0.000394	0.003719	0.003039	0.117533	0.019854	0.00977	0.000312
		0.5	0.0004	0.003719	0.252231	0.141339	0.020615	0.006197	0.000312
		0.9	0.000407	0.003719	0.370247	0.146038	0.014158	0.001924	0.000312
0.5	0.2	0.2	0.00054	0.003719	0.092196	0.467674	0.001516	0.00484	0.000305
		0.5	0.000544	0.003719	0.068415	0.337532	0.000712	0.005632	0.000305
		0.9	0.000551	0.003719	0.400172	0.235772	0.001121	0.006429	0.000305
	0.5	0.2	0.000546	0.013772	0.135945	0.276491	0.009423	0.001197	0.000305
		0.5	0.000551	0.003719	0.18215	0.249434	0.006936	0.001222	0.000305
		0.9	0.000559	0.003719	0.212152	0.207245	0.003718	0.003804	0.000305
	0.9	0.2	0.000555	0.003719	0.003039	0.122801	0.017086	0.008619	0.000305
		0.5	0.000558	0.003719	0.252231	0.14472	0.01746	0.005349	0.000305
		0.9	0.000565	0.003719	0.370247	0.14707	0.012668	0.001569	0.000305
0.7	0.2	0.2	0.00068	0.003719	0.092196	0.422257	6.07E-05	0.00455	0.000284
		0.5	0.000683	0.003719	0.068415	0.3218	0.000189	0.005036	0.000284
		0.9	0.000689	0.003719	0.400172	0.232278	0.001383	0.005773	0.000284
	0.5	0.2	0.000684	0.013772	0.135945	0.2626	0.006869	0.000495	0.000284
		0.5	0.000689	0.003719	0.18215	0.240698	0.005509	0.00136	0.000284
		0.9	0.000696	0.003719	0.212152	0.20511	0.003334	0.003508	0.000284
	0.9	0.2	0.000691	0.003719	0.003039	0.126546	0.014765	0.007337	0.000284
		0.5	0.000694	0.003719	0.252231	0.145195	0.013634	0.004554	0.000284
		0.9	0.0007	0.003719	0.370247	0.147287	0.011224	0.001195	0.000284

TABLE 54: Absolute Bias of various Estimators when n=100 and T=20 for β

			ARMA(1,1) ABSOLUTE BIAS (β)			T=20	n=100	$\delta=0.7$	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.000377	0.003669	0.002731	0.31447	0.001173	0.005602	0.000315
		0.5	0.000386	0.003669	0.422141	0.252309	0.000567	0.006523	0.000315
		0.9	0.000395	0.003669	0.378987	0.196954	0.003264	0.007236	0.000315
	0.5	0.2	0.000388	0.003669	0.002731	0.221222	0.008941	0.00047	0.000315
		0.5	0.000396	0.003669	0.002731	0.205746	0.005163	0.002484	0.000315
		0.9	0.000404	0.003669	0.033337	0.181462	0.000528	0.004915	0.000315
	0.9	0.2	0.000398	0.003669	0.002731	0.108482	0.019202	0.008331	0.000315
		0.5	0.000404	0.003669	0.439197	0.12741	0.016838	0.004137	0.000315
		0.9	0.000409	0.003669	0.002731	0.138293	0.009533	3.27E-05	0.000315
0.5	0.2	0.2	0.00054	0.003669	0.002731	0.299091	0.000648	0.005066	0.000312
		0.5	0.000548	0.003669	0.422141	0.249467	0.000347	0.005919	0.000312
		0.9	0.000556	0.003669	0.378987	0.199433	0.00237	0.00664	0.000312
	0.5	0.2	0.000549	0.003669	0.002731	0.214766	0.007491	0.000144	0.000312
		0.5	0.000556	0.003669	0.002731	0.20371	0.004869	0.002348	0.000312
		0.9	0.000563	0.003669	0.033337	0.183607	0.001344	0.004523	0.000312
	0.9	0.2	0.000558	0.003669	0.002731	0.111947	0.01598	0.00746	0.000312
		0.5	0.000561	0.003669	0.439197	0.129774	0.014746	0.003795	0.000312
		0.9	0.000566	0.003669	0.002731	0.139884	0.009381	3.10E-05	0.000312
0.7	0.2	0.2	0.000679	0.003669	0.002731	0.2824	0.000171	0.004544	0.000299
		0.5	0.000686	0.003669	0.422141	0.244334	0.000643	0.005261	0.000299
		0.9	0.000693	0.003669	0.378987	0.200245	0.00173	0.005982	0.000299
	0.5	0.2	0.000687	0.003669	0.002731	0.207014	0.005869	0.000164	0.000299
		0.5	0.000693	0.003669	0.002731	0.201651	0.003972	0.002121	0.000299
		0.9	0.000699	0.003669	0.033337	0.184384	0.001877	0.004096	0.000299
	0.9	0.2	0.000694	0.003669	0.002731	0.114744	0.013566	0.006571	0.000299
		0.5	0.000696	0.003669	0.439197	0.131086	0.012642	0.003469	0.000299
		0.9	0.0007	0.003669	0.002731	0.140109	0.008168	5.58E-05	0.000299

TABLE 55: RMSE of various Estimators when n=100 and T=20 for δ

			ARMA(1,1)RMSE (δ) T=20, n=100 $\delta=0.3$						
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.072065	0.02978	0.019782	0.021935	0.048759	0.057524	0.02143
		0.5	0.074809	0.02978	0.411353	0.021937	0.049093	0.057667	0.02142
		0.9	0.076741	0.02978	0.427812	0.021938	0.049065	0.057507	0.02144
	0.5	0.2	0.067151	0.02978	0.2585	0.021942	0.050332	0.057609	0.02143
		0.5	0.069738	0.02978	0.370195	0.021941	0.049983	0.058028	0.02145
		0.9	0.072137	0.02978	0.411072	0.02194	0.049405	0.05812	0.02144
	0.9	0.2	0.062928	0.030197	0.375813	0.021947	0.055177	0.059198	0.02143
		0.5	0.064543	0.02978	0.403956	0.021945	0.05459	0.057968	0.02099
		0.9	0.066785	0.02978	0.019782	0.021943	0.053553	0.057698	0.02092
0.5	0.2	0.2	0.075018	0.02978	0.019782	0.021937	0.05153	0.060919	0.02075
		0.5	0.077599	0.02978	0.411353	0.021937	0.0519	0.060995	0.02077
		0.9	0.079317	0.02978	0.427812	0.021938	0.051832	0.06081	0.02073
	0.5	0.2	0.069822	0.02978	0.2585	0.021943	0.05259	0.061043	0.02074
		0.5	0.072416	0.02978	0.370195	0.021941	0.05265	0.061476	0.02073
		0.9	0.07467	0.02978	0.411072	0.02194	0.052248	0.06155	0.02073
	0.9	0.2	0.065349	0.030197	0.375813	0.021947	0.05738	0.062279	0.02074
		0.5	0.066923	0.02978	0.403956	0.021945	0.056479	0.061134	0.02073
		0.9	0.069119	0.02978	0.019782	0.021943	0.055408	0.061083	0.02073
0.7	0.2	0.2	0.077633	0.02978	0.019782	0.021938	0.053799	0.06368	0.018782
		0.5	0.080062	0.02978	0.411353	0.021938	0.054172	0.063688	0.018922
		0.9	0.081581	0.02978	0.427812	0.021939	0.054189	0.063481	0.018512
	0.5	0.2	0.072223	0.02978	0.2585	0.021943	0.054566	0.063832	0.018512
		0.5	0.074762	0.02978	0.370195	0.021941	0.054944	0.064232	0.018512
		0.9	0.076896	0.02978	0.411072	0.02194	0.05452	0.064297	0.018512
	0.9	0.2	0.067411	0.030197	0.375813	0.021947	0.058932	0.06457	0.018512
		0.5	0.068972	0.02978	0.403956	0.021945	0.057952	0.063708	0.018512
		0.9	0.071149	0.02978	0.019782	0.021943	0.057085	0.063846	0.018512

TABLE 56: RMSE of various Estimators when n=100 and T=20 for δ

			ARMA(1,1) RMSE (δ) T=20			n=100		$\delta=0.5$	
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.073899	0.029375	0.207621	0.021937	0.049143	0.057655	0.019825
		0.5	0.0766	0.029375	0.180013	0.021938	0.049035	0.057608	0.019825
		0.9	0.078594	0.029375	0.440958	0.021938	0.048745	0.057311	0.019825
	0.5	0.2	0.068706	0.029997	0.262047	0.021941	0.050763	0.057999	0.019825
		0.5	0.071521	0.029375	0.299958	0.02194	0.049764	0.058209	0.019825
		0.9	0.074143	0.029375	0.320322	0.021939	0.048883	0.058098	0.019825
	0.9	0.2	0.063902	0.029375	0.01959	0.021946	0.055369	0.058697	0.019825
		0.5	0.065939	0.029375	0.349156	0.021944	0.054562	0.057893	0.019825
		0.9	0.068644	0.029375	0.42342	0.021942	0.051253	0.057875	0.019825
0.5	0.2	0.2	0.076849	0.029375	0.207621	0.021937	0.051897	0.061001	0.019612
		0.5	0.079357	0.029375	0.180013	0.021938	0.051782	0.06093	0.019612
		0.9	0.08112	0.029375	0.440958	0.021938	0.051479	0.060604	0.019612
	0.5	0.2	0.071459	0.029997	0.262047	0.021941	0.053235	0.061508	0.019612
		0.5	0.074186	0.029375	0.299958	0.02194	0.052328	0.061652	0.019612
		0.9	0.076662	0.029375	0.320322	0.021939	0.051624	0.061534	0.019612
	0.9	0.2	0.066275	0.029375	0.01959	0.021946	0.057064	0.061793	0.019612
		0.5	0.068366	0.029375	0.349156	0.021944	0.05626	0.061156	0.019612
		0.9	0.071024	0.029375	0.42342	0.021942	0.053677	0.061333	0.019612
0.7	0.2	0.2	0.07947	0.029375	0.207621	0.021938	0.054171	0.063712	0.019614
		0.5	0.081795	0.029375	0.180013	0.021938	0.054081	0.063616	0.019614
		0.9	0.083341	0.029375	0.440958	0.021938	0.053764	0.063272	0.019614
	0.5	0.2	0.073912	0.029997	0.262047	0.021942	0.055124	0.06429	0.019614
		0.5	0.076531	0.029375	0.299958	0.02194	0.054562	0.064406	0.019614
		0.9	0.078868	0.029375	0.320322	0.021939	0.053994	0.06428	0.019614
	0.9	0.2	0.068339	0.029375	0.01959	0.021945	0.058702	0.064183	0.019614
		0.5	0.070448	0.029375	0.349156	0.021943	0.057324	0.06382	0.019614
		0.9	0.073091	0.029375	0.42342	0.021942	0.055822	0.064132	0.019614

TABLE 57: RMSE of various Estimators when n=100 and T=20 for δ

			ARMA(1,1) RMSE (δ) T=20, n=100 $\delta=0.7$						
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.074723	0.028888	0.01949	0.021938	0.04915	0.057576	0.019527
		0.5	0.077502	0.028888	0.456711	0.021938	0.048853	0.057413	0.019527
		0.9	0.079638	0.028888	0.431216	0.021938	0.048215	0.057087	0.019527
	0.5	0.2	0.069483	0.028888	0.01949	0.021941	0.050384	0.05812	0.019527
		0.5	0.072488	0.028888	0.01949	0.02194	0.049231	0.058156	0.019527
		0.9	0.075349	0.028888	0.12669	0.021939	0.048228	0.057962	0.019527
	0.9	0.2	0.064285	0.028888	0.01949	0.021945	0.05448	0.058253	0.019527
		0.5	0.066863	0.028888	0.466427	0.021943	0.052493	0.057753	0.019527
		0.9	0.06981	0.028888	0.01949	0.021941	0.049325	0.05796	0.019527
0.5	0.2	0.2	0.077642	0.028888	0.01949	0.021938	0.05179	0.060917	0.019525
		0.5	0.080218	0.028888	0.456711	0.021938	0.051474	0.060732	0.019525
		0.9	0.082119	0.028888	0.431216	0.021938	0.051013	0.060361	0.019525
	0.5	0.2	0.072231	0.028888	0.01949	0.021941	0.05296	0.061602	0.019525
		0.5	0.075136	0.028888	0.01949	0.02194	0.052026	0.061603	0.019525
		0.9	0.077836	0.028888	0.12669	0.021939	0.051087	0.061375	0.019525
	0.9	0.2	0.066688	0.028888	0.01949	0.021945	0.056301	0.061513	0.019525
		0.5	0.069269	0.028888	0.466427	0.021943	0.054792	0.061141	0.019525
		0.9	0.072164	0.028888	0.01949	0.021941	0.052165	0.061455	0.019525
0.7	0.2	0.2	0.080239	0.028888	0.01949	0.021938	0.053983	0.063622	0.018989
		0.5	0.082621	0.028888	0.456711	0.021938	0.053749	0.06342	0.018989
		0.9	0.084302	0.028888	0.431216	0.021938	0.053358	0.063014	0.018989
	0.5	0.2	0.074669	0.028888	0.01949	0.021941	0.055037	0.064372	0.018989
		0.5	0.077476	0.028888	0.01949	0.021939	0.054174	0.06436	0.018989
		0.9	0.080013	0.028888	0.12669	0.021938	0.053544	0.064116	0.018989
	0.9	0.2	0.068808	0.028888	0.01949	0.021944	0.058027	0.064082	0.018989
		0.5	0.07139	0.028888	0.466427	0.021943	0.05674	0.063884	0.018989
		0.9	0.074216	0.028888	0.01949	0.021941	0.054518	0.064245	0.018989

TABLE 58: RMSE of various Estimators when n=100 and T=20 for β

			ARMA(1,1) RMSE (β) T=20, n=100 $\delta=0.3$						
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.036242	0.02918	0.09436	0.986466	0.03835	0.044908	0.01691
		0.5	0.036226	0.02918	0.412884	0.542718	0.038511	0.044966	0.01691
		0.9	0.036213	0.02918	0.431192	0.301258	0.038581	0.044901	0.01691
	0.5	0.2	0.036227	0.02918	0.268344	0.375944	0.040181	0.045597	0.01691
		0.5	0.036218	0.02918	0.37423	0.320657	0.039686	0.04564	0.01691
		0.9	0.036207	0.02918	0.414933	0.246934	0.038873	0.045591	0.01691
	0.9	0.2	0.036195	0.027674	0.378855	0.131598	0.043345	0.047153	0.01691
		0.5	0.0362	0.02918	0.405752	0.145108	0.043381	0.0463	0.01691
		0.9	0.036202	0.02918	0.09436	0.150935	0.042909	0.045836	0.01691
0.5	0.2	0.2	0.035426	0.02918	0.09436	0.818968	0.037736	0.044312	0.01683
		0.5	0.035409	0.02918	0.412884	0.498421	0.037991	0.044271	0.01683
		0.9	0.035395	0.02918	0.431192	0.289663	0.038069	0.044158	0.01683
	0.5	0.2	0.035415	0.02918	0.268344	0.350905	0.038868	0.044929	0.01683
		0.5	0.035404	0.02918	0.37423	0.307694	0.03883	0.045006	0.01683
		0.9	0.035391	0.02918	0.414933	0.239852	0.038396	0.04494	0.01683
	0.9	0.2	0.035385	0.027674	0.378855	0.13381	0.041658	0.046202	0.01683
		0.5	0.035391	0.02918	0.405752	0.149304	0.041606	0.045463	0.01683
		0.9	0.03539	0.02918	0.09436	0.150954	0.041141	0.045168	0.01683
0.7	0.2	0.2	0.034423	0.02918	0.09436	0.672621	0.036839	0.043307	0.01665
		0.5	0.034406	0.02918	0.412884	0.449805	0.037112	0.043174	0.01665
		0.9	0.034392	0.02918	0.431192	0.275465	0.037299	0.04302	0.01665
	0.5	0.2	0.034415	0.02918	0.268344	0.324123	0.037387	0.043836	0.01665
		0.5	0.034403	0.02918	0.37423	0.291112	0.037791	0.043905	0.01665
		0.9	0.034389	0.02918	0.414933	0.230774	0.037432	0.043832	0.01665
	0.9	0.2	0.034389	0.027674	0.378855	0.134889	0.039775	0.044763	0.01665
		0.5	0.034394	0.02918	0.405752	0.151252	0.039672	0.044218	0.01665
		0.9	0.03439	0.02918	0.09436	0.149907	0.039483	0.044069	0.01665

TABLE 59: RMSE of various Estimators when n=100 and T=20 for β

ARMA(1,1) RMSE (β) T=20, n=100 $\delta=0.5$									
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST.
0.3	0.2	0.2	0.036223	0.029003	0.202796	0.513665	0.038517	0.044968	0.016232
		0.5	0.036217	0.029003	0.179303	0.351116	0.038426	0.044971	0.016232
		0.9	0.036214	0.029003	0.416322	0.238191	0.038392	0.044891	0.016232
	0.5	0.2	0.036214	0.01521	0.251678	0.289181	0.040089	0.045773	0.016232
		0.5	0.036211	0.029003	0.286583	0.256675	0.039085	0.045754	0.016232
		0.9	0.036207	0.029003	0.30629	0.209408	0.038317	0.045667	0.016232
	0.9	0.2	0.036187	0.029003	0.063073	0.119409	0.043239	0.046883	0.016232
		0.5	0.036202	0.029003	0.331865	0.14253	0.043279	0.046173	0.016232
		0.9	0.036206	0.029003	0.401434	0.14687	0.040395	0.0459	0.016232
0.5	0.2	0.2	0.035407	0.029003	0.202796	0.47004	0.037941	0.044296	0.01575
		0.5	0.035401	0.029003	0.179303	0.339268	0.037927	0.044242	0.01575
		0.9	0.035396	0.029003	0.416322	0.237051	0.037873	0.044123	0.01575
	0.5	0.2	0.035401	0.01521	0.251678	0.278249	0.039034	0.045159	0.01575
		0.5	0.035396	0.029003	0.286583	0.250751	0.038354	0.045107	0.01575
		0.9	0.035392	0.029003	0.30629	0.208231	0.037845	0.044993	0.01575
	0.9	0.2	0.035381	0.029003	0.063073	0.124562	0.041466	0.045973	0.01575
		0.5	0.035393	0.029003	0.331865	0.145881	0.04153	0.045416	0.01575
		0.9	0.035392	0.029003	0.401434	0.14791	0.039417	0.045288	0.01575
0.7	0.2	0.2	0.034404	0.029003	0.202796	0.424753	0.037081	0.043218	0.01566
		0.5	0.034398	0.029003	0.179303	0.323621	0.037142	0.043114	0.01566
		0.9	0.034393	0.029003	0.416322	0.233611	0.037102	0.042962	0.01566
	0.5	0.2	0.034401	0.01521	0.251678	0.264403	0.037623	0.044056	0.01566
		0.5	0.034395	0.029003	0.286583	0.242064	0.037395	0.043995	0.01566
		0.9	0.03439	0.029003	0.30629	0.206128	0.037048	0.043868	0.01566
	0.9	0.2	0.034389	0.029003	0.063073	0.128213	0.03978	0.044593	0.01566
		0.5	0.034395	0.029003	0.331865	0.146347	0.03933	0.044242	0.01566
		0.9	0.034392	0.029003	0.401434	0.148135	0.038324	0.044208	0.01566

TABLE 60: RMSE of various Estimators when n=100 and T=20 for β

ARMA(1,1) RMSE (β) T=20, n=100 $\delta=0.7$									
λ	P	θ	OLS	ABGMM1	SYS 1	AH(d)	M-EST.	MM-EST.	P-EST
0.3	0.2	0.2	0.03621	0.028716	0.048747	0.316261	0.038486	0.044933	0.018188
		0.5	0.036212	0.028716	0.436517	0.253642	0.038371	0.044919	0.018188
		0.9	0.036215	0.028716	0.41355	0.197932	0.038048	0.044876	0.018188
	0.5	0.2	0.036204	0.028716	0.048747	0.22257	0.039391	0.045795	0.018188
		0.5	0.036206	0.028716	0.048747	0.206769	0.038532	0.045757	0.018188
		0.9	0.036209	0.028716	0.129234	0.182227	0.037806	0.045692	0.018188
	0.9	0.2	0.036186	0.028716	0.048747	0.109866	0.042538	0.046505	0.018188
		0.5	0.036204	0.028716	0.445736	0.12834	0.041234	0.045982	0.018188
		0.9	0.036209	0.028716	0.048747	0.138938	0.038373	0.045966	0.018188
0.5	0.2	0.2	0.035395	0.028716	0.048747	0.300971	0.037944	0.044237	0.018162
		0.5	0.035395	0.028716	0.436517	0.250854	0.037813	0.044176	0.018162
		0.9	0.035398	0.028716	0.41355	0.200442	0.037571	0.044085	0.018162
	0.5	0.2	0.035391	0.028716	0.048747	0.216153	0.03858	0.045174	0.018162
		0.5	0.035391	0.028716	0.048747	0.204766	0.038107	0.045103	0.018162
		0.9	0.035393	0.028716	0.129234	0.18439	0.037421	0.044994	0.018162
	0.9	0.2	0.035381	0.028716	0.048747	0.113274	0.040763	0.045736	0.018162
		0.5	0.035393	0.028716	0.445736	0.130694	0.040103	0.04533	0.018162
		0.9	0.035395	0.028716	0.048747	0.140536	0.038046	0.045371	0.018162
0.7	0.2	0.2	0.034392	0.028716	0.048747	0.284354	0.037089	0.043136	0.017292
		0.5	0.034392	0.028716	0.436517	0.245773	0.037053	0.043036	0.017292
		0.9	0.034394	0.028716	0.41355	0.201288	0.036809	0.042902	0.017292
	0.5	0.2	0.034391	0.028716	0.048747	0.208437	0.037486	0.044071	0.017292
		0.5	0.03439	0.028716	0.048747	0.202732	0.03716	0.043989	0.017292
		0.9	0.034391	0.028716	0.129234	0.185189	0.036766	0.043851	0.017292
	0.9	0.2	0.034386	0.028716	0.048747	0.11602	0.039132	0.044488	0.017292
		0.5	0.034394	0.028716	0.445736	0.132003	0.03873	0.044224	0.017292
		0.9	0.034396	0.028716	0.048747	0.140772	0.037117	0.044278	0.017292